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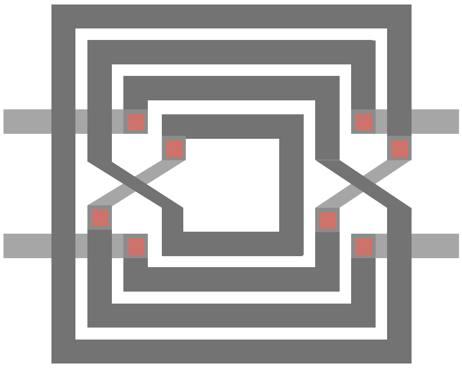

Monolithic RF Transformers on CMOS Technologies

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Dissertação para obtenção do Grau de Mestre em

Engenharia Electrotécnica e de Computadores

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Abstract

This dissertation presents a thorough study of square monolithic transformers on a 0.18µm CMOS technology, for radiofrequency applications. The structures are analyzed with Momentum, a 2.5-D electromagnetic simulator, in terms of dimensions and geometry, focusing in high performance design techniques, i.e., that are known to provide strong magnetic coupling with the least possible insertion losses. Planar transformers with a Metal 5 spiral connected to the spirals present the best behavior, since they generally present higher coupling factor and power gain.

A complete model for planar transformers with metal 5 is presented and tuned, proving to be unfit for one-turn transformers. An alternative model for one-turn transformers is proposed. A simplified model for the calculation of the quality factor of primary and secondary windings is presented.

An inductance calculation computer program for symmetric designs, based on Greenhouse's method, is presented. This program computes the Greenhouse formula in each segment of a symmetric transformer, calculating the self-inductance of the primary and secondary windings. The results obtained were compared with the EM simulations results, and the error is almost negligible for the symmetric structures. This program also provides fairly accurate results for a type of quasi-symmetric transformers, although with higher error.

Index Terms **—** Monolithic transformer, Coupling factor, Power gain, Transformer electrical model, Inductance, Quality factor.

Resumo

Esta dissertação consiste num estudo detalhado de transformadores integrados quadrados para aplicação em radiofrequência, para a tecnologia CMOS 0.18µm. As estruturas são simuladas variando dimensões e geometria, com o Momentum, um simulador electromagnético 2.5-D. São usadas técnicas de desenho já conhecidas na literatura por garantirem melhor performance das estruturas, isto é, melhor acoplamento e baixas perdas de inserção. Os transformadores planares com uma espiral de metal 5 ligada aos enrolamentos são as estruturas mais vantajosas, já que apresentam excelente acoplamento e relativamente baixas perdas.

Propõe-se um modelo eléctrico completo para estes transformadores, que no entanto não se ajusta aos transformadores de uma espira. Para estes é apresentado um modelo alternativo. Propõe-se ainda um modelo simplificado para o cálculo do factor de qualidade de um transformador.

Apresenta-se um programa de computador para o cálculo da indutância dos enrolamentos de um transformador simétrico, baseado no método de Greenhouse. O programa processa a fórmula de Greenhouse em cada segmento do transformador e calcula a indutância do primário e do secundário. Os resultados obtidos apresentam um erro pouco significativo em relação aos resultados das simulações electromagnéticas dos transformadores simétricos. Relativamente às estruturas quase-simétricas estudadas, o erro do programa é já mais significativo.

Palavras Chave — Transformador Integrado, Factor de Acoplamento, Ganho de Potência, Modelo Eléctrico do Transformador, Indutância, Factor de Qualidade.

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List of Symbols and Abbreviations

RF - Radiofrequency

GaAs – Gallium Arsenide

MESFET – Metal Semiconductor Field Effect Transistor

RFIC – Radiofrequency Integrated Circuit

CMOS – Complementary metal-oxide-semiconductor

DC – Direct Current

LNA – Low Noise Amplifier

WLAN – Wireless Local Area Network

UMC – United Microelectronics Corporation

EM - Electromagnetic

ADS – Advanced Design System

$\mathbf{v_1}$ – Voltage in primary terminals

$\mathbf{v_2}$ - Voltage in secondary terminals

$\mathbf{i_1}$ – Current flowing in primary windings

$\mathbf{i_2}$ – Current flowing in secondary windings

$\mathbf{N_1}$ - Number of primary turns

$\mathbf{N_2}$ - Number of secondary turns

$\mathbf{L_P}$ – Primary's self-inductance

$\mathbf{L_S}$ – Secondary's self-inductance

M – Mutual inductance

n - Transformer's turn ratio

$\mathbf{E_a}$ – Stored Energy in transformer's windings

K – Coupling Factor

Z - Impedance

$\boldsymbol{\omega}$ – Angular Frequency

$\boldsymbol{\rho_g}$ – Reflection coefficient of the generator

$\boldsymbol{\rho_c}$ – Reflection coefficient of the load

$\mathbf{G_T}$ – Transduction Gain

MAG – Maximum Available Gain

$\mathbf{L_0}$ - Self-inductance

$\mathbf{Q_M}$ – Mutual Inductance Parameter

$\mathbf{R_P}$ – Primary winding's resistance

R_S – Secondary winding's resistance

L_M – Magnetizing inductance of the transformer

R_{SUB} – Resistance to the substrate

C_{SUB} – Capacitance to the substrate

C_M – Mutual capacitance between primary and secondary windings

C_G – Capacitance between turns in primary winding

C_L – Capacitance between turns in secondary winding

Q – Quality Factor

X – Reactance

s – Spacing between windings

w - Width of the winding

W_T – Length of the winding

1. Introduction

1.1 Overview and Motivation

Radiofrequency communications became possible in 1888, when Heinrich Hertz first discovered and produced radio waves. A few years later, in 1896, Guglielmo Marconi succeeded in sending and receiving radio signals through distances up to two miles, taking a patent in the United Kingdom. At the same time, in the United States, Nikola Tesla patented several key developments in the early radio history.

The invention of the transistor in 1947, replacing the old vacuum tubes, was the first step towards miniaturization. However, 1950's engineers became aware of the impossibility of manually assembly a great number of small components such as transistors, diodes and resistors, without faulty connections. Besides, for large circuits with long interconnections, the signals might not travel fast enough for the circuit to work effectively. But in the late 50's, Jack Kilby, an employee at Texas Instruments, had the idea of making all the components of the circuit out of the same semiconductor block – monolith. This was revolutionary. The circuits could be much smaller, and the assembly process could be automated. Kilby fabricated and tested the very first integrated circuit in 1958.

Fifty years later, the wireless electronics industry is still in substantial growth, responding to the growing needs of computer, military and consumer applications, etc. In these areas compact, reliable and low cost radiofrequency equipment is fundamental.

Traditionally, RF integrated circuits were fabricated in GaAs (Gallium Arsenide) substrates, mostly due to the high bandwidth of the MESFET GaAs transistors, and to the high quality of the passive components.

However, the quest for low cost equipment led to an increasing interest in implementing RFIC's in standard CMOS technology, as it was much improved with the "boom" of microprocessor and memory markets. This technology accepts the integration of both analog and digital components, allowing system-on-chip solutions.

Modern CMOS technology easily provides high frequency active devices for RF applications, but high quality inductors and transformers are still challenging projects, mostly due to substantial losses in the metal traces and in the substrate.

Transformers are essential components in many RF circuits, used for impedance matching, DC isolation, signal coupling, bandwidth enhancement, etc. The integration of transformers leads to less external components, reducing the size of the circuit board. It also implies less interconnections requiring soldering, thus improving the reliability of the final product.

CMOS integrated circuits are faster, smaller, and cheaper than ever before. However, analog circuits are still difficult to implement in this technology due to the poor performance of passive components. Passive inductors and transformers are common in most radiofrequency circuits, but circuit designers often choose

not to integrate these components, given that they still present quality factors and coupling coefficients far from ideal. Besides that, some technologies still don´t include transformers in their component libraries.
Since the introduction of passive components in silicon technologies, much work has been done to improve the performance of these structures, but they still represent the bottleneck of the system.

1.2 State of the Art

The applications of monolithic transformers in nowadays RF technology are numerous. Oscillators [1][2], mixers [3][4], power amplifiers [5][6], low noise amplifiers [7], among others. Multifilament transformers can also be used as power combiners and baluns, employed to couple a balanced circuit to an unbalanced one [8].

The first monolithic transformers were proposed in the early eighties [9]. At the time, investigators worked in alumina, and later in GaAs substrates.

The introduction of spiral inductors on silicon substrates was first reported in [10]. Due to the improvements achieved in silicon technology, passive structures in silicon required much less chip area and a higher quality factor was possible. Inductors and LC filters in silicon for high frequency applications were fabricated and tested. This work was revolutionary. Before this, it was thought that radio circuits were incompatible with silicon technology, partly because of the limited bandwidth it provided, but also because it was believed that tuned circuits and discrete filters weren't suitable for integration, excluding most of RF circuitry [1].
Nguyen and Meyer [10] marked the beginning of many research in improving the performance of RF passive components. Since then, much work has been made to model and optimize monolithic transformers and design techniques have been improved [11][12].

Many different designs of square transformers can be found in previous literature.
The most common design is perhaps the interleaved winding transformer (figure 1-1) [7] [12] [13].

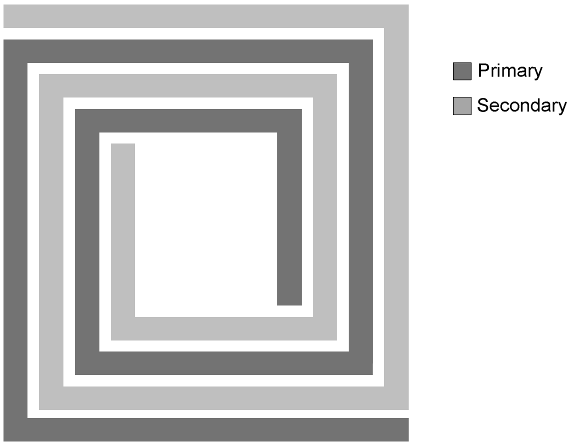


Figure 1-1: Interleaved planar transformer.

This design was proposed in [14], and guarantees full symmetry between spirals and therefore similar electric characteristics in primary and secondary windings. The terminals of this transformer are placed in opposite sides of the design, which simplifies the connections to other components. Although this transformer is not the most efficient concerning occupied area, coupling factors around 0.8 have been reported.

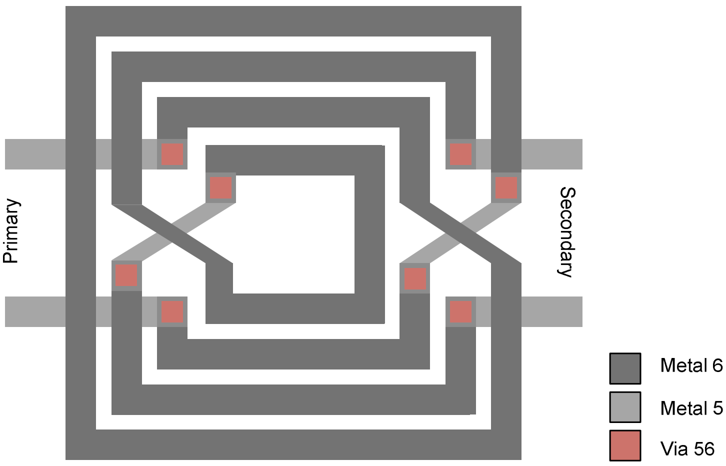


Figure 1-2: Rabjohn planar transformer.

An inventive design of interleaved planar transformers was patented in 1989 [15]. It presents a square and quasi-symmetric[1] layout, in which the four terminals of the transformer are located on the outside edge of the design (figure 1-2). This type of transformer has many applications, although it requires substantial chip area [16][17].

[1] Given that the inner winding (in this case, the secondary) is always shorter than the outer winding, this layout is not fully symmetric. However, its turn ratio can be unitary, guaranteeing electrical symmetry.

The parallel winding transformer, proposed in [9], consists in two parallel conductors interwound (figure 1-3). In this kind of design the total lengths of the primary and secondary windings are significantly different and this asymmetry implies different electrical behavior between the primary and secondary [8]. Also, the location of the ports makes the connection with other circuit components difficult. For these reasons, the applications for this type of transformer are limited.

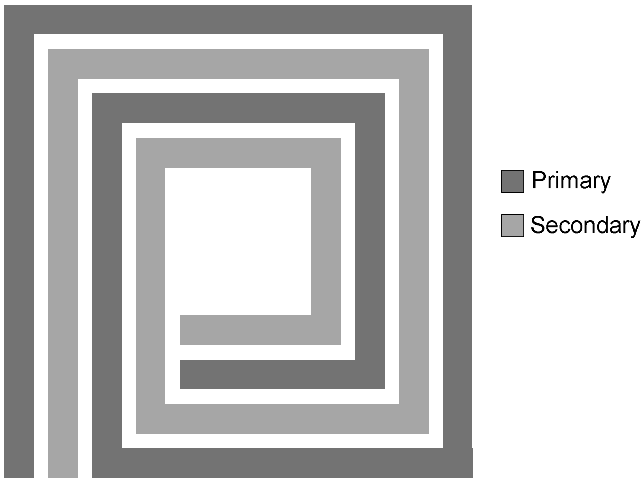


Figure 1-3: Parallel windings transformer.

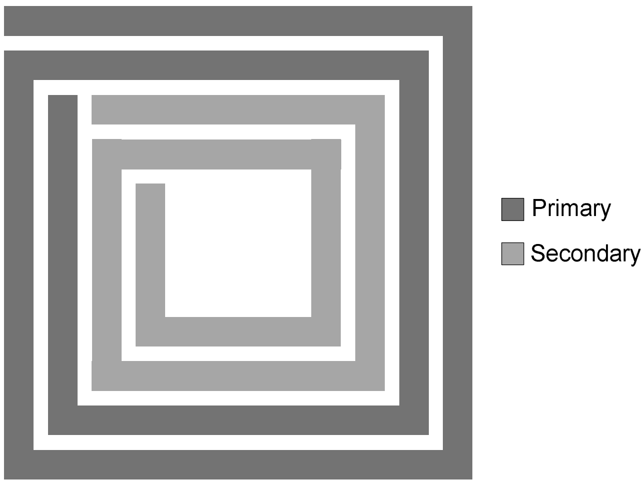


Figure 1-4: Concentric transformer.

The concentric spiral transformer (figure 1-4) is best suited for three port applications. This type of design is also asymmetric, but a diversity of tapping ratios can be implemented [13]. Since the common periphery between windings is just a single turn, the self inductance of each winding is maximized, while the mutual inductance between the windings is very low. This causes a low coupling factor, usually between 0.3 and 0.6 [8].

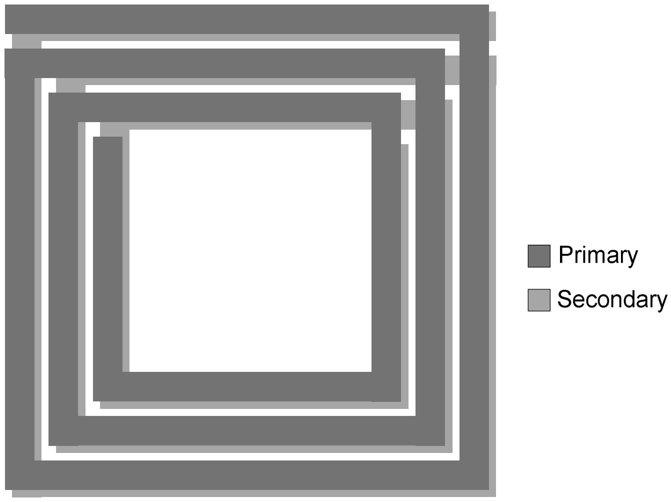


Figure 1-5: Stacked transformer.

A stacked transformer is made of multiple metal layers, in order to obtain vertical magnetic coupling instead of lateral coupling between windings (figure 1-5). This technique was first described in [18] for spiral inductors.

Stacked transformers provide better area efficiency, higher inductance and higher coupling factor (around 0.9) [13]. However, the increase in coupling is also due to capacitive effects between the upper and lower windings, which lead to lower self resonance frequency. This type of design isn't symmetric even though each winding is symmetric itself, because the windings are implemented on different layers of metal with different thickness, resulting in unequal resistances. Moreover, the lower winding isolates the upper winding from the substrate, resulting in unequal parasitic capacitances for each winding [8].

An alternative implementation of stacked transformers is presented in [13]. In this type of design, the top and bottom windings are laterally or diagonally shifted. This way, the capacitance between windings can be reduced, but the coupling factor also diminishes.

Another interesting approach is the stacked-interleaved transformer (figure1-6). The windings are placed on two different metal layers, and cross-placed, so that the design is quasi-symmetric. This transformer presents mixed coupling (vertical and lateral) between primary and secondary windings. Given the fact that inter-layer coupling dominates the overall mutual couplings, this transformer has nearly the same performance as the stacked transformer, being the symmetry the only advantage [19].

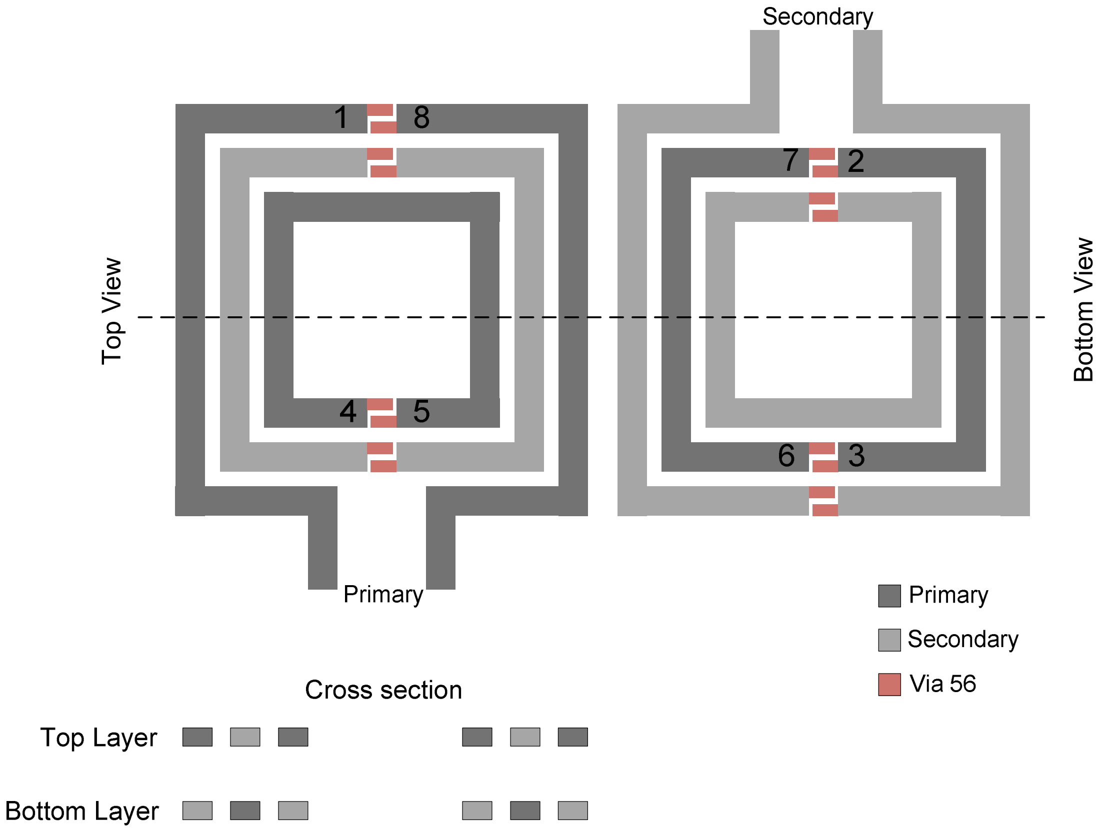


Figure 1-6: Stacked-interleaved transformer.

In order to obtain a transformer with good characteristics for the needed application, the designer should make intelligent compromises. Past researchers have provided a set of design techniques, useful to obtain more efficient transformers.

The first thing to consider is the shape of the transformer. In previous literature [20] there is evidence that circular shaped inductors have less ohmic losses that square inductors with the same inductance. The problem is that circular shapes aren't allowed in most technologies, or are allowed only under special authorizations which require an enormous amount of bureaucracy. But as long as the technology allows 45 degree angles, the closest approximation to a circle without violating any design rule is an octagon. Consequently, some designers prefer to design octagonal shaped inductors and transformers [21][22].
Nevertheless, the square is the most common shape of an integrated transformer. The first reason for this is the tradeoff between the complexity of the design and the improvement of the transformer: designers often prefer to get a slightly worse transformer with a much simpler design. The other reason is that, for the same perimeter, a square transformer requires less chip area than a circular transformer [23].

Considering dimensions and other aspects of monolithic transformers design, the following techniques are currently used in order to maximize coupling and minimize losses.

- Smaller conductor width increases magnetic coupling, and higher coupling factors can be achieved [8].

- Both self and mutual inductance increase with the increase of windings length [25]; thus increasing transformer's length contributes to stronger magnetic coupling.

- The smallest spacing possible should be used, at the expense of increasing interwinding capacitance. However, a high coupling factor is believed to be more important [23].

- A large inner diameter is desirable, given that the innermost windings of planar inductors contribute little to the overall inductance, but contribute substantially to parasitic capacitance [24].

- Interleaving planar metal traces or overlaying conductors maximizes the periphery between windings, which leads to higher mutual inductance, hence higher magnetic coupling [8].

- Multiple layers of metal in each winding reduce ohmic losses and the insertion loss of the transformer, but increase parasitics as inter-layer capacitance appears [13].

- A stacked solution requires much less chip area compared to planar transformers, but also presents higher interwinding capacitance, limiting the frequency response of the transformer [8][23][24].

A summary of the most popular design techniques used to maximize coupling with the least possible losses is presented in Table 1-1.

Table 1-1: Summary of Transformer Design Techniques.

Design Technique		Advantage	Disadvantage
Width	Small	Higher coupling factor	Increases current density
Length	Big	Higher self and mutual inductance, stronger coupling	Increases chip area requirements
Spacing	Small	Higher coupling factor	Increases interwinding capacitance
Inner diameter	Big	Higher self and mutual inductance, stronger coupling	Increases chip area requirements
Interleaved metal traces	Should be used	Maximizes mutual coupling	---
Stacked metal traces	Should be considered cautiously	Less chip area requirements	Limits frequency response

.

1.3 Main Contributions

An ideal transformer is one that can deliver all the power from the source to the load, with 100% magnetic coupling between primary and secondary windings and no losses. However, monolithic transformers have finite inductance, ohmic losses through the windings and substrate, and often low magnetic coupling.

The main contributions of this dissertation are the simulation and modeling of high performance integrated transformers for WLAN applications, on 0.18 µm CMOS technology. It is expected to obtain area efficient transformers with high coupling factor and low insertion losses.

Considering the most usual applications of monolithic transformers, this study focuses on transformers with unitary turn ratio, i.e., the number of primary turns equals the number of secondary turns.

1.4 Outline

This work is organized as follows: Chapter 2 presents some theoretical concepts necessary to understand the functioning and simulation process of monolithic transformers, as well as the current problems limiting their use. Chapter 3 presents the results of the simulation of several types of monolithic transformers, in order to achieve optimal performance. A model for monolithic transformers is discussed in Chapter 4, and a transformer's inductance calculation Matlab program is presented. Finally, Chapter 5 presents this thesis' conclusions, as well as some future work suggestions.

2. The Monolithic Transformer

The present chapter of this dissertation introduces the fundamental concepts to understand the simulation process as well as the functioning of monolithic transformers.

Chapter two is divided in six sections. Section 2.1 describes the design technology employed in this work, in terms of metal and dielectric layers thickness and electrical properties. Section 2.2 briefly introduces the electromagnetic simulator used. The theoretical characterization of the transformer is presented in section 2.3. Section 2.4 presents the inductance calculation method used, and a detailed model of the transformer is described in section 2.5. At last, section 2.6 presents a study of the quality factor of the transformer.

2.1 Silicon-based UMC 0.18 µm Mixed Mode/RF technology

This technology provides six layers of metal with different properties, allowing the design and simulation of complex electrical circuits. In order to perform electromagnetic simulations it is necessary to characterize the different layers of metal in terms of thickness and electrical properties. Figure 2-1 presents the technology profile provided by the foundry.

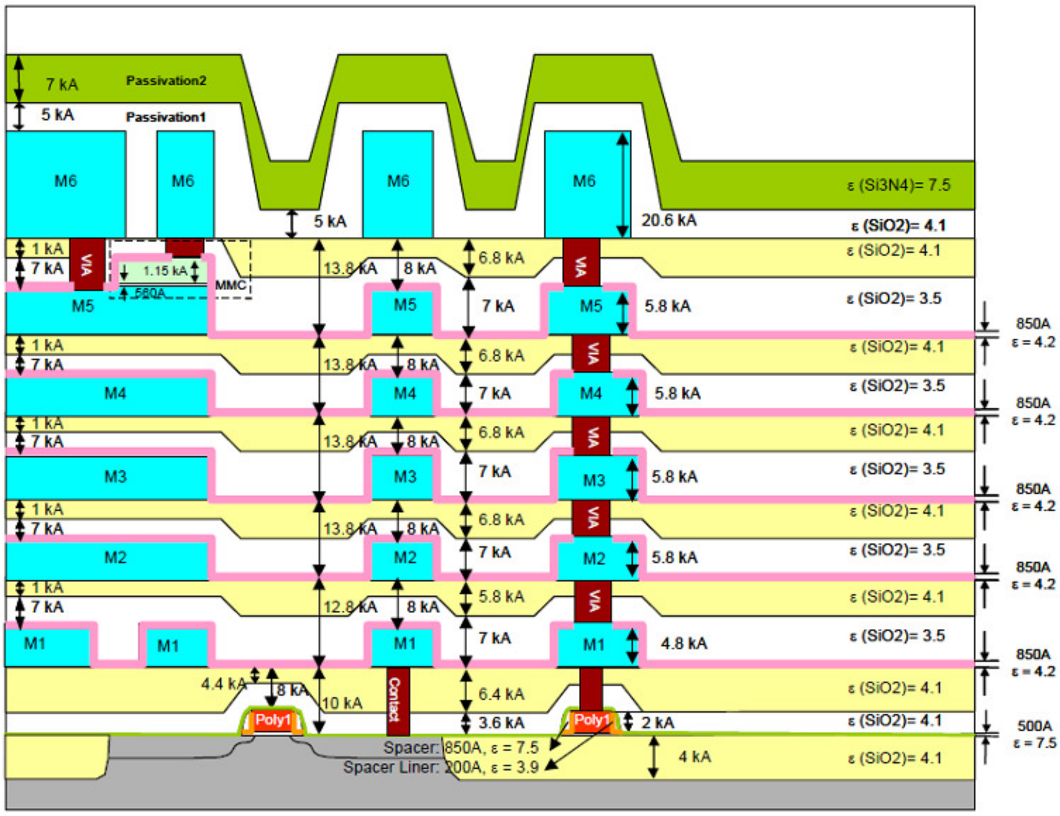


Figure 2-1: Technology profile provided by UMC, with all dimensions in Angstrom.

In order to simulate passive structures with a 2.5-D EM simulator, it is necessary to simplify the scheme on Figure 2-1, and to estimate some values not provided by the foundry. Figure 2-2 presents a simplified characterization of UMC 0.18 µm technology. The values in red are estimated by comparison with other technologies.

The resistivity of each metal is presented on Table 2-1.

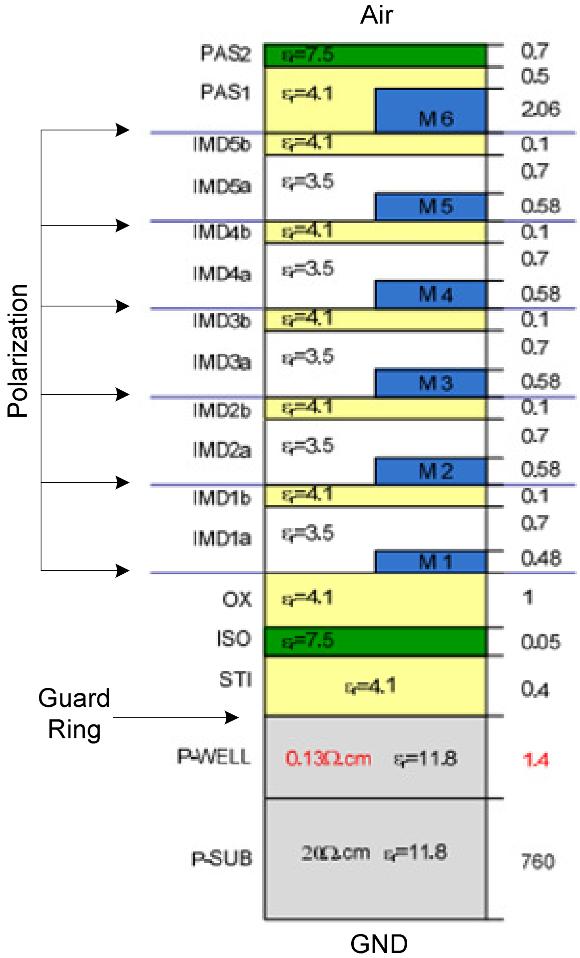


Figure 2-2: Simplified profile of the technology introduced in the EM Simulator.

Table 2-1: Metals' Resistivity

	M1	M2	M3	M4	M5	M6
Resistivity (mΩ/sq)	77	62	62	62	62	20

2.2 2.5-D EM Simulator

An electromagnetic simulator was used to simulate physical structures of monolithic transformers. It was decided to work with Momentum, an EM Tool embedded in Agilent's Advanced Design System.

Momentum is an advanced Method of Moments three-dimensional planar electromagnetic simulator, for passive circuit analysis. Because this kind of simulator considers current in z axis but not function of z, it is named 2.5-D. The structures must have a z direction dimension much less than the wavelength in order to get accurate results.

Momentum accurately simulates current distribution both in metal conductors and vias, as well as complex EM effects such as coupling and parasitic. It offers the possibility of adaptive frequency analysis for faster results, as well as optimization tools that allow modifications in circuit geometry in order to meet the specifications.

2.3 Electrical Characteristics

2.3.1 Ideal Transformer

The basic principle of a transformer is the transfer of electrical energy from one circuit to another through magnetically coupled inductors. A scheme of a single-core transformer is presented on Figure 2-3.

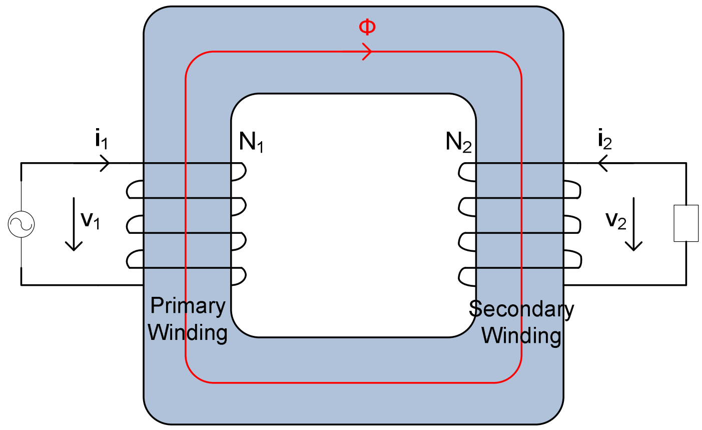


Figure 2-3: Single-core transformer.

A variable current i_1 passing the primary winding generates a magnetic field, and thus a variable magnetic flux Φ through the core of the transformer. This flux induces a voltage v_2 between the terminals of the secondary winding.

The core of the transformer is usually made of ferromagnetic material, with high magnetic permeability, so that most of the magnetic flux passes both the primary and secondary windings.

In an ideal transformer, the primary and secondary coils have perfect magnetic coupling. This transformer does not accumulate energy, and does not present any losses, capacitive and inductive effects [26].

In an ideal transformer, the incoming electric power equals the outgoing electric power, and the following conditions are verified:

$$V_1 = n \times V_2 \quad (1)$$

$$-I_2 = n \times I_1 \quad (2)$$

In which n is the turn ratio of the transformer, defined as

$$n = \frac{N_1}{N_2} \quad (3)$$

Though engineers often use this model to predict transformers' behavior, the ideal transformer does not exist. However, large transformers with superconductive windings and high permeability cores can achieve over 99% efficiency.

2.3.2 Real Transformer

A simple model for a real transformer can be obtained from the analysis of a pair of coupled inductors (Figure 2-4).

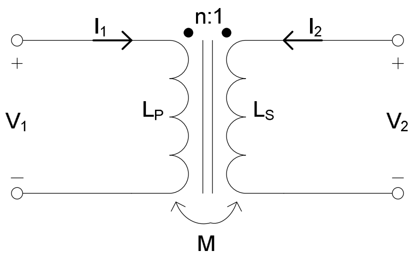


Figure 2-4: Coupled Inductors.

The description of a two-port network can be obtained from the analysis of the electric circuit of Figure 2-4, in which L_P is the self-inductance of the primary coil, L_S is the self-inductance of the secondary coil, and M represents the mutual coupling between them.

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} L_P & M \\ M & L_S \end{bmatrix} \times \begin{bmatrix} \frac{di_1(t)}{dt} \\ \frac{di_2(t)}{dt} \end{bmatrix} \quad (4)$$

The magnetic energy stored in the two-port is given by equation 5 [27].

$$E_a = \int_0^T [v_1(t) \times i_1(t) + v_2(t) \times i_2(t)]dt \quad (5)$$

Assuming $i_1(0)=i_2(0)=0$, the energy stored in the transformer is

$$E_a = \frac{1}{2}[L_P i_1(T)^2 + 2Mi_1(T)i_2(T) + L_S i_2(T)^2] \tag{6}$$

Due to physical reasons, the energy stored in the transformer is positive, which implies the following conditions on the inductance [27]:

$$\begin{cases} L_P > 0 \\ L_S > 0 \\ M^2 < L_P \times L_S \end{cases} \tag{7}$$

It is important to note that the sign of the mutual inductance M depends on the assigned direction of the currents, being positive if the secondary current creates a magnetic field of the same direction of the primary.

The coupling coefficient k measures the strength of the magnetic coupling between primary and secondary windings, being defined as

$$k \triangleq \frac{M}{\sqrt{L_P \times L_S}} \tag{8}$$

Where $0 < |k| < 1$.

A more complex model of a non-ideal transformer is presented in Figure 2-5, in which $n = \frac{L_S}{M}, L_m = \frac{M^2}{L_S}$ and $L_A = L_P - \frac{M^2}{L_S}$.

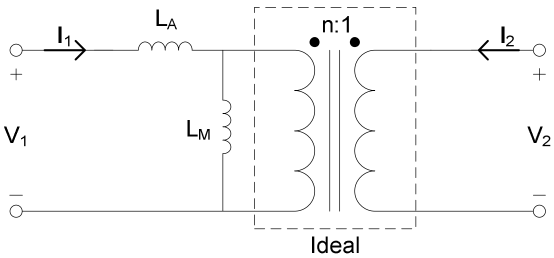


Figure 2-5: Real transformer model including leakage inductance.

In the core of this electric circuit is an ideal transformer. The inductor L_A represents the leakage inductance of the primary winding, and L_M represents the imperfect coupling between primary and secondary. The electric circuit of Figure 2-6 additionally accounts for the ohmic losses in the windings.

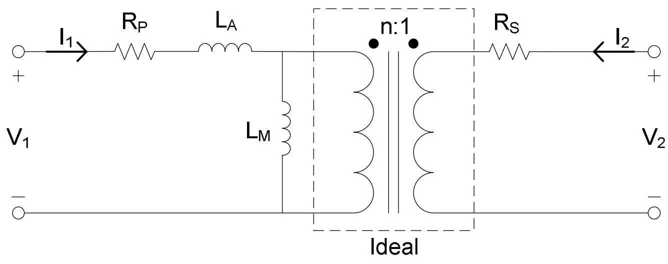


Figure 2-6: Real transformer model.

The linear behavior of this model allows its description in a two-port Z-parameter matrix (equation 9).

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{9}$$

Applying this equation to the model of Figure 2-6:

$$\begin{cases} Z_{11} = R_p + j\omega(L_a + L_m) \\ Z_{12} = j\omega n L_m \\ Z_{21} = j\omega n L_m \\ Z_{22} = R_S + j\omega L_m n^2 \end{cases} \tag{10}$$

Z-parameters can be expressed as

$$\begin{cases} Z_{11} = R_p + j\omega(L_a + L_m) = R_p + j\omega L_{11} \\ Z_{21} = Z_{12} = j\omega M \\ Z_{22} = R_S + j\omega L_m n^2 = R_S + j\omega L_{22} \end{cases} \tag{11}$$

In high frequencies, Z_{21} and Z_{12} may not be pure imaginary numbers. In this case,

$$Z_{21} = Z_{12} = R_M + j\omega M \tag{12}$$

In which R_M represents the mutual resistance. This causes the appearance of a resistive dimension of the coupling factor. The coupling factor can now be expressed as

$$k_{Im} \triangleq \frac{M}{\sqrt{L_P \times L_S}} \ \ and \ \ k_{Rm} \triangleq \frac{R_M}{\sqrt{R_P \times R_S}} \tag{13}$$

The gain of a transformer can be determined from its S-parameters and reflection coefficients.
It is common to use the S-parameters (or scattering parameters) to characterize multi-port devices operating in microwave frequencies, since they provide the description of a linear network in terms of power, which is much easily quantifiable in high frequencies than voltage or current.

The Transduction Gain of a multi-port network is the ratio between the power delivered to the load and the available power in the generator, and it is given by equation 14, in which ρ_g is the reflection coefficient of the generator and ρ_c is the reflection coefficient of the load.

$$G_T \triangleq \frac{P_L}{P_{ag}} = \frac{|S_{21}|^2 \times (1 - |\rho_g|^2)(1 - |\rho_c|^2)}{|(1 - S_{11}\rho_g)(1 - S_{22}\rho_c) - S_{12}\rho_g S_{21}\rho_c|^2} \tag{14}$$

This gain depends on the reflection coefficients of the device, and also on the input and output terminations, thus being representative of the matching level in input and output ports.

The Maximum Available Gain of a multi-port network is the transduction gain of the network when simultaneous conjugate matching is possible. Equation 15 shows that MAG can only be defined when K ≥ |1|, being K the stability factor of the device. Because the transformer is a passive device, it is always stable.

$$MAG = \left|\frac{S_{12}}{S_{21}}\right| \left(K - \sqrt{K^2 - 1}\right) \tag{15}$$

The Power Gain of a multi-port network is defined as the ratio between the power delivered to the load and the power delivered to the input of the two-port network, as shown in equation 16, in which Δ is the determinant of the S-parameters' matrix.

$$G_P \triangleq \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 \times (1 - |\rho_c|^2)}{|(1 - S_{22}\rho_c)|^2 - |(1 - S_{11}\Delta\rho_c)|^2} \tag{16}$$

This gain does not depend on the generator's impedance, therefore mismatch losses between the generator and the input port are not considered.

2.4 Winding's Self and Mutual Inductance Calculation Method

In order to compute the self and mutual inductance in a monolithic transformer, the Greenhouse method was used [25]. In 1976, Greenhouse proposed an algorithm for the processing of a square spiral's inductance, dividing the inductor into rectangular sections and separately processing the self and mutual inductance of each section. In this work, Greenhouse's algorithm was adapted for the design of square planar transformers, with one to four turns in primary and secondary windings.

The self-inductance of an inductor with rectangular cross-section, with length *l*, width *w* and thickness *t,* is given by equation 17,

$$L_0 = 0.0002 \times l \times \left[\ln\left(\frac{2l}{w + t}\right) + 0.50049 + \frac{w + t}{3l}\right] \tag{17}$$

where the self-inductance L_0 units are nH and all the dimensions are in µm.
The inductance of each section is given by the sum of the self-inductance of the section with its mutual inductance with the nearest sections.
When the current flow is in the same direction in two segments of the spiral, the mutual inductance between them is positive. When the current flows in opposite directions, the mutual inductance between the segments is negative. So, the general equation for a coil (or a part of any coil) is

$$L_T = L_0 + \sum (M^+ - M^-) \tag{18}$$

in which L_T is the total inductance, L_0 is the sum of the self inductances of the straight segments, and M^+ and M^- are the positive and negative mutual inductances. It should be noted that the mutual inductance between segment *j* and segment *k* is equal to the mutual inductance between segment *k* and segment *j*.
The mutual inductance between two parallel conductors is given by equation 19. Two orthogonal conductors do not present mutual inductance between them.

$$M = 0.0002 \times l \times Q_M \tag{19}$$

Q_M represents the mutual inductance parameter, and is given by

$$Q_M = \ln\left(\frac{l}{GMD} + \sqrt{1 + \left(\frac{l}{GMD}\right)^2}\right) - \sqrt{1 + \left(\frac{l}{GMD}\right)^2} + \frac{GMD}{l} \tag{20}$$

In which GMD stands for the geometric mean distance. GMD is approximately equal to the distance *d* between the center of the segments, and its exact value can be obtained from a mathematic formula in [25]. Replacing GMD for *d*, and re-writing equation 19, it becomes

$$M = 0.0002 \times l \times \left[\ln\left(\frac{l}{d} + \sqrt{1 + \left(\frac{l}{d}\right)^2}\right) - \sqrt{1 + \left(\frac{l}{d}\right)^2} + \frac{d}{l}\right] \tag{21}$$

where *M* units are nH, and all the dimensions are in µm.
The computer program adapted from Greenhouse's algorithm is a simple *Matlab* script, in which the user should insert the number of turns *N*, the width of the winding *w*, the length of the winding W_T, the spacing *s* between adjacent windings, and the thickness of the metal strip. The program processes this data using the equations described above, and returns primary self-inductance *Lp*, secondary self-inductance *Ls*, and mutual inductance *Lm* between primary and secondary windings, thus providing a preliminary model of the transformer (equation 4), without the need for electromagnetic simulation.
The source code of the script is included in the Appendix A.

2.5 Monolithic Transformers' Electric Model

From the physical layout of a monolithic transformer an electrical model can be derived, with accurate simulation results for 2.5-D simulators, without significant processing time and memory requirements, as computing technology was much improved in the last few years.

The compact model shown in Figure 2-7 was used in this dissertation. This model was validated by many authors such as [8], [16] or [17].

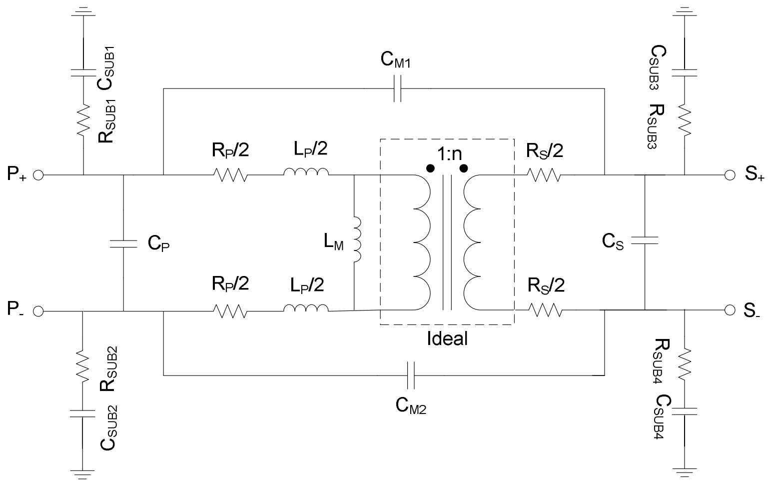


Figure 2-7: 1:n Transformer Compact Model

In the core of the model of Figure 2-7 is an ideal transformer with turn ratio 1:n, with a magnetizing inductance L_M.

The inductance L_P in series with the primary models the imperfect coupling or leakage of magnetic flux between turns [8]. Resistances R_P and R_S represent the ohmic losses in both primary and secondary windings of the transformer.

Components L_P, R_P and R_S are represented by two components of half their magnitude in parallel, symbolizing the longitudinal symmetry of the transformer.

Capacitances C_{M1} and C_{M2} model the mutual capacitance between primary and secondary windings, and C_P and C_S characterize the undesired capacitive coupling between winding turns.

Capacitive parasitic effects between each winding and the substrate are represented by the four C_{SUB} capacitors, and the ohmic losses through the substrate are modeled by the four R_{SUB} resistors.

2.6 Quality Factor

One of the most important parameters to describe an inductor is its quality factor. However, in the scope of this work towards the characterization of sub-micron monolithic transformers, this parameter has been neglected. Let's give it some thought.

The quality factor (or Q-factor) is by definition the ratio between the magnetic energy stored and dissipated in the component. Consequently, a component with high quality factor component has low power losses.

In a simple approach, the quality factor of an inductor is given by equation 22,

$$Q = \frac{\omega L}{R} \tag{22}$$

Where L is the inductance and R is the series resistance of the metal strips.

Given a circuit's complex impedance Z=R+jX, quality factor can be obtained from

$$Q = \left|\frac{X}{R}\right| \tag{23}$$

Another common definition of the quality factor is in terms of the phase derivative of the impedance θ [1]:

$$Q = -\frac{\omega_0}{L}\frac{d\theta}{d\omega}\bigg|_{\omega=\omega_0} \tag{24}$$

Inductors with high quality factor can be used in low noise amplifiers to increase their gain [7], or as matching elements to reduce their noise figure. They can improve phase noise performance of oscillators and reduce their power consumption. In summary, high-Q components can generally provide low power and low noise systems.

Consider two equal inductors with self-inductance L, internal resistance R and quality factor Q. If we connect the pair of inductors in series without magnetic coupling between them, their self-inductance is given by L_T=2xL, and their internal resistance R_T=2xR. Therefore, from equation 22, the quality factor of the series is $Q_T = \frac{\omega \times 2L}{2 \times R} = \frac{\omega L}{R} = Q$.

On the other hand, if the same pair of inductors have magnetic coupling between them, their self inductance is L_T > 2xL, and the internal resistance of the series is still R_T=2xR. The total quality factor is now $Q_T > Q$. This is due to the mutual inductance between the pair of inductors, which contributes to the self-inductance of the pair.

Considering a transformer as a pair of coupled inductors, its quality factor is expected to be higher than the Q-factor of the series of two equivalent independent inductors [7].

3. Simulation of Monolithic Transformers

This chapter presents the results of the simulations of several monolithic transformers designed with Agilent ADS, using a 0.18µm CMOS technology.

Due to the applications of the transformers in project, it was decided to limit this study to quasi-symmetric transformers only, with the number of turns in the primary winding equal to the number of turns in the secondary. Also all the transformers shape is square.

The main goals of this part of the work are to attain maximum coupling factor, and maximum power gain, with the least possible losses.

This chapter is divided in four different sections. In section 3.1 a special focus is given to the study of the optimal dimensions and spacing for a one-turn monolithic transformer, and several geometries of one-turn transformers are studied. Section 3.2 presents a basic study of the optimal dimensions of two-turn monolithic transformers, and several geometries of two-turn transformers are studied. Sections 3.3 and 3.4 present a study of several geometries of three and four-turn monolithic transformers.

3.1 One turn

As mentioned previously, the transformers in this section have one winding on the primary and secondary side.

- **Planar structures**

Several quasi-symmetric transformers were designed with different dimensions and spacing, in order to analyze these parameters. Figure 3-1 shows a generic illustration of the layouts simulated, in which W_T is the square side length, W is the strip width, and S is the spacing between windings.

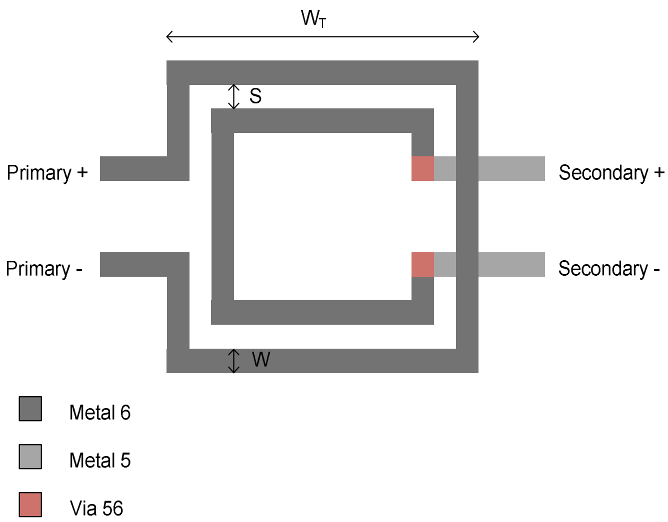


Figure 3-1: Example layout of the single turn planar transformer.

The first simulations were performed varying the spacing between windings of the transformers for the same length (W_T=200µm) and width (W=6µm). The results are presented in Figure 3-2 to Figure 3-5.

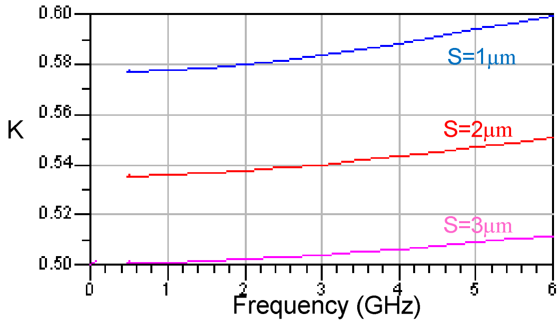
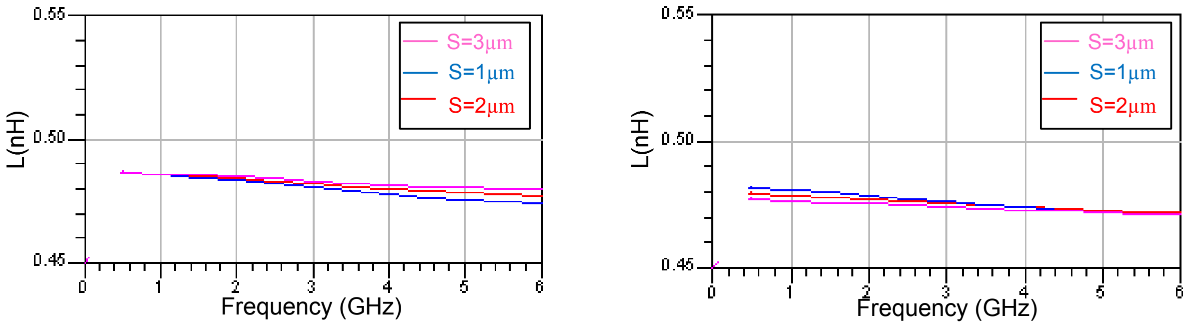
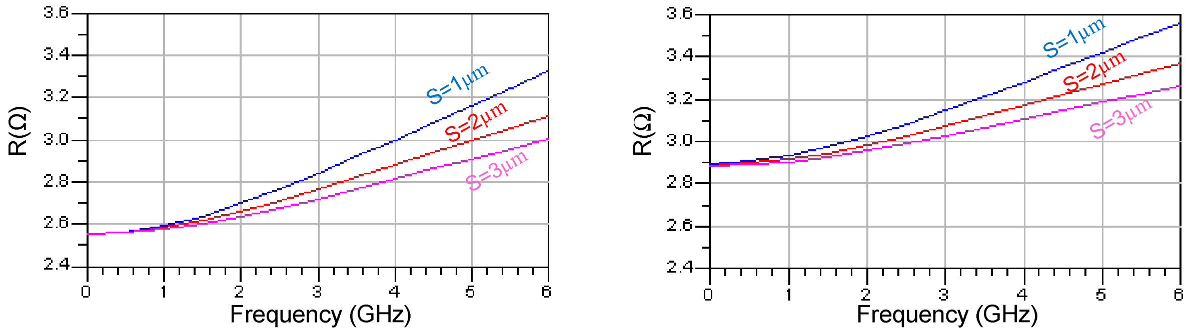


Figure 3-2: Coupling coefficient.



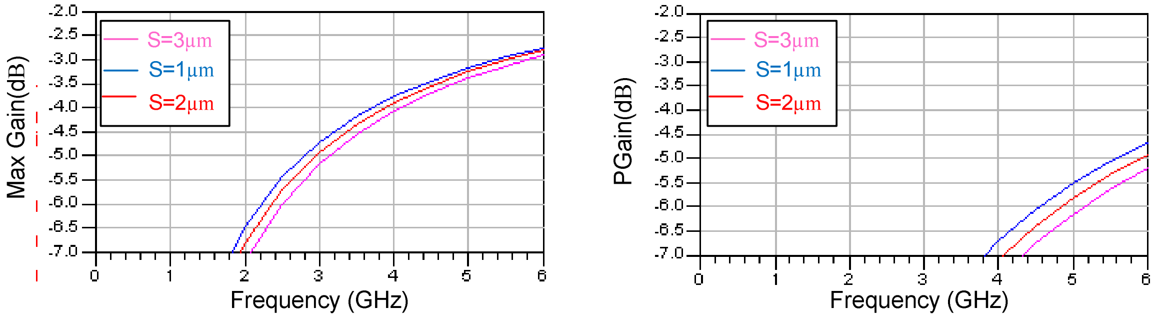
(a) (b)

Figure 3-3: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-4: (a) Primary Resistance; (b) Secondary Resistance.



(a) (b)

Figure 3-5: (a) Maximum Gain; (b) Power Gain.

As illustrated in Figure 3-2 and Figure 3-5, in order to obtain maximum coupling between spirals and maximum power gain, the smallest spacing possible should be used in transformers design. This procedure, however, leads to higher winding resistance, particularly in higher frequencies (Figure 3-4). Figure 3-3 shows that the spacing between windings does not influence particularly the inductance of one-turn transformers. It is also visible a decrease of the winding inductance for high frequencies.

This study shows that choosing the spacing value implies a compromise between the maximum resistance value expected, and the maximum gain achieved.

In the following simulations, the spacing S was kept 1µm and the width of the spirals 5µm. The total length of the squares was varied between 150µm and 250µm. The results are presented in Figure 3-6, Figure 3-7, Figure 3-8 and Figure 3-9.

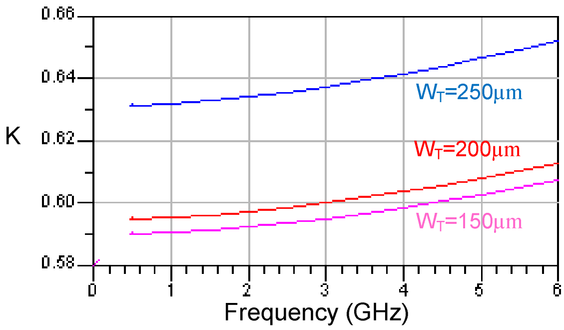
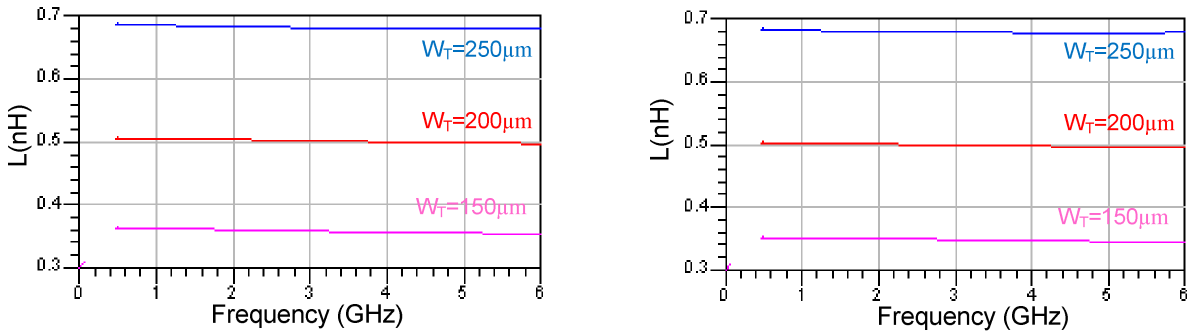
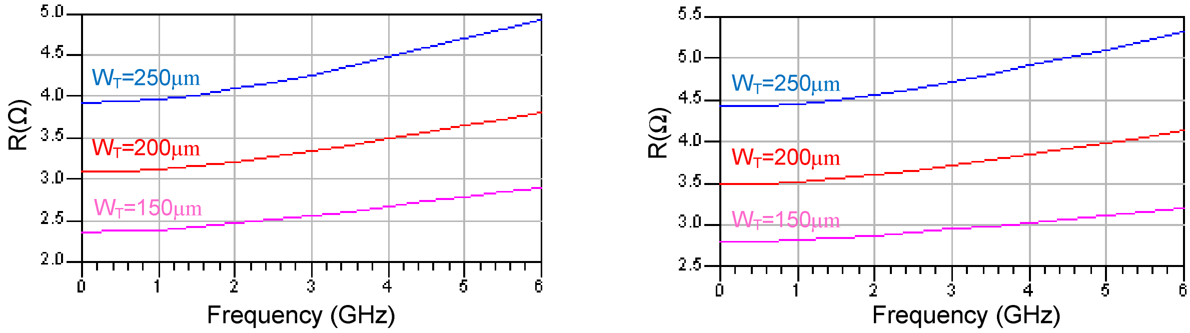


Figure 3-6: Coupling coefficient.



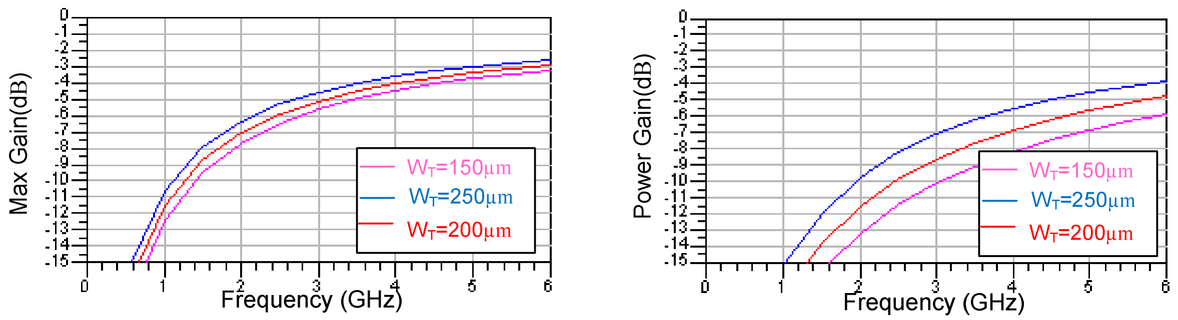
(a) (b)

Figure 3-7: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-8: (a) Primary Resistance; (b) Secondary Resistance.



(a) (b)

Figure 3-9: (a) Maximum Gain; (b) Power Gain.

These results show that the transformers achieve better behavior for greater values of length. The coupling factor exhibits a significant rise for greater lengths (Figure 3-6). Figure 3-7 and Figure 3-8 show that longer windings have higher inductance, but also higher resistance, as expected.

From this study it is clear that, when choosing the size of the transformer, the designer should compromise between the length of the windings (and consequently the chip area consumption), the maximum resistance value expected, and the maximum gain to achieve.

In the following simulations the spacing S was kept 1µm and the total length of the square spirals 200µm. The width of the windings was varied from 4µm to 7µm. Figure 3-10, Figure 3-11, Figure 3-12 and Figure 3-13 show the results of this experience.

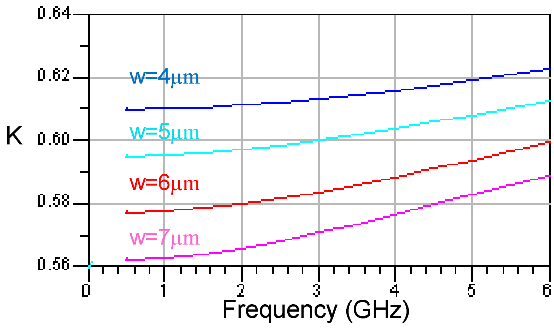


Figure 3-10: Coupling Coefficient.

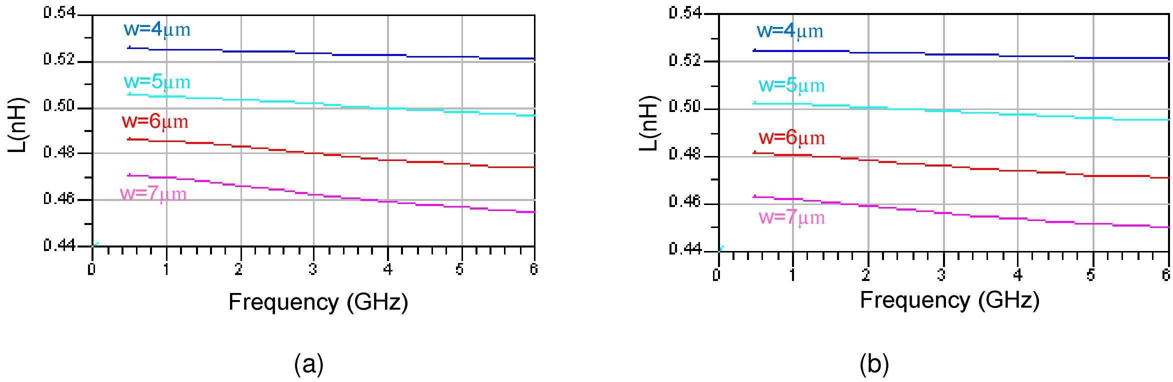


Figure 3-11: (a) Primary Inductance; (b) Secondary Inductance.

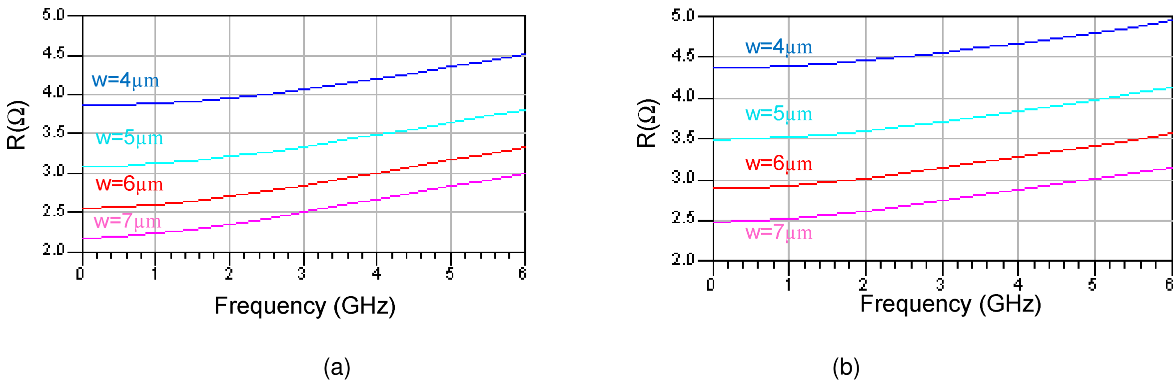
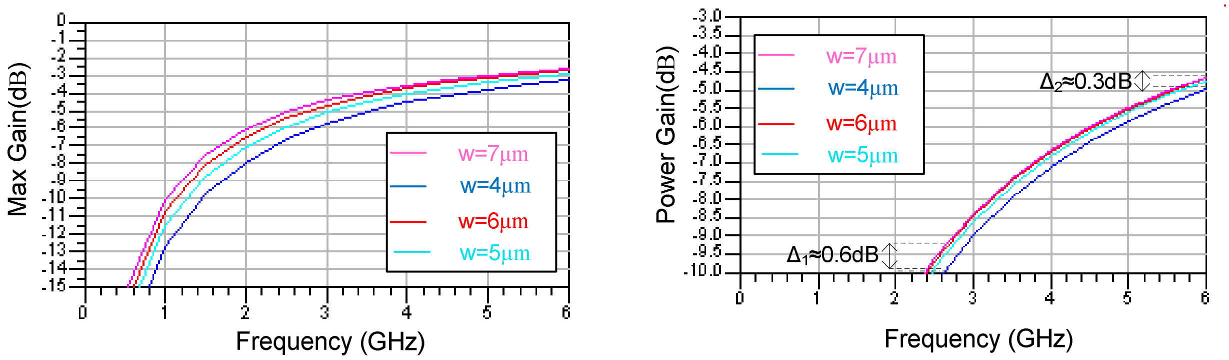


Figure 3-12: (a) Primary Resistance; (b) Secondary Resistance.



(a) (b)

Figure 3-13: (a) Maximum Gain; (b) Power Gain.

These results show that transformers can achieve higher coupling factor, inductance and resistance for smaller values of width.

Despite the fact that thinner windings have the best coupling, Figure 3-13 shows that thicker windings have higher power gain and maximum gain. This behavior is more pronounced in lower frequencies, as Δ_2 is half the value of Δ_1. This is due to the increase in spiral resistance that overrides the increase in the coupling factor.

From the previous simulations, it was decided that a transformer with w=5µm, W_T=200µm and S=1µm provides a good trade-off between chip area, magnetic coupling and power gain. At 6 GHz this transformer can achieve a K-factor of 0.613 and -4.749 dB of power gain.

The referred planar transformer was simulated and compared with another transformer of the same dimensions, with an additional spiral of Metal 5 underneath it. This spiral was connected in parallel to both primary and secondary. Several vias were placed along the metal strips to connect the spirals.

This procedure was expected to improve the coupling factor, providing vertical coupling, and simultaneously reducing the resistance of the windings.

The results of this simulation are presented in Figure 3-14, Figure 3-15, Figure 3-16 and Figure 3-17.

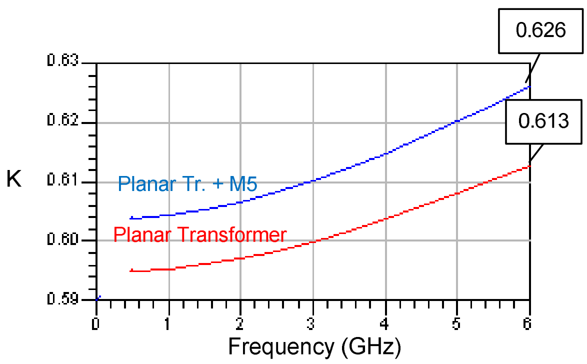
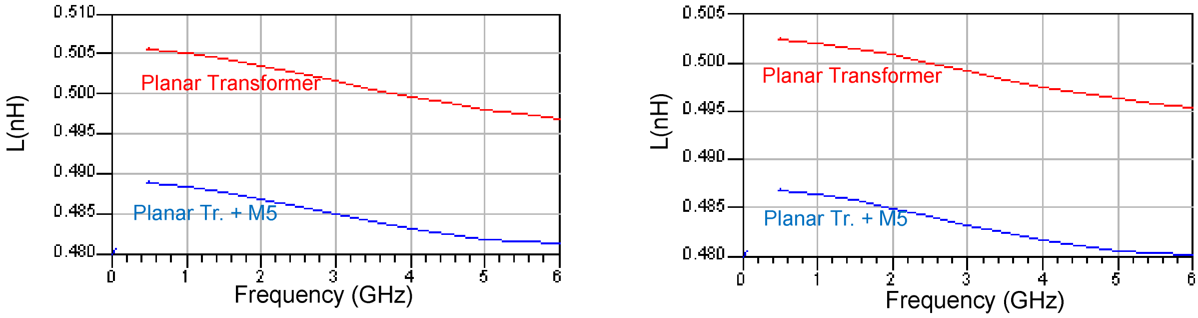
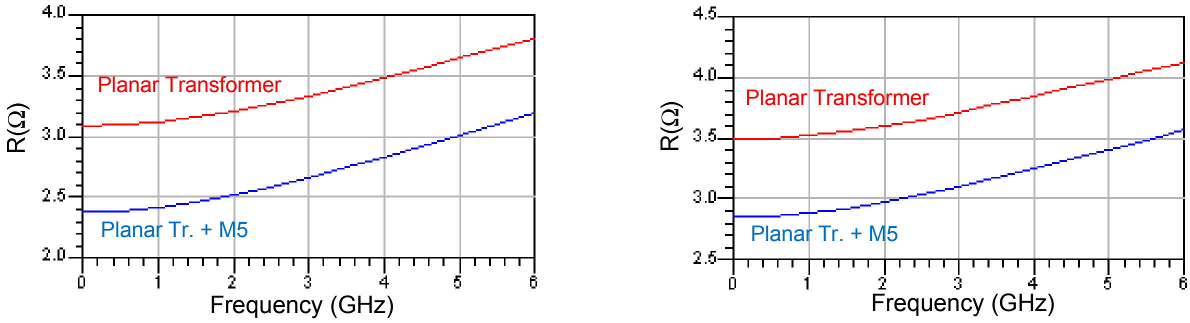


Figure 3-14: Coupling Coefficient.



(a) (b)

Figure 3-15: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-16: (a) Primary Resistance; (b) Secondary Resistance.

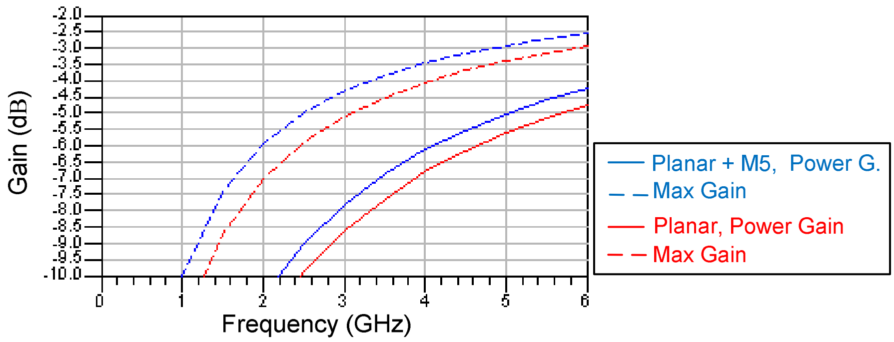


Figure 3-17: Maximum Gain and Power Gain of Planar Transformers.

These results show that adding a Metal 5 spiral under a planar transformer can slightly improve its coupling factor and lower its inductance (Figure 3-14 and 3-15). The main advantage is to reduce windings resistance (Figure 3-16). As a consequence the maximum gain and power gain of this transformer are about 0.5 dB higher than for the planar device.

- **Stacked structures**

In order to further investigate the effects of the superposition of metal on the transformers, a different kind of layout presented in Figure 3-18 was created.

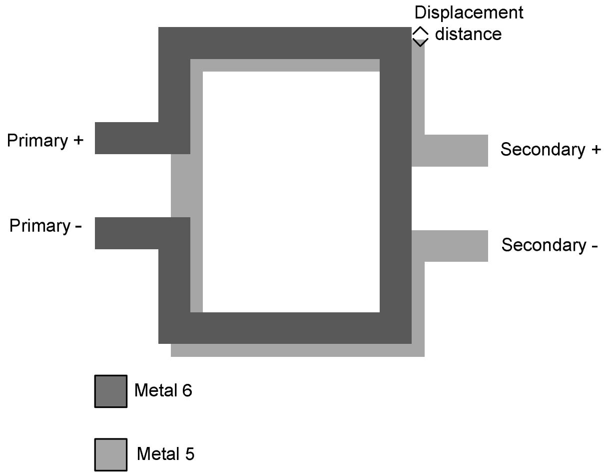


Figure 3-18: Example layout of a stacked transformer.

Single turn stacked transformers were designed, with a width of 5µm, total length of 200µm and 1µm of spacing. A transformer with the primary winding precisely above the secondary was studied and compared with a transformer with a displacement of 1µm between primary and secondary windings. The results are shown in Figure 3-19, Figure 3-20, Figure 3-21 and Figure 3-22.

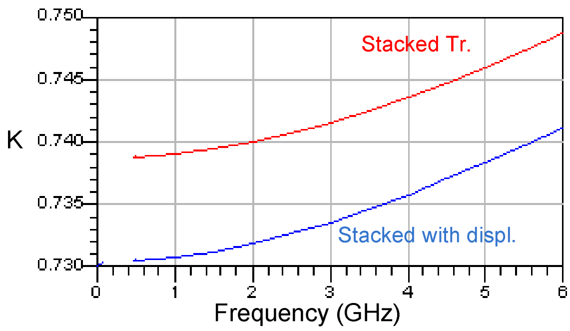
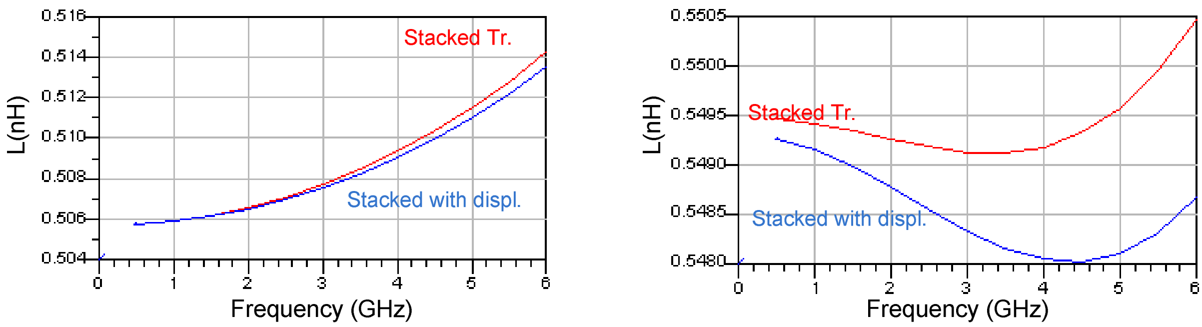
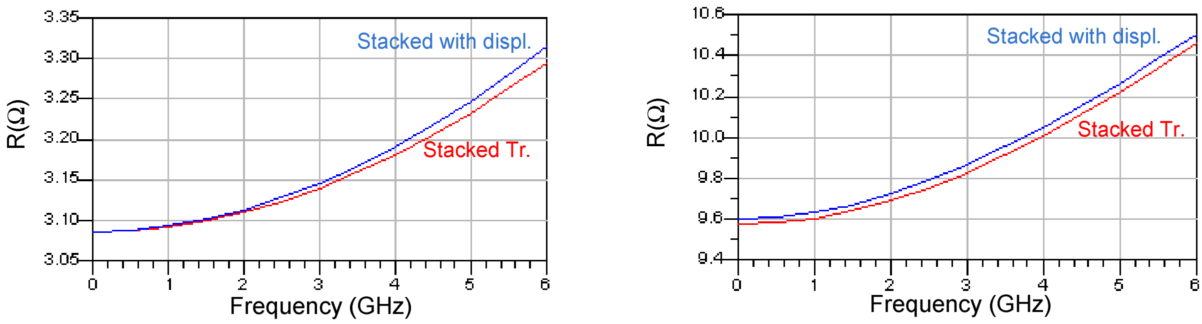


Figure 3-19: Coupling Coefficient.



(a) (b)

Figure 3-20: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-21: (a) Primary Resistance; (b) Secondary Resistance.

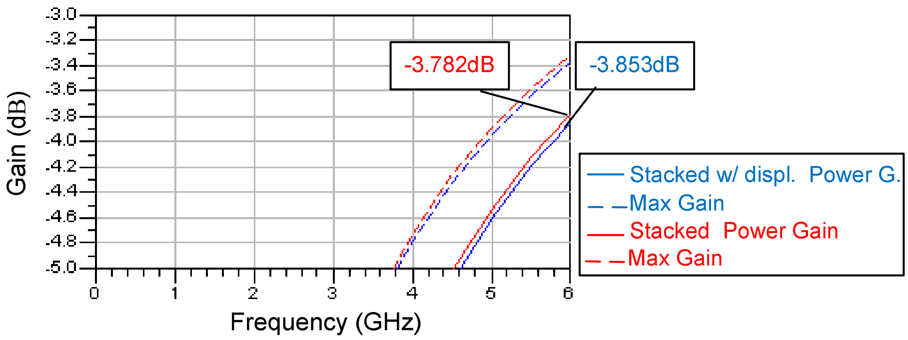


Figure 3-22: Maximum Gain and Power Gain of Stacked Transformers.

Figure 3-19 and Figure 3-22 show that a stacked transformer can achieve significantly higher K-factor and power gain than the planar transformer discussed above: At 6 GHz, K is now of 0.749 and Power Gain is -3.782 dB. The resistance of the secondary side, however, rose from 4 to 10.5Ω at the same frequency (Figure 3-21). In contrast to previous results, stacking primary and secondary windings causes an increase of the transformer's inductance with the frequency (Figure 3-20). The variation of inductance in the secondary winding is different from the primary, since the primary and secondary are now in different layers of metal, with different characteristics.

Concerning the displacement between windings, the results show that this design technique does not improve the behavior of a monolithic transformer. The inherent asymmetry of the displaced transformer causes an increase of the leakage flux phenomena, adding energy losses.

To reduce the undesired value of secondary resistance in stacked transformers, another design was created. In this transformer a spiral of Metal 4 underneath the secondary was added, connected to it in parallel. The results of this experience are presented in Figure 3-23, Figure 3-24, Figure 3-25 and Figure 3-26.

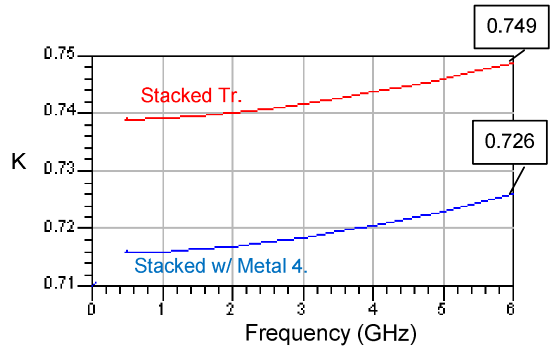
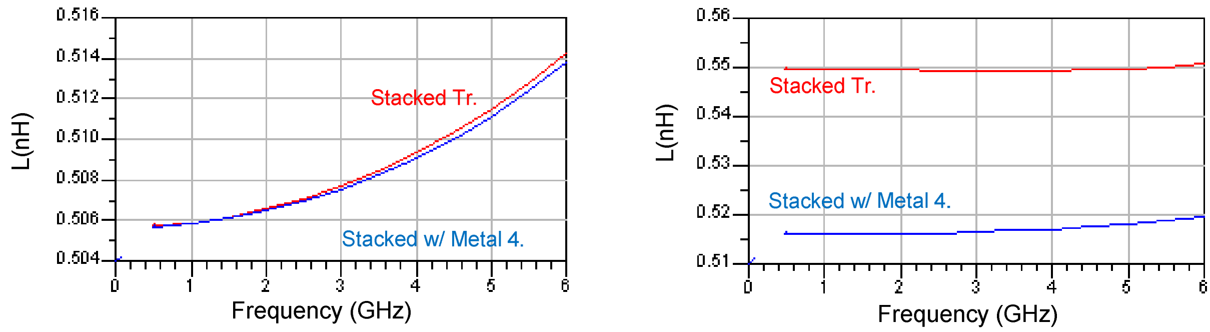
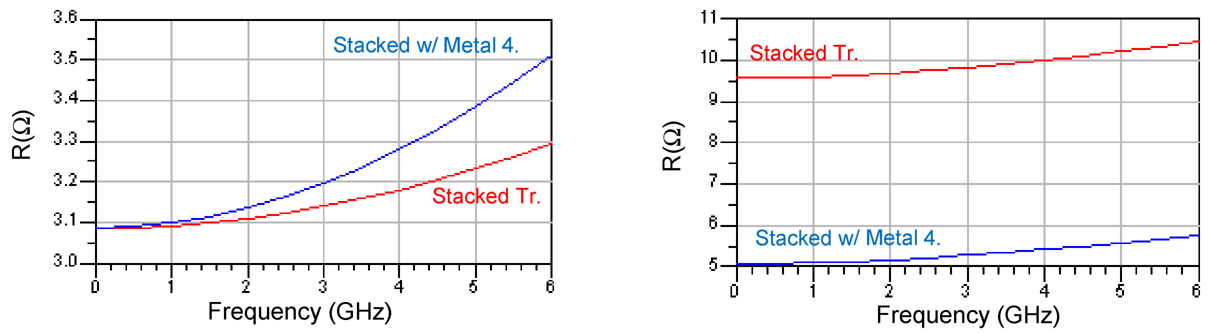


Figure 3-23: Coupling Coefficient.



(a) (b)

Figure 3-24: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-25: (a) Primary Resistance; (b) Secondary Resistance.

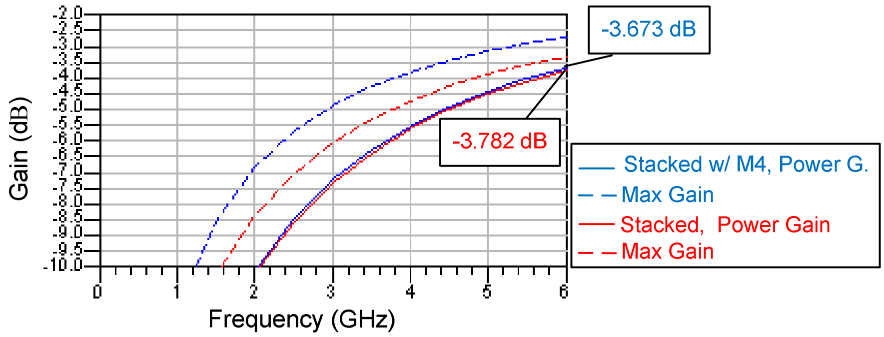


Figure 3-26: Maximum Gain and Power Gain of Stacked Transformers.

Figure 3-25 confirms that a spiral of Metal 4 connected in parallel to the secondary of the transformer reduces significantly the secondary winding resistance. This extra layer of metal, however, causes a slight increase of the resistance of the primary winding, more pronounced for higher frequencies. The magnetic coupling of this transformer is lower (Figure 3-23), as result of a lower secondary inductance, but a higher power gain and maximum gain can be achieved (Figure 3-26).

The results achieved with stacked transformers suggest a more thorough investigation of this topology. Figure 3-27 presents an alternative layout of a stacked transformer, in which the ports of the primary and secondary are placed in the corners of the square spirals.

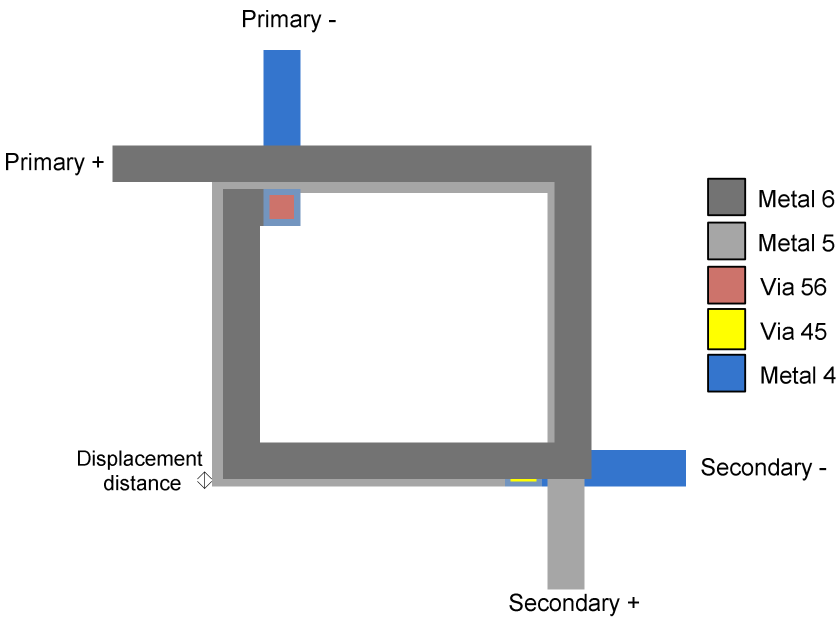


Figure 3-27: Example layout of a stacked transformer with ports placed in the corners.

The first experience was made with a transformer with 5µm of width, and total length of 200µm. The spirals are placed precisely one above the other. Figure 3-28 to Figure 3-31 represent the results of this simulation, in which the transformer of Figure 3-27 is referred to as *Stacked II* and the transformer of Figure 3-18 as *Stacked I*.

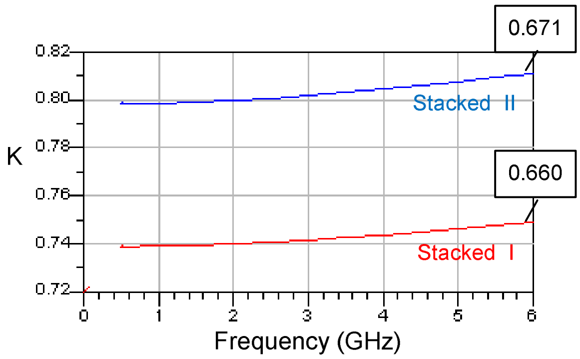
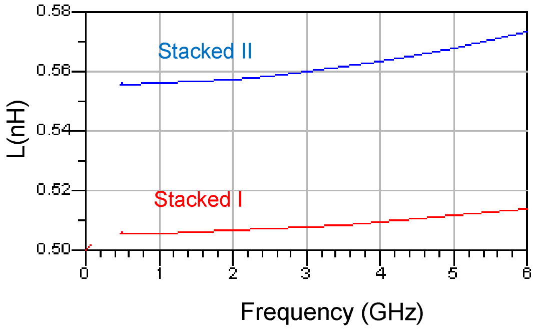
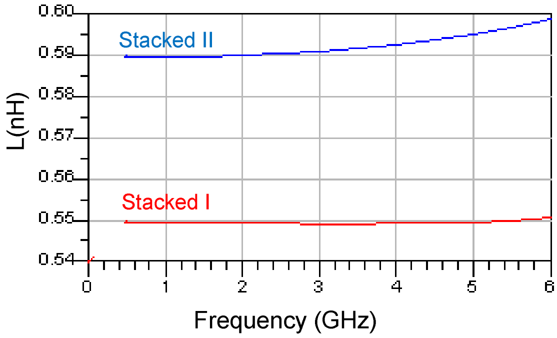


Figure 3-28: Coupling Coefficient.

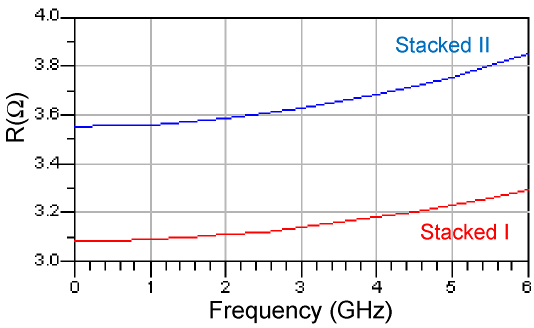


(a)

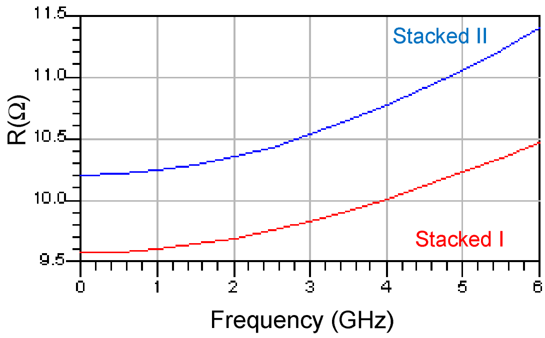


(b)

Figure 3-29: (a) Primary Inductance; (b) Secondary Inductance.



(a)



(b)

Figure 3-30: (a) Primary Resistance; (b) Secondary Resistance.

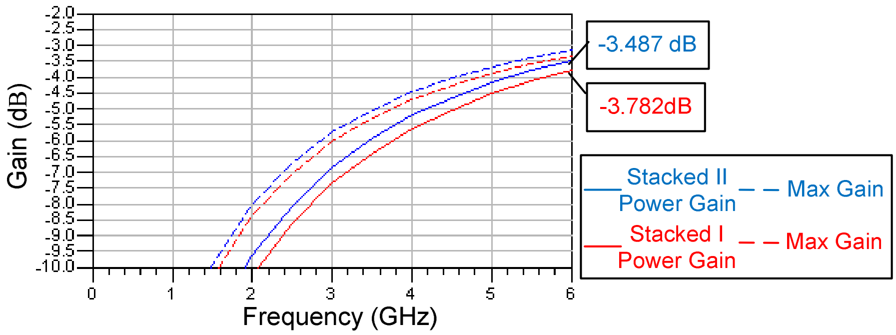


Figure 3-31: Maximum Gain and Power Gain of type I and II Stacked Transformers.

When using a stacked transformer with corner ports (*Stacked II*), the results show an improvement in the coupling factor (Figure 3-28), as well as a considerable increase in the inductance (Figure 3-29) and resistance of the windings (Figure 3-30). Figure 3-31 shows that both power and maximum gain of this transformer are about 0.5 dB higher.

There is not an obvious explanation for this improvement in the transformer's behavior. However, in high frequency circuits, a ninety-degree corner in a signal path can represent a discontinuity, increasing signal reflections [28]. This distribution of the ports means the signal is to be injected in the corners of the spirals, and so, the signal finds in its path three corners less than in *Stacked I* transformer. This geometry causes less signal reflections, which can prevent losses and signal degradation. This geometry also presents slightly larger coupling area between spirals, which contributes to the coupling factor.

In order to achieve lower values of secondary resistance, the same procedure used for *Stacked I* transformers was adopted: a new layout was created with a Metal 4 spiral under the secondary, connected to it in parallel. The results are shown in Figure 3-32 to Figure 3-35.

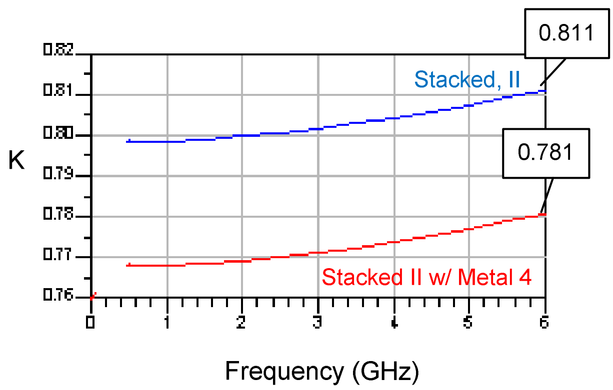
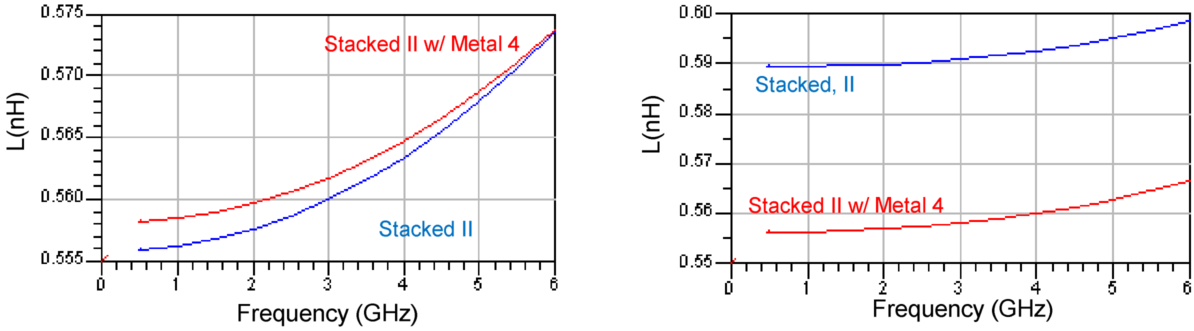
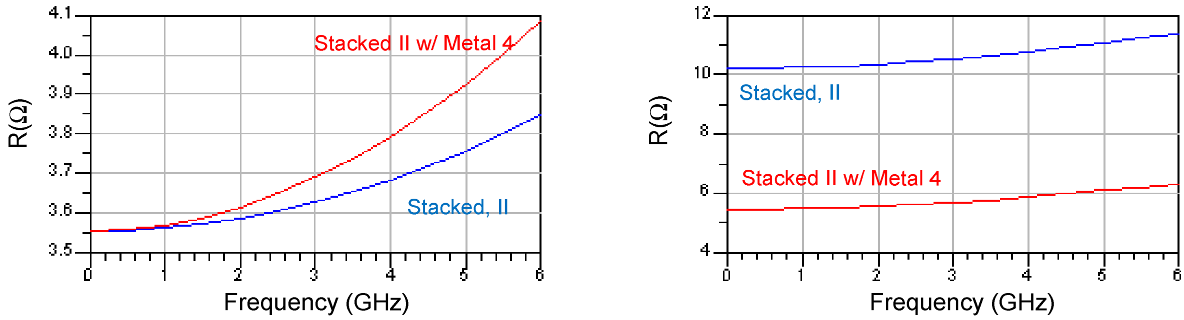


Figure 3-32: Coupling Coefficient.



(a) (b)

Figure 3-33: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-34: (a) Primary Resistance; (b) Secondary Resistance.

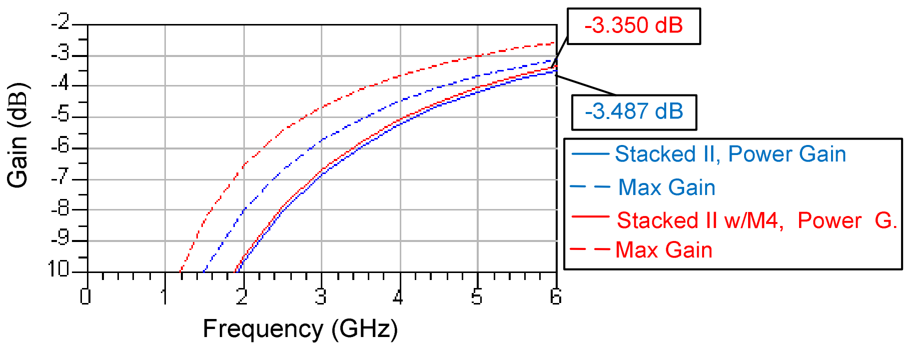


Figure 3-35: Maximum and Power Gain of type II Stacked Transformers

As expected, the use of the Metal 4 spiral connected to the secondary causes a significant decrease of the secondary resistance (Figure 3-34). This procedure also leads to lower inductance values (Figure 3-33), which contributes to the reduction of the coupling factor, from 0.811 to 0.781 at 6GHz (Figure 3-32). Figure 3-35 shows an improvement in the power and maximum gain.

A summary of notable parameters of one-turn transformers at 6GHz is presented on table 3-1.

Table 3-1: Parameters of one-turn transformers at 6 GHz.

	Planar	Planar w/ M5	Stacked I	Stacked I w/ M4	Stacked II	Stacked II w/ M4
K	0.613	0.626	0.749	0.726	0.811	0.781
Power Gain (dB)	-4.749	-4.247	-3.782	-3.673	-3.487	-3.350
Maximum Gain (dB)	-2.944	-2.547	-3.312	-2.714	-3.135	-2.588
Primary Resistance (Ω)	3.801	3.191	3.295	3.511	3.848	4.804
Sec. Resistance (Ω)	4.126	3.568	10.465	5.755	11.392	6.327
Primary Inductance (nH)	0.497	0.481	0.514	0.514	0.574	0.574
Sec. Inductance (nH)	0.495	0.480	0.550	0.520	0.599	0.567

One-turn planar transformers do not present strong magnetic coupling, given that the coupling factor is around 0.6. Adding the Metal 5 spiral immediately improves the coupling factor and gain, reducing the winding resistance.

One turn stacked structures present good magnetic coupling, although its secondary resistance is significantly higher. This can be reduced with the addition of a metal 4 spiral connected under the secondary of the transformer.

Although type II stacked structures present good performance, the designer should keep in mind this type of transformer is difficult to connect to other components of the circuit because of the location of the ports.

3.2 Two turns

Several symmetric two-turn transformers were designed with different dimensions and spacing, in order to analyze these parameters.

- **Planar Structures**

Figure 3-36 presents a generic illustration of the layouts simulated, in which W_T is the square side length, W is the strip width, and S is the spacing between windings.

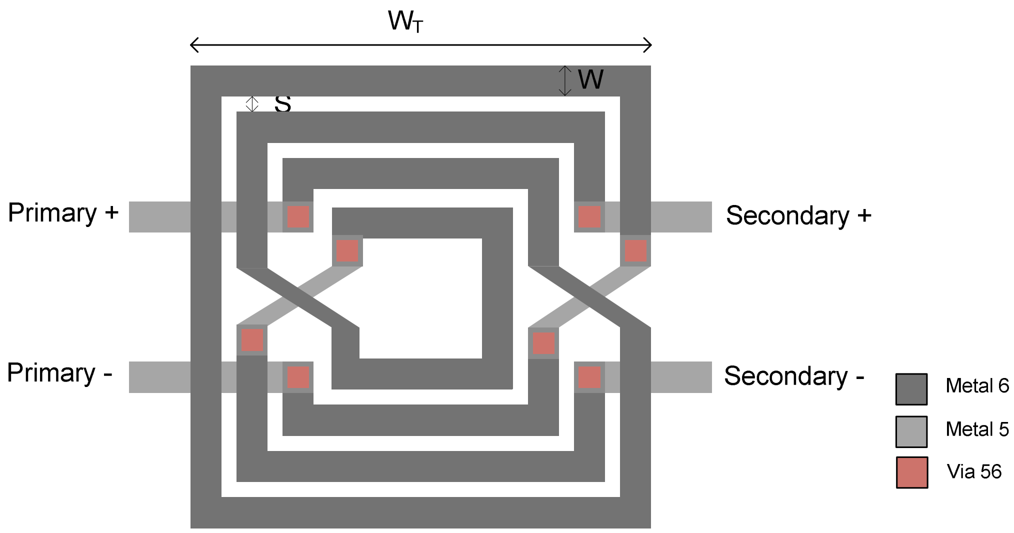


Figure 3-36: Example layout of the two-turn planar transformer.

The first simulations were performed varying the spacing, width and length of the windings. The results of this experience are presented in Figure 3-37 to Figure 3-40.

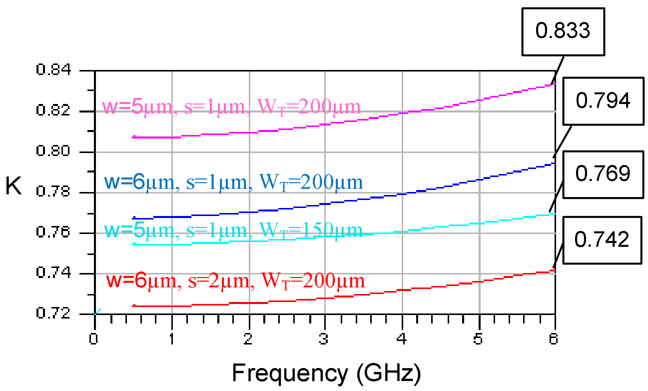
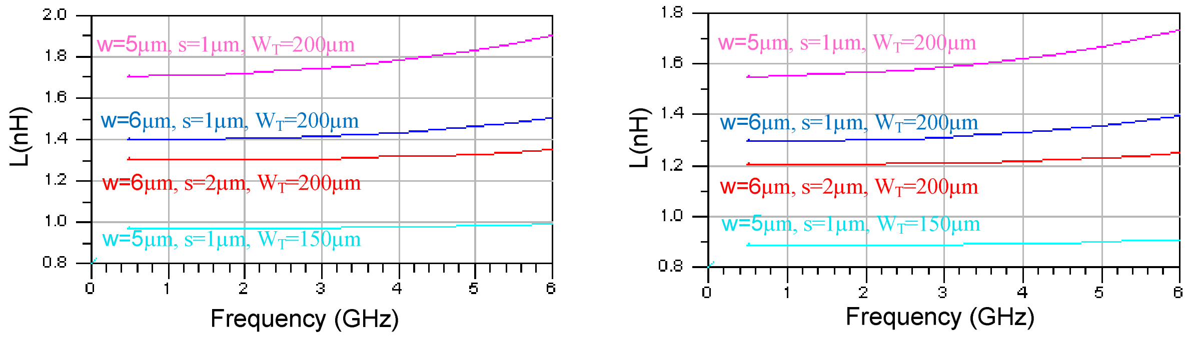
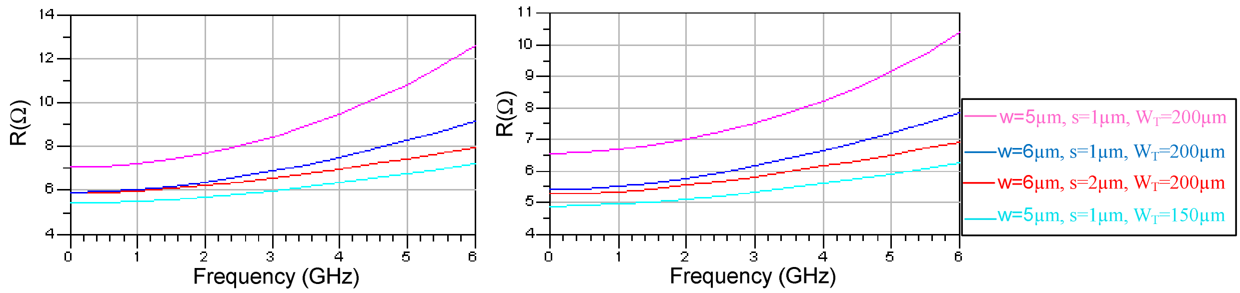


Figure 3-37: Coupling Coefficient.



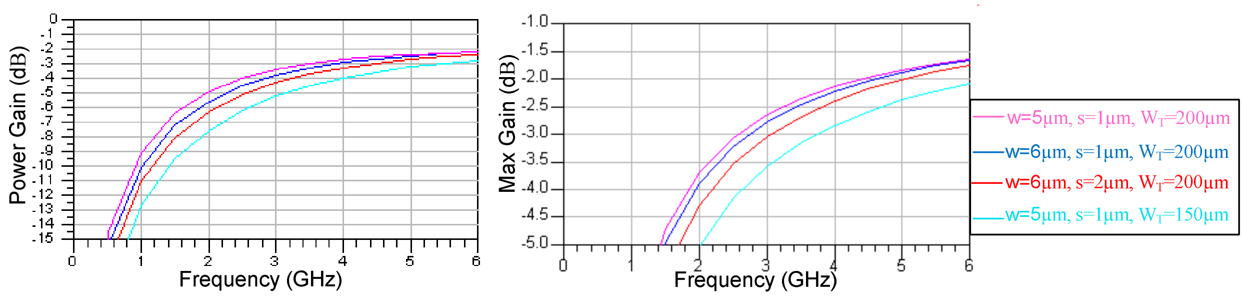
(a) (b)

Figure 3-38: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-39: (a) Primary Resistance; (b) Secondary Resistance.



(a) (b)

Figure 3-40: (a) Power Gain; (b) Maximum Gain.

The results show that using the smallest spacing S and width W and large values of length W_T lead to higher values of coupling coefficient and power gain. However, both primary and secondary resistances become higher. The inductance growth with frequency (Figure 3-38) was not observed for the one-turn planar transformers. This is a typical behavior in monolithic transformers, which appears with more significance at high frequencies when metal area is higher [29]. Some authors, such as Grover [30], believe this behavior is due to skin effect.

As expected, as more metal is used in the design, the winding inductance increases significantly, and thinner windings can provide stronger magnetic coupling and higher gain (Figure 3-37 and Figure 3-40). However, higher ohmic losses should also be taken into account, given their significance for higher frequencies. Overall, the designer should compromise between the needed coupling and power gain, and the tolerable losses, taking into consideration that the smallest width available may not always be the appropriate choice.

A planar transformer with a width of 5µm, total length of 200µm and 1µm of spacing was compared with a second transformer of the same dimensions, with a spiral of Metal 5 connected in parallel underneath. As mentioned previously, with this procedure a simultaneous improvement in the coupling factor and reduction of the windings resistance is expected.
The results of this simulation are presented in Figure 3-41 to Figure 3-44.

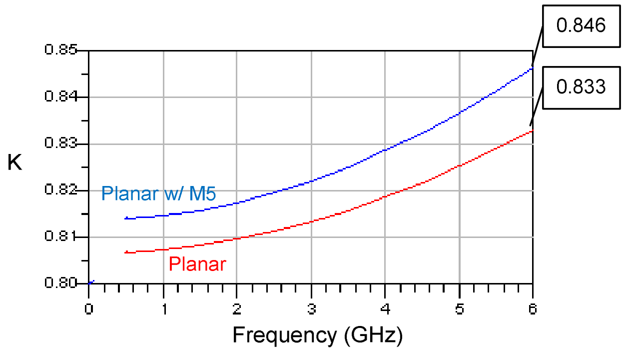
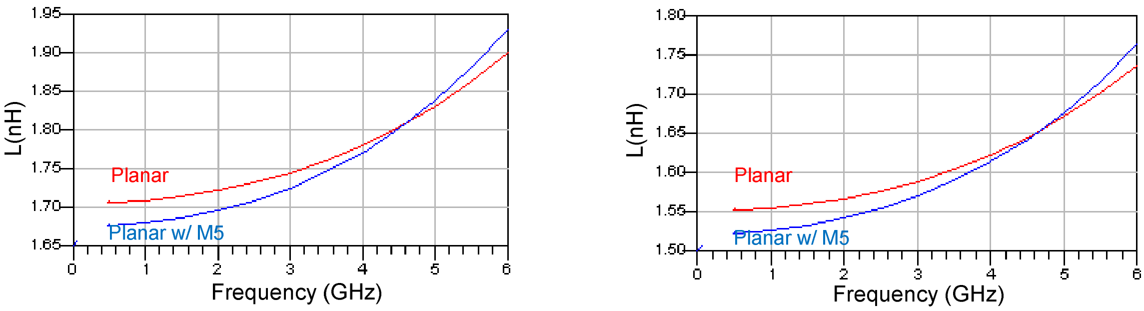
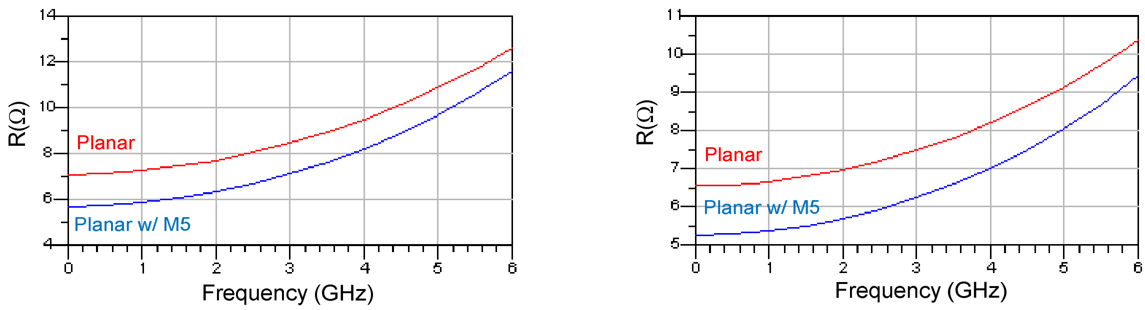


Figure 3-41: Coupling Coefficient.



(a) (b)

Figure 3-42: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-43: (a) Primary Resistance; (b) Secondary Resistance.

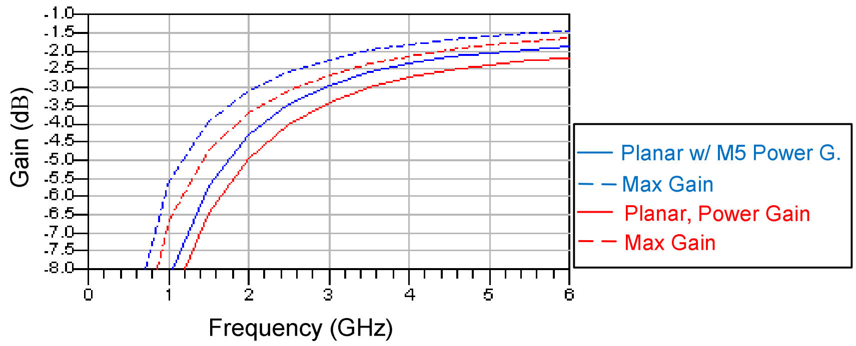


Figure 3-44: Maximum Gain and Power Gain of Planar Transformers.

These results show that adding a Metal 5 spiral under the transformer brings improvement to the coupling factor and both maximum and power gain. This transformer presents lower primary and secondary winding resistance (Figure 3-43).

Figure 3-42 shows that, for lower frequencies, the inductance of the planar transformer is higher than the inductance of the transformer with Metal 5. However, above 4.5 GHz, the inductance of the Metal 5 transformer exceeds it. This abrupt rise of the inductance is mainly due to non-ideal capacitive effects that appear at high frequencies, between overlaid traces of metal.

- **Stacked structures**

In order to study the effects of superposition of windings, the layout of Figure 3-45 was created.

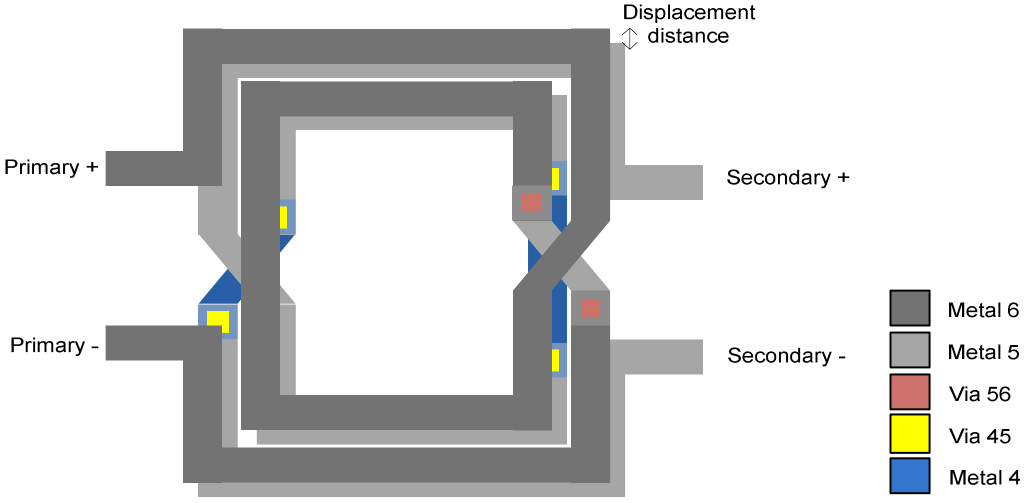


Figure 3-45: Example layout of the two-turn stacked transformer.

A stacked transformer as illustrated on Figure 3-45 was simulated. The dimensions of this device were 5µm of width, total length of 200µm and 1µm of spacing, and the primary winding was designed precisely above the secondary. This transformer was simulated and compared with another stacked transformer with 1 µm of displacement between primary and secondary. Figure 3-46 to Figure 3-49 present the results of this analysis.

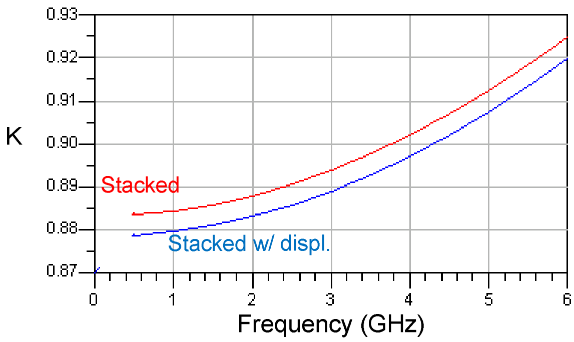
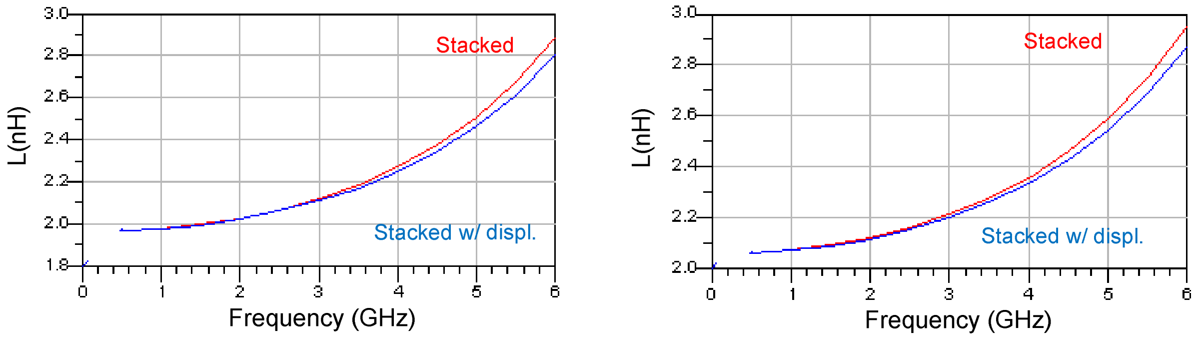
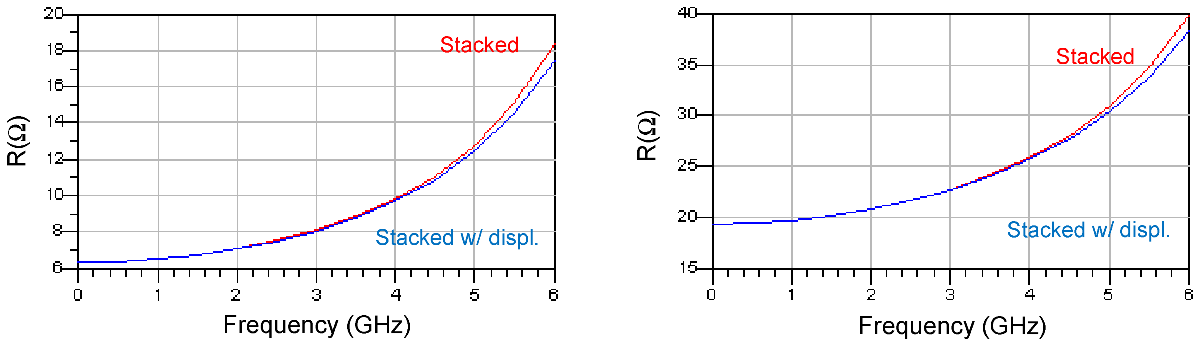


Figure 3-46: Coupling Coefficient.



(a) (b)

Figure 3-47: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-48: (a) Primary Resistance; (b) Secondary Resistance.

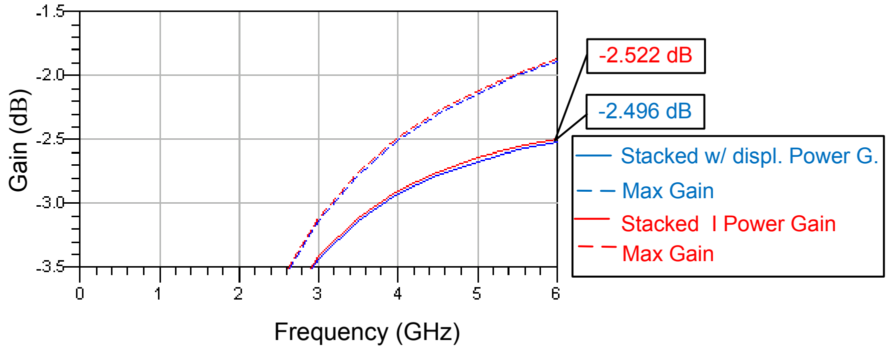


Figure 3-49: Maximum Gain and Power Gain of Stacked Transformers

As seen in Figure 3-46 Figure 3-49, displacing the windings of a stacked transformer does not enhance its behavior, confirming the results achieved in the previous section.

The results show that the stacked transformer has considerably higher K-factor and power gain than the planar transformer discussed above: At 6 GHz, K is now of 0.925 and has a power gain of -2.496 dB. The resistance of the secondary side, however, rose from about 10 Ω to 40 Ω at the same frequency.

A stacked transformer as seen on Figure 3-45 was simulated and compared to a stacked transformer of the same dimensions, which was added a Metal 4 spiral under the secondary, connected to it in parallel, in order to achieve lower resistance values in the secondary.

In this study, the displacement distance between primary and secondary windings is null. The results are shown in Figure 3-50 to Figure 3-53.

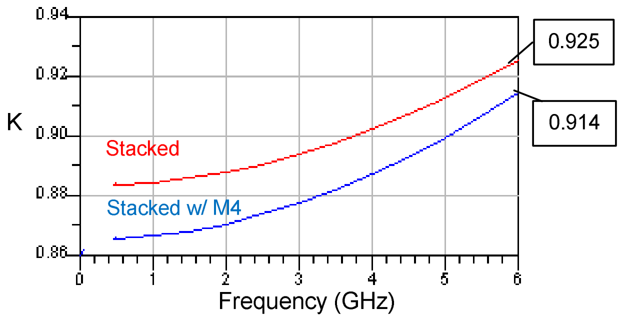
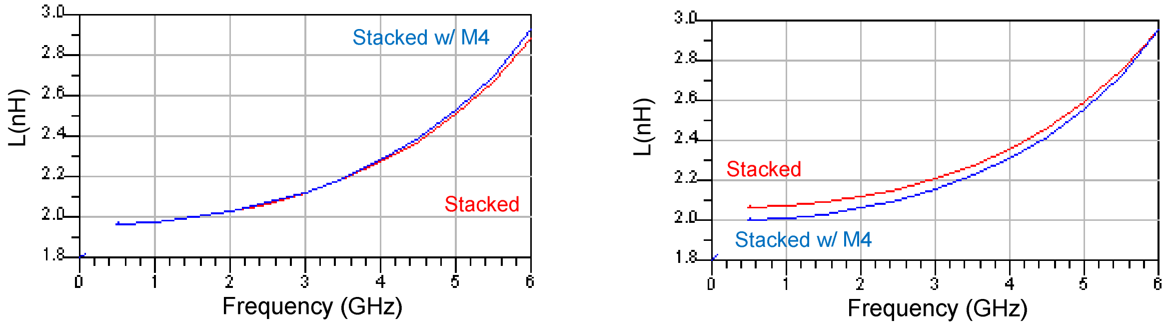
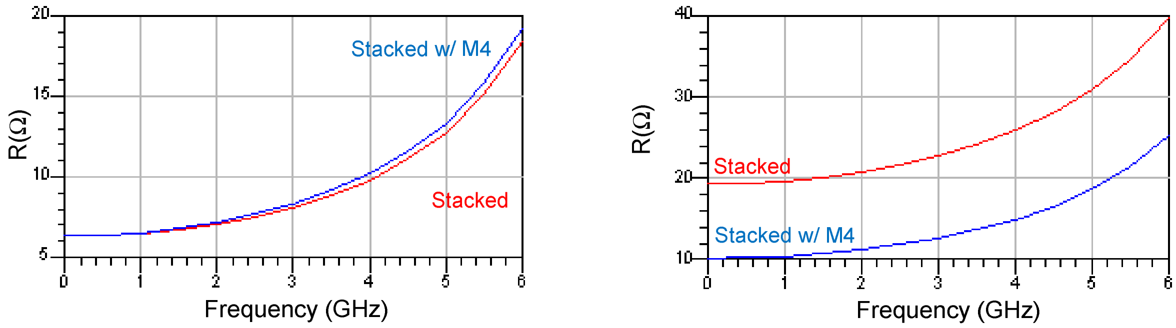


Figure 3-50: Coupling Coefficient.



(a) (b)

Figure 3-51: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-52: (a) Primary Resistance; (b) Secondary Resistance.

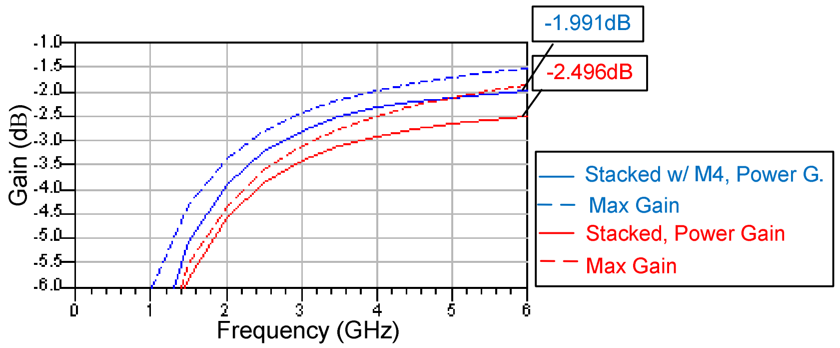


Figure 3-53: Maximum Gain and Power Gain of Stacked Transformers.

From Figure 3-52 it is clear that a spiral of Metal 4 connected in parallel to the secondary of the transformer reduces significantly the secondary winding resistance, which confirms the results observed for one-turn transformers. This extra layer of metal, however, causes a slight increase of the resistance of the primary winding, more pronounced for higher frequencies. The magnetic coupling of this transformer decreases as a result of the decrease of the inductance, but a higher power gain and maximum gain can be achieved (Figure 3-53).

Figure 3-54 presents an alternative design of a two-turn stacked transformer, in which the ports are placed on the corners of the square windings.

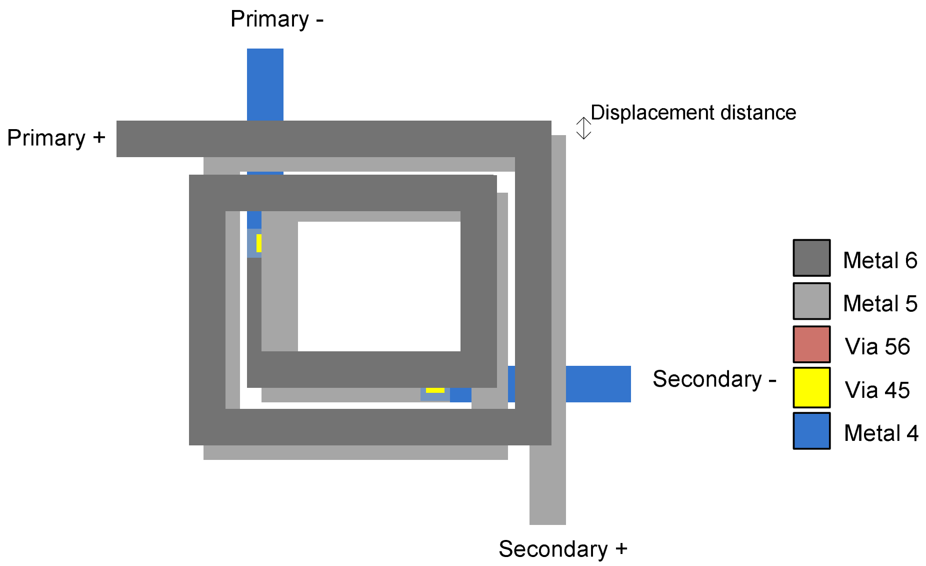
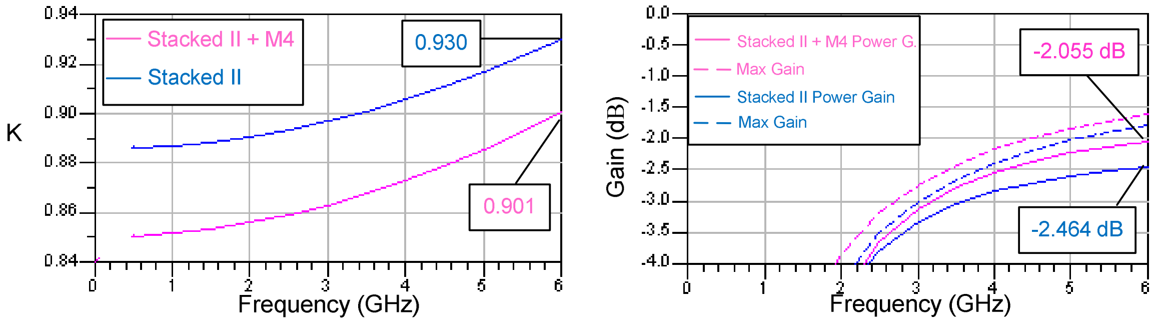


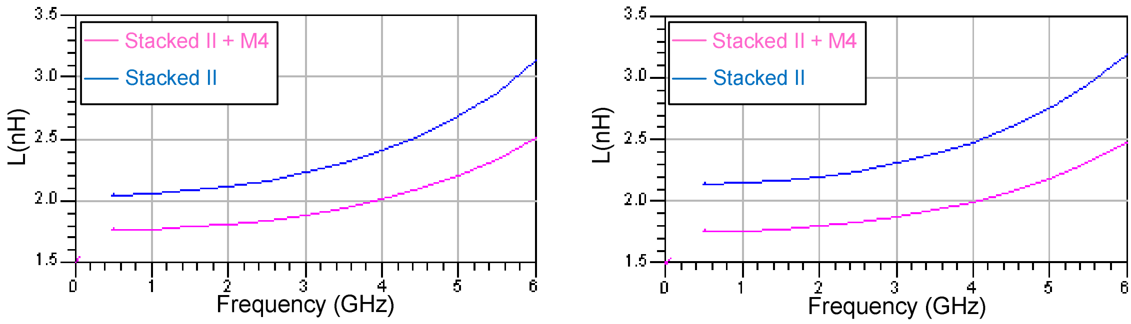
Figure 3-54: Two-turn stacked transformer with corner ports

The transformer illustrated in Figure 3-54 (referred to as *Stacked II*) was simulated and compared to a transformer with the same dimensions plus an additional spiral of Metal 4 connected under the secondary. Figure 3-55 to Figure 3-57 present the results of this analysis.



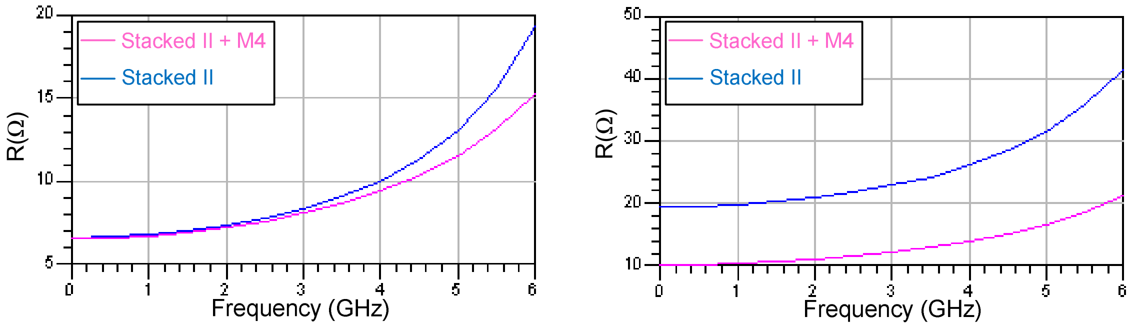
(a) (b)

Figure 3-55: (a) Coupling Coefficient; (b) Power and Maximum Gain.



(a) (b)

Figure 3-56: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-57: (a) Primary Resistance; (b) Secondary Resistance.

As verified in the previous section, these simulation results show an improvement both in coupling coefficient and in power gain using the *Stacked II* transformer, in opposition to the first type of stacked transformer.

The ordinary *Stacked II* transformer, however, presents lower power gain than the device added with a Metal 4 spiral (Figure 3-55). On the other hand, the Metal 4 solution provides lower primary winding inductance, hence lower coupling factor. It also presents higher primary resistance for high frequencies, but its secondary resistance lowers significantly (Figure 3-57).

A summary of notable parameters of two-turn transformers at 6 GHz is presented in table 3-2.

Table 3-2: Parameters of two-turn transformers at 6 GHz.

	Planar	**Planar w/ M5**	**Stacked I**	**Stacked I w/ M4**	**Stacked II**	**Stacked II w/ M4**
K	0.833	0.846	0.925	0.914	0.930	0.901
Power Gain (dB)	-2.190	-1.889	-2.496	-1.991	-2.464	-2.055
Maximum Gain (dB)	-1.650	-1.437	-1.872	-1.533	-1.779	-1.611
Primary Resistance (Ω)	12.628	11.629	18.335	19.190	19.266	15.269
Sec. Resistance (Ω)	9.720	9.438	39.784	25.356	41.557	21.280
Primary Inductance(nH)	1.901	1.931	2.882	2.924	3.136	2.504
Sec. Inductance (nH)	1.737	1.765	2.953	2.951	3.197	2.476

Two-turn planar transformers present stronger magnetic coupling, with a coupling factor of over 0.83. Adding the metal 5 spiral immediately improves the coupling factor and gain, reducing the winding resistance, as expected.

The stacked structures present excellent magnetic coupling, although their secondary resistance is significantly higher. This can be reduced with the addition of a metal 4 spiral connected under the secondary of the transformer.

3.3 Three Turns

- **Planar structures**

Figure 3-58 presents an example of the three-turn planar transformers in study in which square side length, W_T, is 200 µm, windings width is 5µm, and spacing between windings, S, is 1µm.

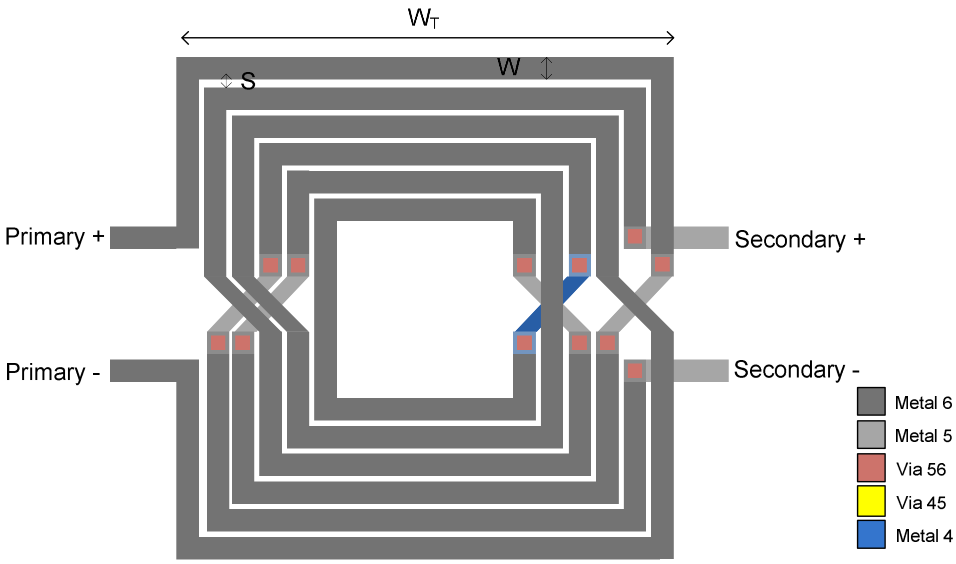
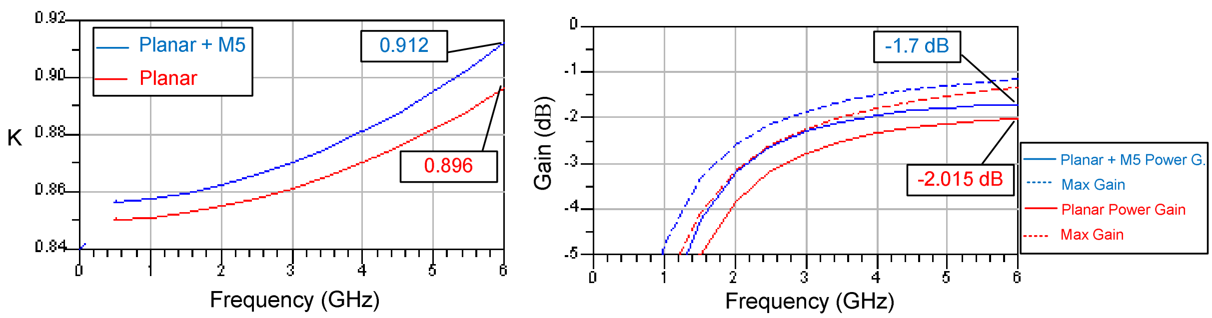


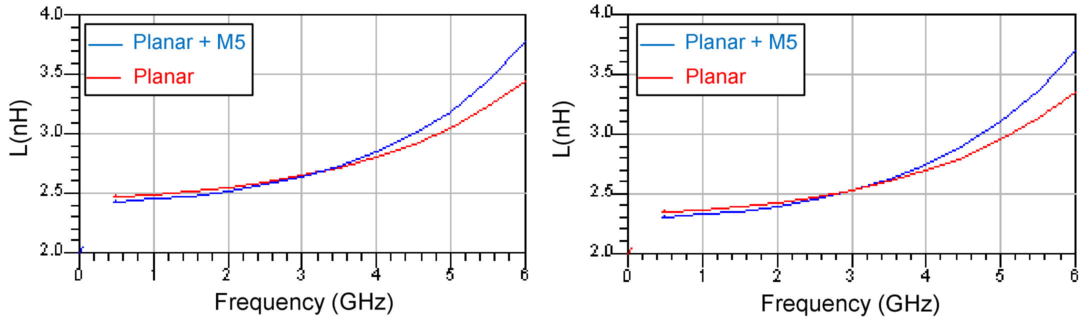
Figure 3-58: Example layout of the three-turn planar transformer.

The planar transformer discussed above was simulated and compared to a similar transformer, with an additional spiral of Metal 5 underneath it, connected to both primary and secondary windings.
The results of these simulations are presented in Figure 3-59 to Figure 3-61.



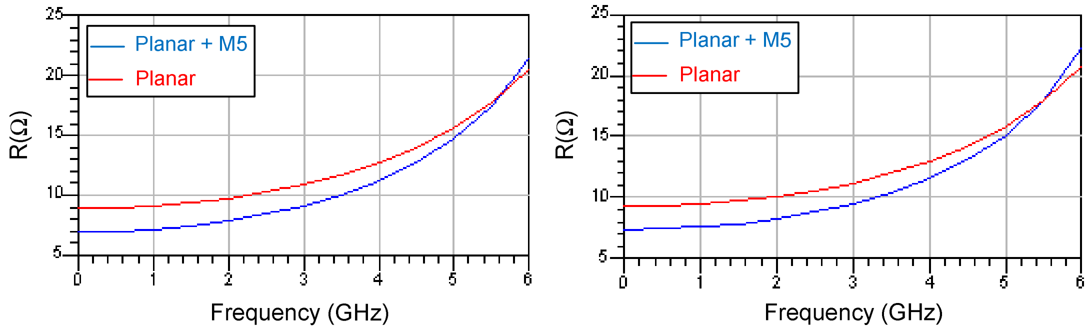
a) (b)

Figure 3-59: (a) Coupling Coefficient; (b) Power and Maximum Gain.



a) (b)

Figure 3-60: (a) Primary Inductance; (b) Secondary Inductance.



a) (b)

Figure 3-61: (a) Primary Resistance; (b) Secondary Resistance.

The results of this study show that the transformer with additional Metal 5 presents lower winding resistance than the planar transformer, except for frequencies above 5.5 GHz (Figure 3-61). For frequencies below 3 GHz, the Metal 5 transformer also presents lower winding inductance (Figure 3-60). From the results of fFigure 3-59 it is clear that the additional spiral under the transformer causes an increase of both magnetic coupling and gain.

- **Stacked Structures**

In order to verify the effects of windings superposition for three-turn transformers, the stacked transformer of Figure 3-62 was simulated and compared to a similar transformer with an additional winding of Metal 4 under the secondary, connected to it in parallel.

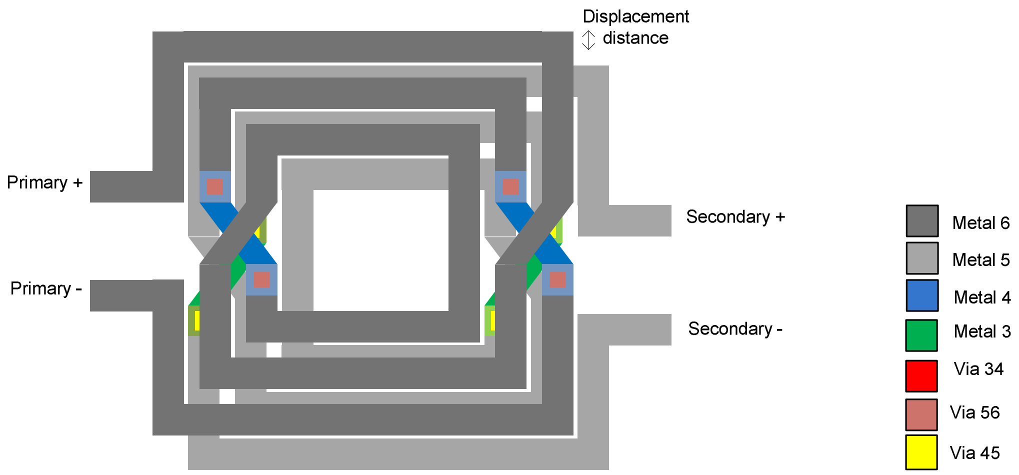


Figure 3-62: Example layout of the three-turn stacked transformer

The results of this experience are shown in Figure 3-63 to 3-65.

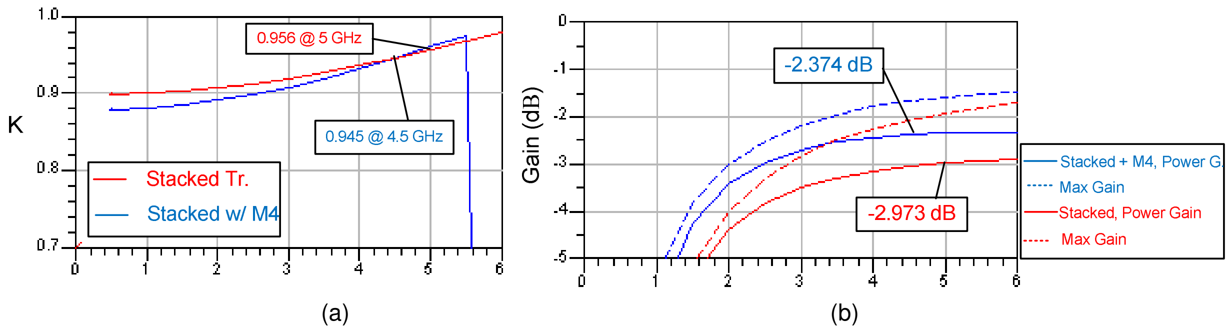


Figure 3-63: (a) Coupling Coefficient; (b) Power and Maximum Gain.

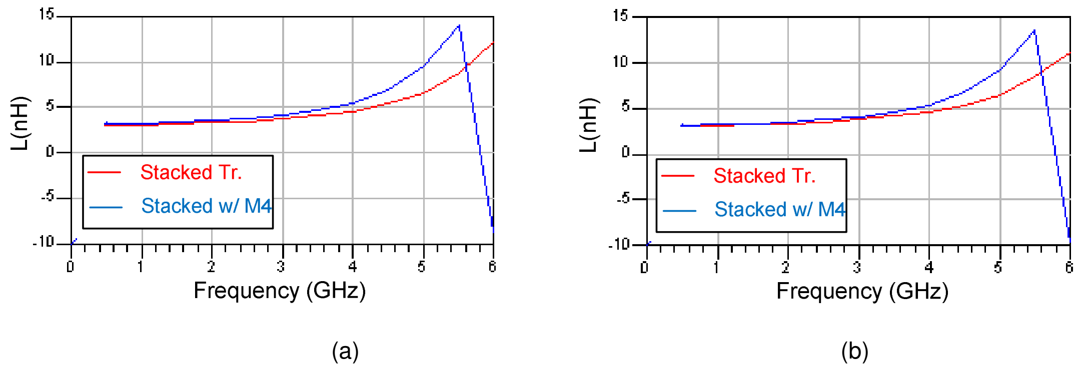
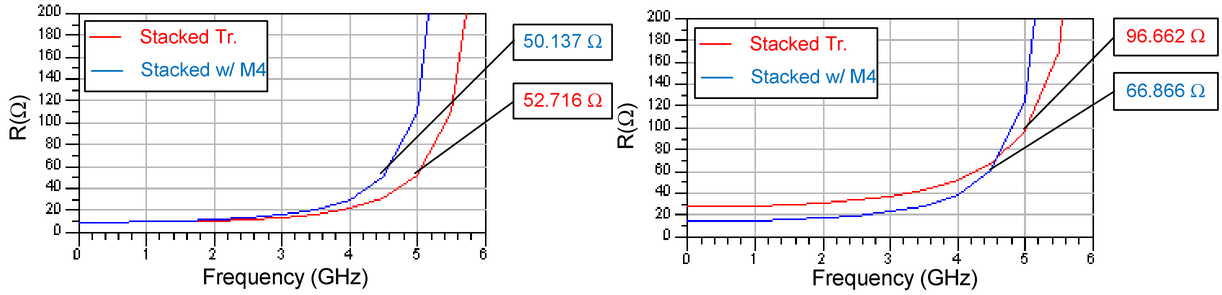


Figure 3-64: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-65: (a) Primary Resistance; (b) Secondary Resistance.

These results show that both stacked transformers achieve stronger magnetic coupling than the planar, at the expense of higher secondary resistance.

The stacked transformer approaches resonance below 6 GHz. Figure 3-63 shows that the stacked transformer with Metal 4 can achieve higher magnetic coupling and higher gain than the simple stacked transformer in high frequency. It should be mentioned, however, that this behavior is more determined by the influence of high frequency parasitic effects than by the strength of the magnetic coupling itself, since it did not appear in transformers with smaller amounts of metal.

As expected, the Metal 4 spiral under the secondary contributes to a significant reduction of secondary resistance, but faintly increasing primary resistance (Figure 3-65). This device approaches resonance at a lower frequency than the simple stacked transformer (5 GHz), since it already presents a substantial quantity of metal: The primary and secondary add a total of 6 windings, plus other 6 windings of metal 4.

Figure 3-66 presents an alternative layout of a stacked transformer, in which the primary and secondary ports are placed in the corners of the square spirals.

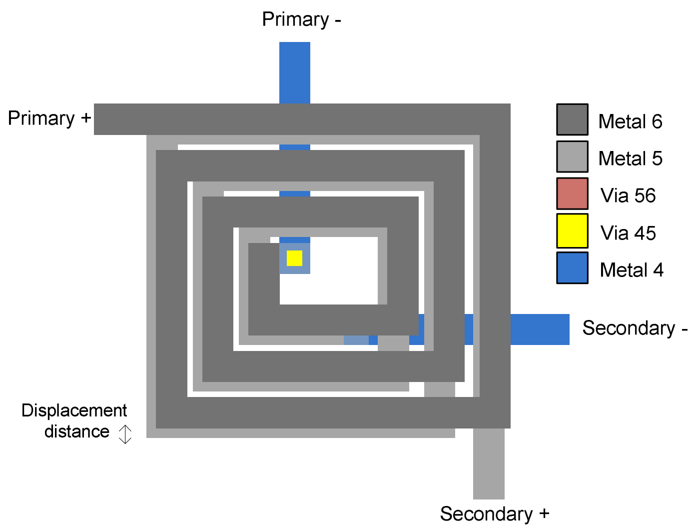
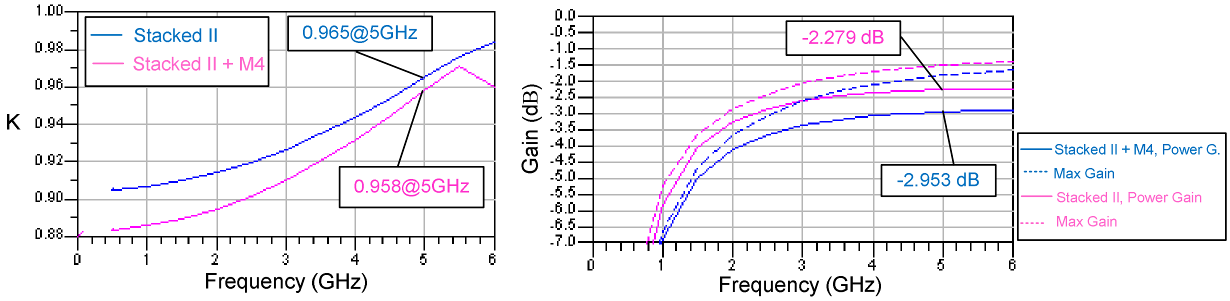


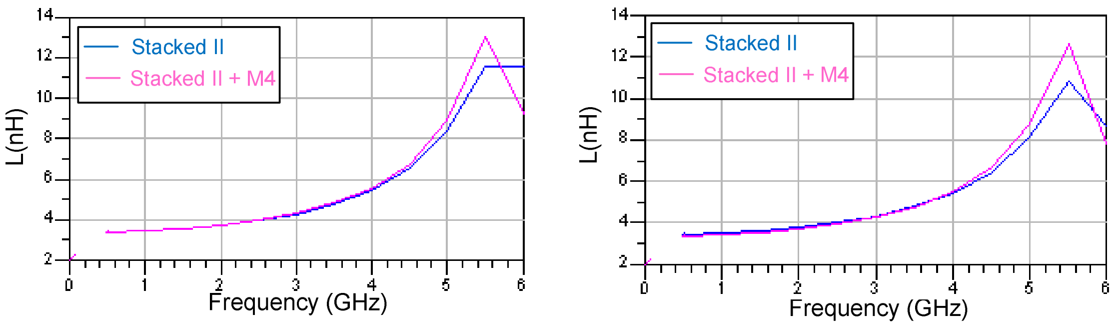
Figure 3-66: Example layout of the three-turn stacked transformer with corner ports.

The stacked transformer of Figure 3-66 (*Stacked II*) was simulated and compared to a similar transformer, with an additional Metal 4 spiral under the secondary. The results are shown in Figure 3-67 to 3-69.



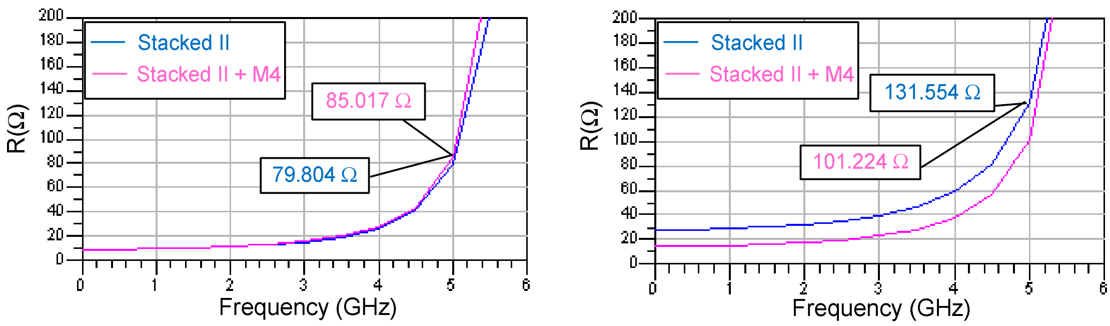
(a) (b)

Figure 3-67: (a) Coupling Coefficient; (b) Power and Maximum Gain.



(a) (b)

Figure 3-68: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-69: (a) Primary Resistance; (b) Secondary Resistance.

The obtained results show that both *Stacked II* transformers approach resonance above 5.5 GHz. Nevertheless, the coupling factor can reach 0.965 at 5 GHz. This type of transformer can achieve higher values of inductance.

The Metal 4 transformer can achieve higher coupling factor, and higher maximum and power gain that the two-layer stacked transformer (Figure 3-67). The higher coupling factor, however, is mostly due to high frequency parasitic effects that also affect windings inductance (Figure 3-68).

The transformer with Metal 4 presents lower secondary resistance, but its primary resistance is higher than for the two-layer stacked transformer, particularly in high frequency (Figure 3-69).

A summary of notable parameters of three-turn transformers is presented in table 3-3.

Table 3-3: Parameters of three-turn transformers.

	Planar At 6 GHz	**Planar w/M5 At 6GHz**	**Stacked I At 5 GHz**	**Stacked I w/ M4 At 4.5 GHz**	**Stacked II At 5 GHz**	**Stacked II w/ M4 At 5 GHz**
K	0.896	0.912	0.956	0.945	0.965	0.958
Power Gain (dB)	-2.015	-1.700	-2.973	-2.374	-2.953	-2.279
Maximum Gain (dB)	-1.342	-1.162	-1.910	-1.662	-1.826	-1.499
Primary Res. (Ω)	20.490	21.437	52.716	50.137	79.804	85.017
Sec. Resistance (Ω)	20.739	22.160	96.662	60.735	131.554	101.224
Primary Ind. (nH)	3.440	3.769	6.611	6.861	8.370	8.894
Sec. Inductance (nH)	3.353	3.701	6.533	6.784	8.160	8.738

Three-turn planar transformers present very good magnetic coupling, with a coupling factor of about 0.9. Adding the metal 5 spiral immediately improves the coupling factor and gain, as expected, although the winding resistance starts to increase in high frequency due to the significant parasitics. This is the most profitable topology to use, guaranteeing small insertion losses.

The stacked structures present excellent magnetic coupling, but approach resonance at much lower frequencies.

3.4 Four turns

- **Planar Structures**

Figure 3-70 illustrates an example design of the four-turn planar transformer.

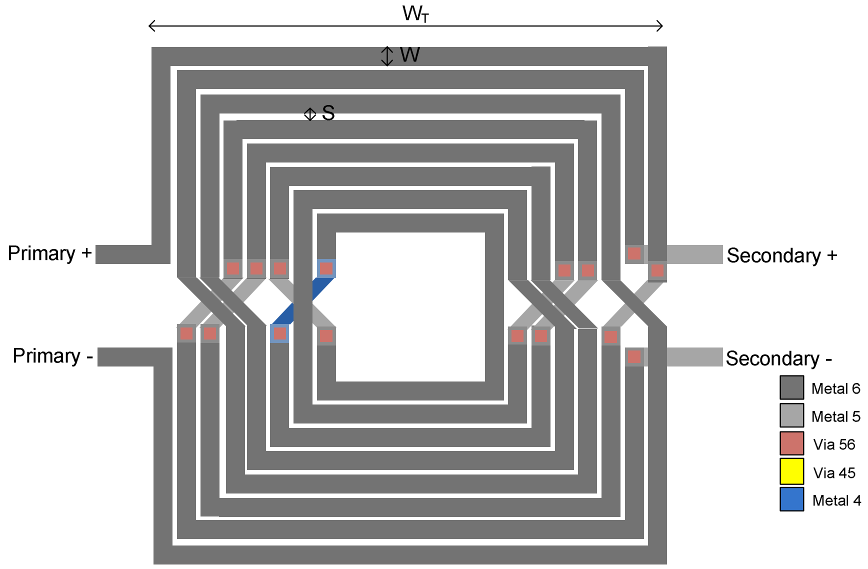


Figure 3-70: Example layout of the four-turn planar transformer.

The transformer presented in Figure 3-70 was simulated and compared to a similar transformer, with an additional spiral of Metal 5 underneath it, connected to both primary and secondary windings.

The chosen dimensions of the windings were 200µm of length, 5µm of width and 1µm of spacing between turns. The results of this study are shown in Figure 3-71, Figure 3-72 and Figure 3-73.

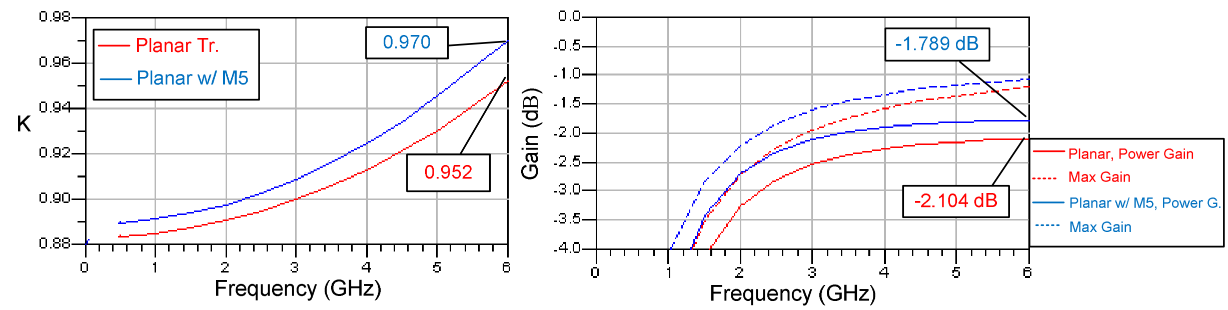
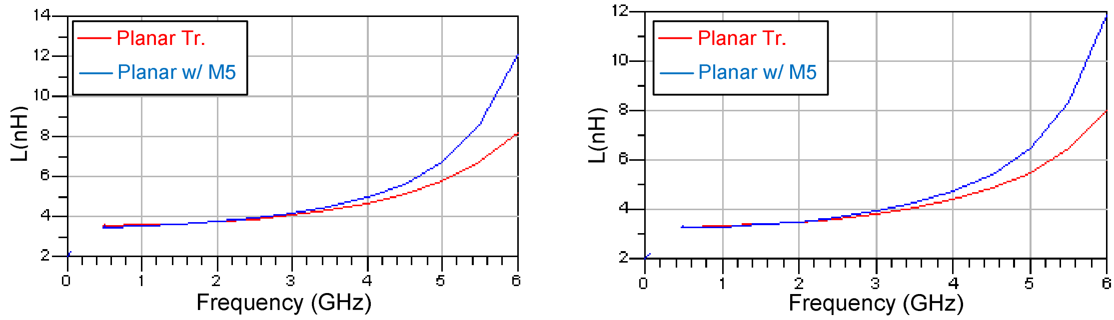
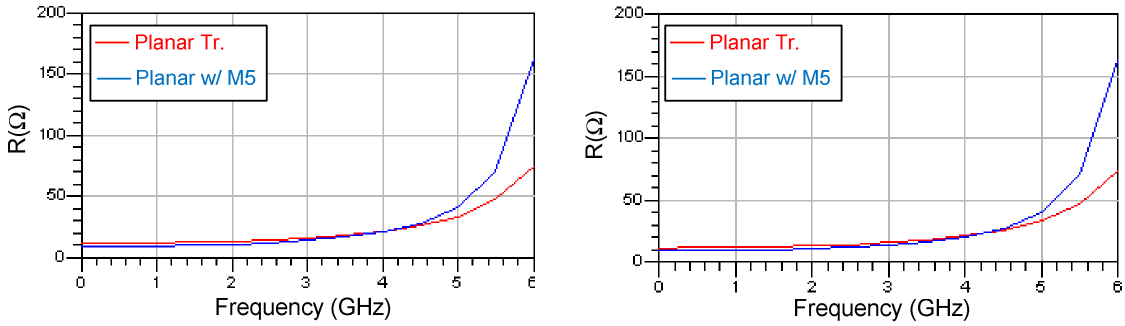


Figure 3-71: (a) Coupling Coefficient; (b) Power Gain and Maximum Gain.



(a) (b)

Figure 3-72: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-73: (a) Primary Resistance; (b) Secondary Resistance.

The results show that a four-turn transformer leads to stronger magnetic coupling and higher gain than others with fewer turns, at the expense of a higher winding resistance.

Adding a Metal 5 spiral under the transformer allows an even higher coupling factor and gain (Figure 3-71), but this procedure reduces winding resistance only below 4 GHz. For higher frequencies, both primary and secondary resistance are superior to the resistance achieved with the planar transformer alone (Figure 3-73). Above 3 GHz, the Metal 5 transformer has higher primary and secondary inductance than the planar transformer (Figure 3-72).

- **Stacked Structures**

In order to analyze the effects of superposition of the windings, the transformer of Figure 3-74 was designed, using the dimensions referred above.

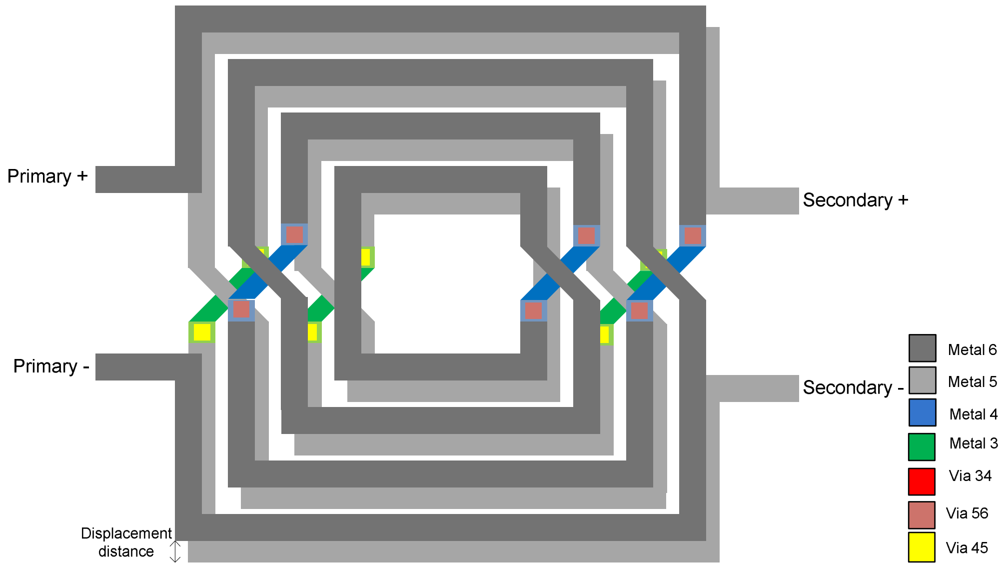


Figure 3-74: Example layout of the four-turn stacked transformer.

The stacked transformer was simulated and compared with a similar transformer, with an additional spiral of Metal 4 under the secondary, connected to it in parallel. Figure 3-75 to Figure 3-78 present the results of this experiment.

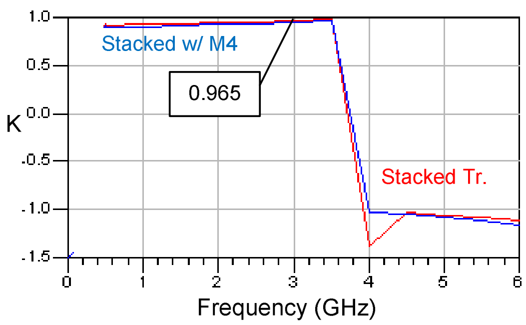


Figure 3-75: Coupling Coefficient.

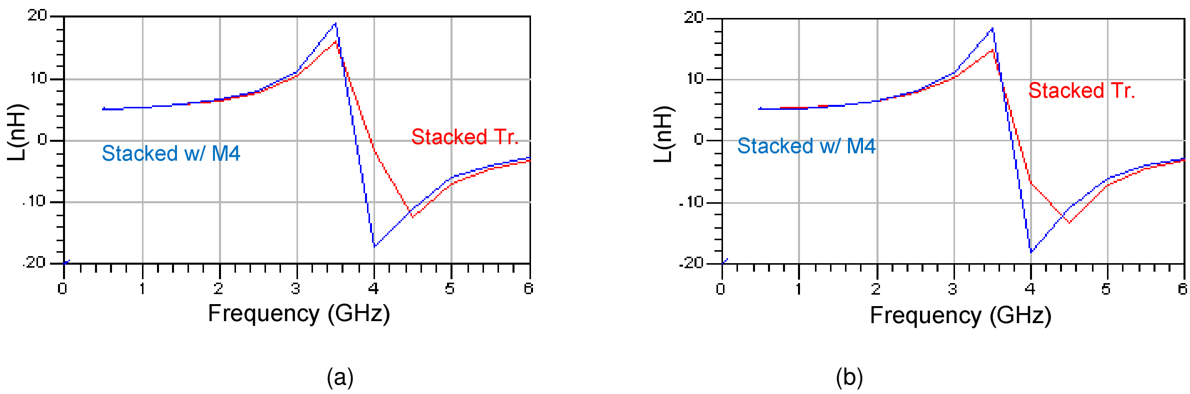
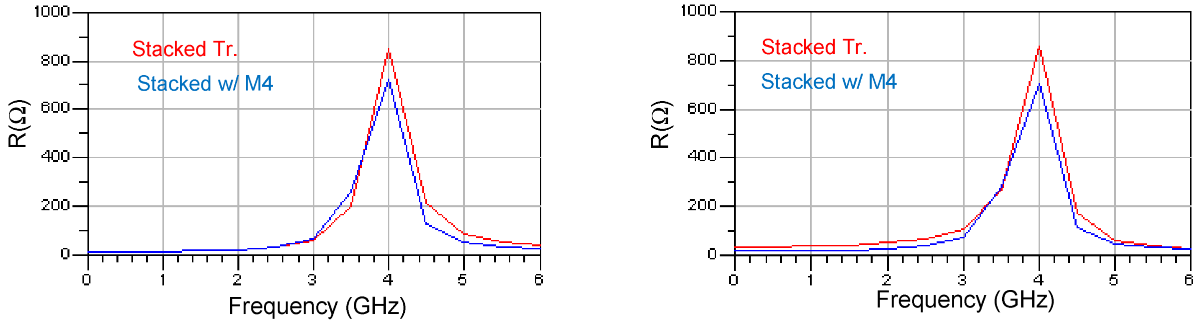


Figure 3-76: (a) Primary Inductance; (b) Secondary Inductance.



(a) (b)

Figure 3-77: (a) Primary Resistance; (b) Secondary Resistance.

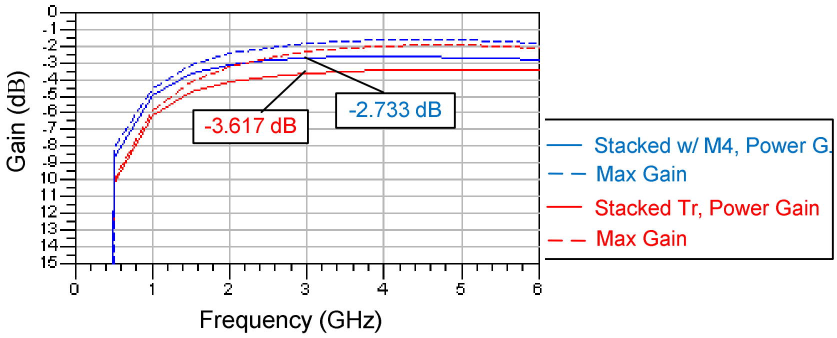


Figure 3-78: Maximum Gain and Power Gain of Stacked Transformers

These results show that both the stacked transformers reach resonant frequency around 4 GHz (Figure 3-77). However, below that frequency, they present a very strong magnetic coupling, evident in Figure 3-80. As verified previously for the three-turn transformers, the gain of the stacked transformers is inferior to the gain achieved with a planar one, but the stacked transformer added with a Metal 4 spiral under the secondary can achieve higher gain than the two-layer stacked one (Figure 3-78).
The Metal 4 transformer presents lower secondary resistance, but higher primary resistance and higher inductance for near-resonance frequencies, which validates previously achieved results.

Figure 3-79 illustrates an alternative design of a four-turn stacked transformer, in which primary and secondary ports are placed in the corners of the square spirals.

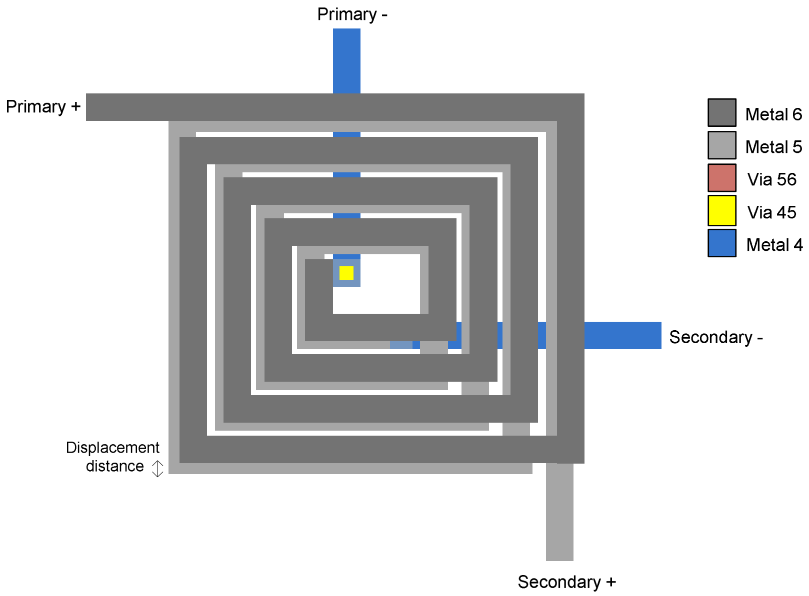
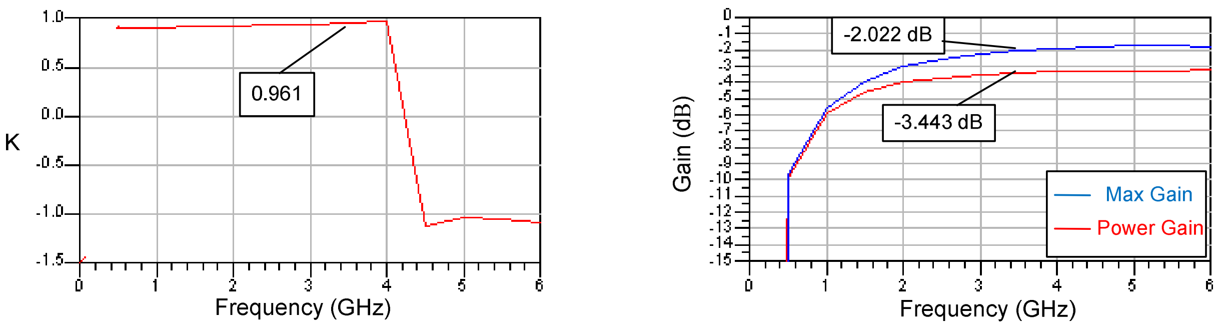


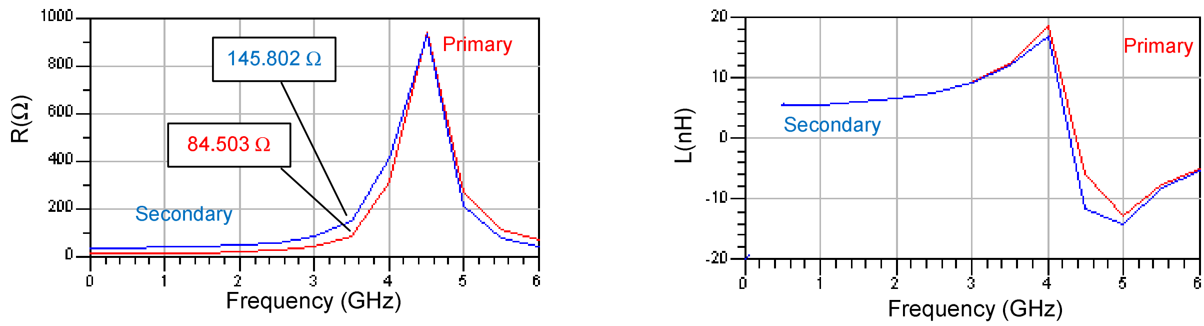
Figure 3-79: Example layout of the four-turn stacked transformer with corner ports.

The transformer presented in Figure 3-79 was designed and simulated, using the dimensions discussed above. The results of study are presented in Figure 3-80 and Figure 3-81.



(a) (b)

Figure 3-80: (a) Coupling Coefficient; (b) Maximum Gain and Power Gain.



(a) (b)

Figure 3-81: (a) Primary and Secondary Resistance; (b) Primary and Sec. Inductance.

Figure 3-80 and Figure 3-81 show that this kind of stacked transformer reaches resonance at 4.5 GHz. The behavior of this transformer below that frequency is acceptable, since it presents strong magnetic coupling. However, its winding resistance is very high, limiting the gain of the device.

A summary of notable parameters of four-turn transformers is presented in table 3-4.

Table 3-4: Parameters of four-turn transformers.

	Planar At 6 GHz	**Planar w/ M5 At 6 GHz**	**Stacked I At 3 GHz**	**Stacked I w/ M4 At 3 GHz**	**Stacked II At 3.5 GHz**
K	0.952	0.970	0.965	0.957	0.961
Power Gain (dB)	-2.104	-1.789	-3.617	-2.733	-3.443
Maximum Gain (dB)	-1.217	-1.081	-2.353	-1.851	-2.022
Primary Resistance (Ω)	74.154	161.753	58.171	62.966	84.503
Sec. Resistance (Ω)	73.953	162.856	105.490	76.346	145.802
Primary Inductance (nH)	8.211	12.091	10.478	11.270	12.485
Sec. Inductance (nH)	7.893	11.879	10.181	11.095	12.060

Four-turn planar transformers present excellent magnetic coupling, with a coupling factor over 0.95. Adding the metal 5 spiral immediately improves the coupling factor and gain, but the winding resistance increases in high frequency due to the parasitics. Nevertheless, this is the most profitable topology to use, guaranteeing small insertion losses.

The stacked structures present excellent magnetic coupling, but as expected, they approach resonance at much lower frequencies.

4. Monolithic Transformer Electric Model

This chapter of the dissertation is divided in two sections. In section 4.1 a comparison between the results from the inductance calculation program and the EM simulation results is presented. Section 4.2 describes the models obtained for the planar structure with Metal 5 added, for transformers with one to four turns. The obtained values for the models components are presented.

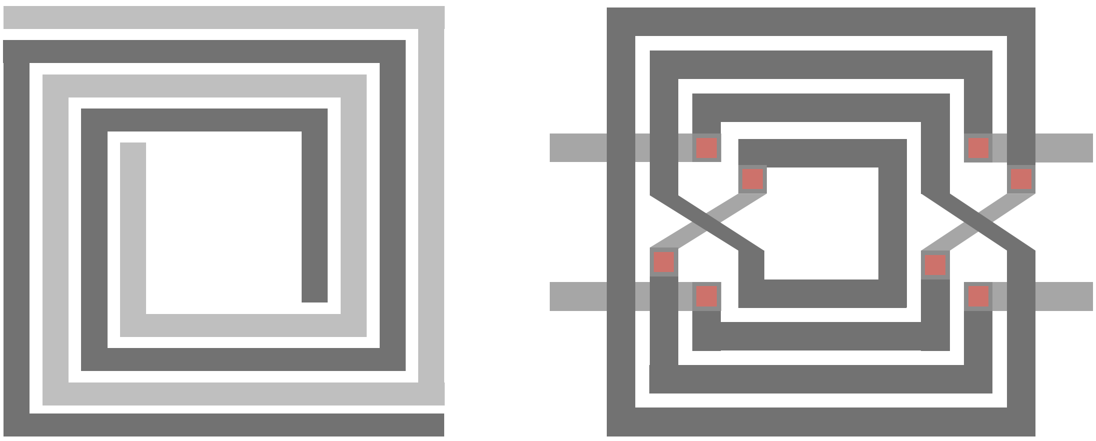
4.1 Inductance Calculation Program

As referred in Chapter 2, a program for inductance calculation was created in the scope of this dissertation. The objective was to develop a program able to calculate a simple transformer model for a given set of dimensions. The input dimensions are W_T, W, S and N, already described, and the output values are L_P, L_S and L_M described in section 2.3.2.

This software consists in a simple *Matlab* script which implements the Greenhouse algorithm [25], applied to square planar transformers.

The inductance script was initially developed to calculate the spirals self and mutual inductances of a symmetric transformer as the one in figure 4.1 (a).

When tested, it was verified it provides relatively accurate results to *Rabjohn* transformer type too (Figure 4-1 (b)).



(a) (b)

Figure 4-1: (a) Symmetric transformers: (a) Interwound winding; (b) *Rabjohn* transformer;

Table 4-1 presents a comparison between the inductance results obtained with the Matlab script and the results obtained by electromagnetic simulation with Momentum for both transformers of Figure 4-1, with W_T = *outer_diameter* = 200 µm, w = *width* = 5 µm, s = *spacing* = 1 µm and *metal_thickness* = 2.06 µm (Thickness of a Metal 6 strip, from figure 2-2).

Table 4-1: Script's inductance results versus electromagnetic simulation results

			Momentum Results	
	Inductance (nH)	**Matlab Results**	*Interwound Transformer*	*Rabjohn Transformer*
N=1	Lp	0.6021	0.545	0.503
	Ls	0.6021	0.546	0.506
	Lm	0.3145	0.355	0.300
N=2	Lp	1.6668	1.508	1.706
	Ls	1.6668	1.499	1.552
	Lm	1.7111	1.210	1.313
N=3	Lp	2.8050	2.631	2.476
	Ls	2.8050	2.615	2.346
	Lm	3.3826	2.247	2.049
N=4	Lp	3.7946	3.729	3.546
	Ls	3.7946	3.706	3.297
	Lm	4.3483	3.277	3.020

The results of Table 4-1 show that the program provides a fairly accurate estimation for primary and secondary inductance (L_S and L_P), given that the maximum error of the program is about 7% for the interwound transformer. For the *Rabjohn* transformer, the program presents a maximum error of 17%.
The difference between the computed and simulated mutual inductance (L_M) increases with the transformer number of turns: for one-turn transformers the program estimation is almost precise, but for three and four turns, the difference is over 30% for both transformer types. The *Rabjohn* transformer always presents higher error, because the program was conceived to fully symmetric transformers, which is not the case[2]. The Metal 5 segments to the ports and the 45º crossing segments also contribute to this error.
This behavior was expectable, since the program considers only the variation of the dimensions of the metal strips, ignoring several non idealities of transformers such as nonlinear current distributions, capacitive effects, losses, etc.
Because the calculations assume symmetric transformers, primary inductance is always equal to the secondary.
A higher number of turns leads to higher positive contribution for mutual inductance. The negative contribution for mutual inductance is lower, since the distance between opposite sections increases rapidly, and mutual inductance decreases with distance. Therefore, for a considerable number of turns,

[2] Even though this transformer can have unitary turn ratio and thus being symmetric from an electric point of view (Chapter 1).

the mutual inductance rapidly increases. Also, the Greenhouse formula provides optimistic values, given that the metal strips are considered to be on free space.

The EM simulation tool, however, considers non-idealities of the transformer, resulting in a primary self-inductance different from the secondary self-inductance even for fully symmetric transformers.

In order to understand the behavior of the mutual inductance, let's take a closer look at the Z-parameter's extraction:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \times \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{25}$$

As mentioned in chapter 2, the primary inductance is extracted from the Z_{11} parameter. From equation 1, comes:

$$Z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0} \tag{26}$$

The secondary inductance is extracted from Z_{22}. Applying equation 1,

$$Z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0} \tag{27}$$

And the mutual inductance comes from Z_{12}=Z_{21}. Again from equation 1,

$$Z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0} = \left.\frac{v_2}{i_1}\right|_{i_2=0} \tag{28}$$

So, as shown in equations 2 and 3, the primary (secondary) inductance in obtained considering the secondary (primary) in open circuit, thus current flowing only in the primary (secondary) windings of the transformer. So, for a N-turn transformer (being N the number of primary windings, equal to the number of secondary windings), the EM simulator considers the current flow in N-windings.

Applying the same reasoning to equation 4, the EM simulator now needs to process 2*N windings, since it is measuring the influence of the primary (secondary) current flow in the secondary (primary) winding.

As mentioned in the previous chapter, the non-idealities considered by the electromagnetic simulator have stronger effects in devices where more metal is used, given that undesired capacitive effects between windings (and to the substrate) appear in high frequency. In this case, when processing Z_{12} (or Z_{21}), twice the number of turns is considered, leading to significant differences in the obtained values of mutual inductance.

4.2 Monolithic transformer electric model

In this section electric models for some of the transformers already studied will be presented. The model topology was already presented in chapter 2 and is adequate for monolithic transformers on Silicon substrate. The model parameters were obtained by fit to electromagnetic simulation results.

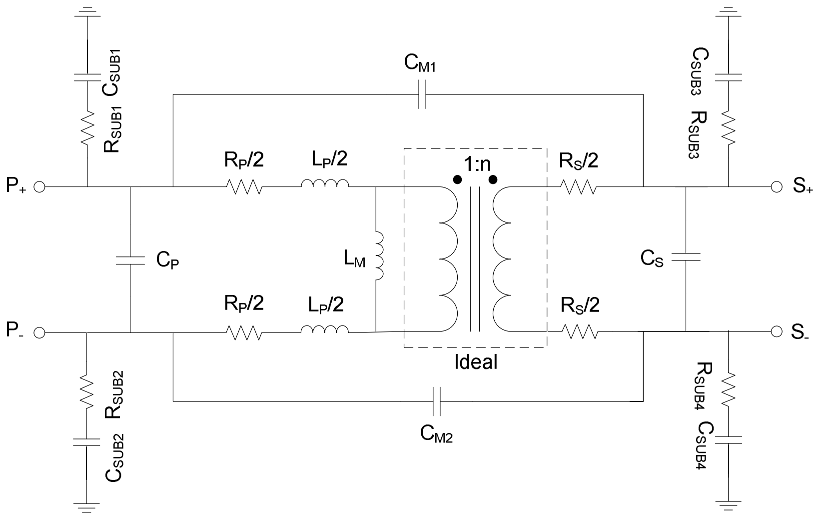


Figure 4-2: Monolithic transformer electric model

Several layouts of monolithic transformers were presented in Chapter 3, but not all showed satisfactory performance in terms of maximum gain, power gain, coupling factor or winding resistance. Generally the planar transformer topology, when a Metal 5 spiral is added underneath it, shows very good performance, so this type of transformer was chosen to model. The results for transformers with two to four turns are shown in table 4-2.

Table 4-2: Model Parameters for Planar Transformers with Metal 5 (2 to 4 turns)

	N=2	N=3	N=4
N	1.17111	1.137	1.0848
$R_P/2$	2.849 Ω	3.483 Ω	4.399 Ω
$R_S/2$	2.625 Ω	3.672 Ω	4.518 Ω
$L_P/2$	0.28268 nH	0.324 nH	0.365 nH
L_M	1.11025 nH	1.782 nH	2.760 nH
C_{M1}	30 fF	100 fF	100 fF
C_{M2}	30 fF	100 fF	100 fF
C_P	58 fF	60 fF	67 fF
C_S	34 fF	60 fF	96 fF
R_{SUB1}	1250 Ω	2000 Ω	1900 Ω
C_{SUB1}	210 fF	60 fF	21 fF
R_{SUB2}	1250 Ω	2000 Ω	1900 Ω
C_{SUB2}	210 fF	60 fF	21 fF
R_{SUB3}	1250 Ω	2000 Ω	1900 Ω
C_{SUB3}	210 fF	60 fF	21 fF
R_{SUB4}	1250 Ω	2000 Ω	1900 Ω
C_{SUB4}	210 fF	60 fF	21 fF

It can be seen that for a greater number of turns, the winding resistance increases, but so does the transformer's magnetizing inductance. The interwinding capacitance increases significantly, but the parasitic capacitive coupling to the substrate decreases in the same proportion. The ohmic losses to the substrate increase for a greater number of turns, but tend to stabilize around 2kΩ.

The model of Figure 4-2 fits the referred transformer topology with precision, which is supported by the figures presented in the Appendix B. The highlighted values in the figures represent the points where the most significant differences between the simulations and the model were found.

This model, however, is not suitable for one-turn transformers. As previously mentioned in Chapter 3, although this transformer provides weak coupling, low primary and secondary self-inductance, and low gain, the small amount of metal used provides fewer parasitic effects even in high frequency. Considering this, a simpler model can be used (Figure 4-3).

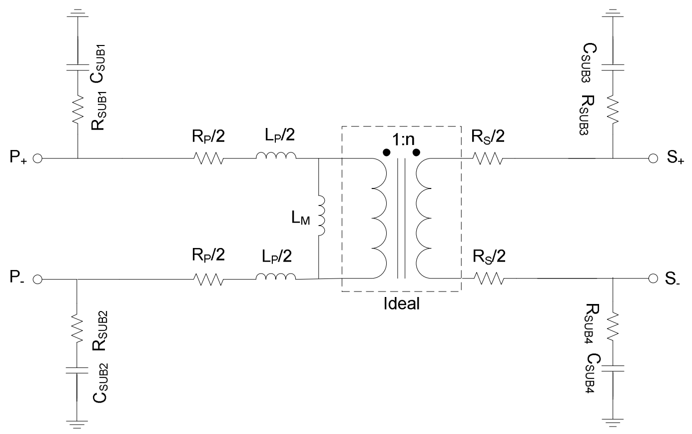


Figure 4-3: One-Turn Transformer Compact Model.

The components' values obtained by simulation and tuning to this model are shown in Table 4-3.

Table 4-3: Model Parameters for one-turn Planar Transformer with Metal 5

	N=1
N	1.65285
R_P/2	1.191 Ω
R_S/2	1.4238 Ω
L_P/2	0.155374 nH
L_M	0.17819 nH
R_{SUB1}	500 Ω
C_{SUB1}	350 fF
R_{SUB2}	500 Ω
C_{SUB2}	350 fF
R_{SUB3}	500 Ω
C_{SUB3}	350 fF
R_{SUB4}	500 Ω
C_{SUB4}	350 fF

This model does not take into account the mutual capacitance between primary and secondary nor the capacitive coupling between winding turns. As expected, the ohmic losses in the windings of this transformer are smaller than the verified for greater number of turns, and so are the self-inductance of the primary and the magnetizing inductance. The ohmic losses through the substrate are also much smaller, but the capacitive effects to the substrate are higher.

The model of Figure 4-3 fits the one-turn planar transformer with metal 5 with precision, with the exception of the winding resistance, in which this model proves to be insufficient. This behavior can be verified in the Appendix B.

4.3 Quality Factor Calculation

Like with spiral inductors the quality factor Q can be used as a figure of merit for transformers. For its calculation the simplified model presented in Figure 4-4 will be used. This model is valid if the substrate effect and mutual capacitance between primary and secondary can be neglected.

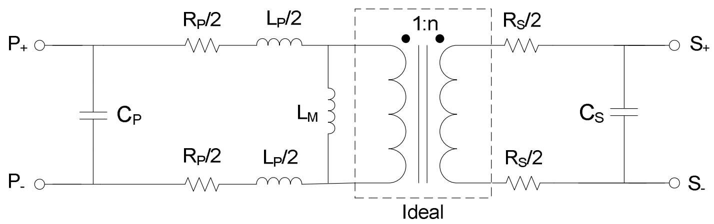


Figure 4-4: Simplified model for Q calculation.

The primary equivalent impedance is given by the parallel of $\mathbf{C_P}$ with the series of $\mathbf{R_P}$, $\mathbf{L_P}$ and $\mathbf{L_M}$. The current flow in the secondary is negligible, given that C_S is a very small capacitor.

$$Z_{EQ_{Primary}} = [R_P + j\omega(L_P + L_M)]\ //\ \frac{1}{j\omega C_P} \tag{29}$$

$$Z_{EQ_{Primary}} = \frac{\frac{(L_P + L_M)}{C_P} - \frac{jR_P}{\omega C_P}}{R_P + j\omega(L_P + L_M - \frac{1}{\omega^2 C_P})} \tag{30}$$

The secondary equivalent impedance is given by the parallel of $\mathbf{C_S}$ with the series of $\mathbf{R_S}$ and $\mathbf{n \times L_M}$. The current flow in the primary is negligible, given that C_P is a very small capacitor.

$$Z_{EQ_{Secondary}} = (R_S + j\omega n L_M)\ //\ \frac{1}{j\omega C_S} \tag{31}$$

$$Z_{EQ_{Secondary}} = \frac{\frac{n \times L_M}{C_S} - \frac{jR_S}{\omega C_S}}{R_S + j\omega(n \times L_M - \frac{1}{\omega^2 C_S})} \tag{32}$$

The quality factor of a monolithic transformer can be computed from the complex impedance Z = R + jX, being $Q = \left|\frac{X}{R}\right|$ (eq. 26), as shown in Chapter 2.

Table 4-4 presents the quality factor of the transformers obtained in the previous section, using the components values shown in Table 4-2. The model used for the one-turn transformer did not fit to its resistance values; therefore its Q-factor was not computed.

Table 4-4: Expected Quality Factor of 2 to 4-Turn Planar Transformers with Metal 5

	Quality factor		
	2-Turn Transformer	**3-Turn Transformer**	**4-Turn Transformer**
Primary	4.3315	5.0796	5.6549
Secondary	3.6944	4.0387	4.6569

The predicted Q-factor of Planar Transformers with Metal 5 is among the typical values for monolithic transformers.

5. Conclusions and Future Work

5.1 Conclusions

This work verified that one-turn transformers are generally unfit to use. They present poor performance for all the simulated frequencies, with weak magnetic coupling and low power gain. Two-turn planar transformers show better performance, but for two turns the stronger magnetic coupling is obtained with the *Stacked* topology (in particular *Stacked II*), although with slight degradation of the gain. With three and four-turn planar transformers excellent performance can be achieved, but the use of stacked windings lowers the resonance frequency, and generally provides higher losses and lower gain.

Considering the studied geometries, this work concludes that the stacked topologies provide the highest coupling factor, because of additional inter-layer coupling. But given that these transformers also have significant losses, they generally have lower gain and lower resonance frequency than planar transformers.

In order to reduce the secondary resistance of these transformers, stacked transformers with metal 4 under the metal 5 secondary were tested. This geometry proved it can reduce ohmic losses in the secondary winding, increasing the power gain, but also providing lower coupling factor.

As mentioned before, planar transformers only have high performance when using more that 2 turns per winding. However, using a metal 5 spiral connected under a planar transformer, the designer can significantly reduce resistive losses, simultaneously introducing vertical magnetic coupling (and increasing coupling factor).

Concerning transformer dimensions, as many authors already described, the designer should use long metal segments, taking into account chip occupied area. The spacing between windings should be small. The windings should be narrow, but using small widths leads to higher ohmic losses and also low maximum current density, which can be a limiting factor in power applications.

The electrical model studied proved to be very precise for 2 to 4 turn planar transformers with metal 5. However, it did not fit the one-turn transformer. For this transformer a simpler model was presented, which proved to be accurate, although insufficient to model this transformer winding losses.

The inductance calculation software presented can provide a first guess preliminary model for symmetric transformers, without the need for electromagnetic simulation. For three and four-turn transformers the computed and the simulated mutual inductance diverge, due to significant non-idealities in current distribution not considered in the computer program.

Generally, some of the simulated monolithic transformers show high performance near 6 GHz, but none of them shows excellent behavior in frequencies around 2.4 GHZ (for WLAN applications). For instance, the four-turn planar transformer with metal 5 presents a power gain of -2.315dB at 2.4GHz, which represent losses of approximately 41%.

However, most of the transformers in this dissertation show much higher performance than those reported in earlier technologies. This suggests that evolution in Silicon RF IC technologies will soon allow the production of excellent monolithic transformers for general applications.

5.2 Future Work

The fabrication and measure of the proposed monolithic transformers, particularly the planar transformers added with a metal 5 spiral, is the last step to validate the conclusions drawn in this thesis. This was not possible to achieve in the time frame stipulated for the development of this work.

Further investigations should be done in the design of monolithic transformers. A state-of-the-art technology with lower resistivity metal layers, such as UMC 40nm (with up to 12 layers of copper), might offer high performance devices for the desired operation frequency.

Concerning the inductance calculation program, future work should be done in order to improve its performance. For instance, the processing of primary and secondary windings with different number of turns, or the computing of any possible number of turns per winding can be implemented. Also the strips ohmic losses can be calculated and as a consequence the quality factors.

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Appendix A

Source Code of the Inductance Calculation Script

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%   Inductance calculation for square   %%%
%%%%%        planar transformers        %%%%%
%%%%      based on Greenhouse method     %%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear;
clc;

%%% Please insert number of windings and
dimensions of the transformer %%%

N=3                   %   Number of primary and
secondary windigs
                      %   (balanced transformers
only, 4 turns maximum)
OUTER_DIAMETER=200  %   Total length of the
exterior windings, in um
WIDTH=5               %   Width of the
windings, in um
SPACING=1             %   Spacing between
windings, in um
METAL_THICKNESS=1   %   Thickness of the
metal strip, in um

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if N==1
    np=1;    %numero de espiras no primario
    ns=1;    %numero de espiras no secundario
    Zp=4;    %numero de troços no primario
    Zs=4;    %numero de troços no secundario
elseif N==2
    np=2;
    ns=2;
    Zp=8;
    Zs=8;
elseif N==3
    np=3;
    ns=3;
    Zp=12;
    Zs=12;
elseif N==4
    np=4;
    ns=4;
    Zp=16;
    Zs=16;
end

lp(1)=OUTER_DIAMETER;
lp(2)=OUTER_DIAMETER;
ls(1)=OUTER_DIAMETER;
ls(2)=OUTER_DIAMETER;

w=WIDTH;
t=METAL_THICKNESS;
s=SPACING;
d= s+w;

%   Calculo dos comprimentos dos troços do
primario e secundario

for y= 3:Zp,
    lp(y)=lp(y-1)-d;
end

for y= 3:Zs,
    ls(y)=ls(y-1)-d;
end

%   Calculo das indutancias proprias dos
troços do primario e secundario

for j= 1:Zp,

Lp(j)=0.0002*lp(j)*(log(2*lp(j)/(w+t))+0.5004
9+(w+t)/(3*lp(j)));
end

for j= 1:Zs,

Ls(j)=0.0002*ls(j)*(log(2*ls(j)/(w+t))+0.5004
9+(w+t)/(3*ls(j)));
end

Lp0 = sum (Lp)
Ls0 = sum (Ls)
L= cat(2, Lp, Ls);
l=cat(2,lp,ls);

%   Cálculo das distâncias entre os segmentos

if np==1        %   Transformador com 1
espira

for k=1:(Zp+Zs)/4
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= l(k+1)+d;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)=d;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
     end
 end

  for k=(Zp+Zs)/4+1:(Zp+Zs)/2
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=d;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)= l(k-1)-d;
            d_ef(j,k)=d_ef(k,j);
        end
```

```
        i=i+1;
      end
    end

    for k=(Zp+Zs)/2+1:Zp+Zs
      for j=k+2:2:Zp+Zs
          d_ef(k,j)=l(j-1);
          d_ef(j,k)=d_ef(k,j);
      end
    end

  elseif np==2    %   Transformador com 2
  espiras

      for k=1:2
       i=0;
       for j=k+2:2:Zp+Zs
          if i==0
              d_ef(k,j)=l(k+1);
              d_ef(j,k)=d_ef(k,j);
          elseif i==1
              d_ef(k,j)= 2*d;
              d_ef(j,k)=d_ef(k,j);
          elseif (i==2)
              d_ef(k,j)=l(j-1)+2*d;
              d_ef(j,k)=d_ef(k,j);
          elseif (i==3)
              d_ef(k,j)=l(k+1)+d;
              d_ef(j,k)=d_ef(k,j);
          elseif (i==4)
              d_ef(k,j)=d;
              d_ef(j,k)=d_ef(k,j);
           elseif (i==5)
              d_ef(k,j)=l(j-1)+d;
              d_ef(j,k)=d_ef(k,j);
           else
              d_ef(k,j)=3*d;
              d_ef(j,k)=d_ef(k,j);
          end
          i=i+1;
       end
      end

    for k=3:4
       i=0;
       for j=k+2:2:Zp+Zs
          if i==0
              d_ef(k,j)=l(k+1);
              d_ef(j,k)=d_ef(k,j);
          elseif i==1
              d_ef(k,j)= 2*d;
              d_ef(j,k)=d_ef(k,j);
          elseif i==2
              d_ef(k,j)= d;
              d_ef(j,k)=d_ef(k,j);
          elseif i==3
              d_ef(k,j)=l(k+1)+d ;
              d_ef(j,k)=d_ef(k,j);
          elseif i==4
              d_ef(k,j)= d;
              d_ef(j,k)=d_ef(k,j);
          else
              d_ef(k,j)= l(j-1)+d;
              d_ef(j,k)=d_ef(k,j);
          end
          i=i+1;
       end
    end

  for k=5:6
   i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= l(k-1)+d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)= d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==3
            d_ef(k,j)=l(k+1)+d ;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)= d;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
     end
   end

     for k=7:8
         i=0;
         for j=k+2:2:Zp+Zs
            if i==0
                d_ef(k,j)=3*d;
                d_ef(j,k)=d_ef(k,j);
            elseif i==1
                d_ef(k,j)= l(k-1)+d;
                d_ef(j,k)=d_ef(k,j);
            elseif i==2
                d_ef(k,j)= d;
                d_ef(j,k)=d_ef(k,j);
            else
                d_ef(k,j)=l(k-1)-d ;
                d_ef(j,k)=d_ef(k,j);
            end
            i=i+1;
         end
      end

      for k=9:10
         i=0;
         for j=k+2:2:Zp+Zs
            if i==0
                d_ef(k,j)=l(k+1);
                d_ef(j,k)=d_ef(k,j);
            elseif i==1
                d_ef(k,j)= 2*d;
                d_ef(j,k)=d_ef(k,j);
            else
                d_ef(k,j)=l(k+1)-2*d ;
                d_ef(j,k)=d_ef(k,j);
            end
            i=i+1;
         end
       end

     for k=11:12
         i=0;
         for j=k+2:2:Zp+Zs
            if i==0
                d_ef(k,j)=l(k+1);
                d_ef(j,k)=d_ef(k,j);
            else
                d_ef(k,j)= 2*d;
                d_ef(j,k)=d_ef(k,j);
            end
            i=i+1;
         end
       end
```

```
  for k=13:14
    for j=k+2:2:Zp+Zs
        d_ef(k,j)=l(j-1);
        d_ef(j,k)=d_ef(k,j);
     end
  end

elseif np==3    %   Transformador com 3
espiras

    for k=1:2
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif (i==2)
            d_ef(k,j)=l(j-1)+2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif (i==3)
            d_ef(k,j)=4*d;
            d_ef(j,k)=d_ef(k,j);
        elseif (i==4)
            d_ef(k,j)=l(j-1)+4*d;
            d_ef(j,k)=d_ef(k,j);
         elseif (i==5)
            d_ef(k,j)=l(k+1)+d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==6
            d_ef(k,j)=d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==7
            d_ef(k,j)=l(j-1)+d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==8
            d_ef(k,j)=3*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==9
            d_ef(k,j)=l(j-1)+3*d;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)=5*d;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
     end
 end

  for k=3:4
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)= l(j-1)+2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==3
            d_ef(k,j)=4*d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==4||i==6
            d_ef(k,j)= d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==5
            d_ef(k,j)= l(k+1)+d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==7
            d_ef(k,j)= l(j-1)+d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==8
            d_ef(k,j)= 3*d;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)= l(j-1)+3*d;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
     end
  end

for k=5:6
 i=0;
 for j=k+2:2:Zp+Zs
    if i==0
        d_ef(k,j)=l(k+1);
        d_ef(j,k)=d_ef(k,j);
    elseif i==1
        d_ef(k,j)= 2*d;
        d_ef(j,k)=d_ef(k,j);
    elseif i==2
        d_ef(k,j)= l(j-1)+2*d;
        d_ef(j,k)=d_ef(k,j);
    elseif i==3
        d_ef(k,j)=l(k-1)+d ;
        d_ef(j,k)=d_ef(k,j);
    elseif i==4||i==6
        d_ef(k,j)= d;
        d_ef(j,k)=d_ef(k,j);
    elseif i==5
        d_ef(k,j)= l(k+1)+d;
        d_ef(j,k)=d_ef(k,j);
    elseif i==7
        d_ef(k,j)= l(j-1)+d;
        d_ef(j,k)=d_ef(k,j);
    else
        d_ef(k,j)= 3*d;
        d_ef(j,k)=d_ef(k,j);
    end
    i=i+1;
 end
end

 for k=7:8
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)= 3*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==3
            d_ef(k,j)=l(k-1)+d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==4||i==6
            d_ef(k,j)=d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==5
            d_ef(k,j)=l(k+1)+d ;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)=l(j-1)+d ;
```

```
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
    end
 end

 for k=9:10
    i=0;
    for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= l(k-1)+3*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)=3*d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==3
            d_ef(k,j)=l(k-1)+d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==4||i==6
            d_ef(k,j)=d ;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)=l(k+1)+d ;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
    end
  end

for k=11:12
    i=0;
    for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=5*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= l(k-1)+3*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)=3*d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==3
            d_ef(k,j)=l(k-1)+d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==4
            d_ef(k,j)=d ;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)=l(k-1)-d ;
            d_ef(j,k)=d_ef(k,j);
        end
    i=i+1;
    end
end

 for k=13:14
    i=0;
    for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)=2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)=l(j-1)+2*d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==3
            d_ef(k,j)=4*d ;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)=l(j-1)+4*d ;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
    end
 end

for k=15:16
    i=0;
    for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)=l(j-1)+2*d ;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)=4*d ;
            d_ef(j,k)=d_ef(k,j);
        end
     i=i+1;
    end
end

 for k=17:18
    i=0;
    for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)=l(j-1)+2*d ;
            d_ef(j,k)=d_ef(k,j);
        end
    i=i+1;
    end
 end

  for k=19:20
    i=0;
    for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        end
    i=i+1;
    end
  end

  for k=21:22
    i=0;
    for j=k+2:2:Zp+Zs
        d_ef(k,j)=l(k+1);
        d_ef(j,k)=d_ef(k,j);
    end
    i=i+1;
  end
```

```
 elseif np==4    %   Transformador com 4
espiras

    for k=1:2
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif (i==2)
            d_ef(k,j)=l(j-1)+2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif (i==3)
            d_ef(k,j)=4*d;
            d_ef(j,k)=d_ef(k,j);
        elseif (i==4)
            d_ef(k,j)=l(j-1)+4*d;
            d_ef(j,k)=d_ef(k,j);
        elseif (i==5)
            d_ef(k,j)=6*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==6
            d_ef(k,j)=l(j-1)+6*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==7
            d_ef(k,j)=l(k+1)+d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==8
            d_ef(k,j)=d;
            d_ef(j,k)=d_ef(k,j);
         elseif i==9
            d_ef(k,j)=l(j-1)+d;
            d_ef(j,k)=d_ef(k,j);
         elseif i==10
            d_ef(k,j)=3*d;
            d_ef(j,k)=d_ef(k,j);
         elseif i==11
            d_ef(k,j)=l(j-1)+3*d ;
            d_ef(j,k)=d_ef(k,j);
          elseif i==12
            d_ef(k,j)=5*d;
            d_ef(j,k)=d_ef(k,j);
         elseif i==13
            d_ef(k,j)=l(j-1)+5*d;
            d_ef(j,k)=d_ef(k,j);
         else
            d_ef(k,j)=7*d ;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
     end
 end

 for k=3:4
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)=l(j-1)+2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==3
            d_ef(k,j)=4*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==4
            d_ef(k,j)=l(j-1)+4*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==5
            d_ef(k,j)=6*d;
            d_ef(j,k)=d_ef(k,j);
         elseif i==6 ||i==8
            d_ef(k,j)= d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==7
            d_ef(k,j)=l(k+1)+d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==9
            d_ef(k,j)=l(j-1)+d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==10
            d_ef(k,j)=3*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==11
            d_ef(k,j)=l(j-1)+3*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==12
            d_ef(k,j)=5*d;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)=l(j-1)+5*d;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
     end
 end

 for k=5:6
  i=0;
  for j=k+2:2:Zp+Zs
     if i==0
         d_ef(k,j)=l(k+1);
         d_ef(j,k)=d_ef(k,j);
     elseif i==1
         d_ef(k,j)=2*d;
         d_ef(j,k)=d_ef(k,j);
     elseif i==2
         d_ef(k,j)=l(j-1)+2*d;
         d_ef(j,k)=d_ef(k,j);
     elseif i==3
         d_ef(k,j)=4*d;
         d_ef(j,k)=d_ef(k,j);
     elseif i==4
         d_ef(k,j)=l(j-1)+4*d;
         d_ef(j,k)=d_ef(k,j);
      elseif i==5
         d_ef(k,j)=l(k+1)+3*d;
         d_ef(j,k)=d_ef(k,j);
     elseif i==6||i==8
         d_ef(k,j)=d;
         d_ef(j,k)=d_ef(k,j);
     elseif i==7
         d_ef(k,j)=l(k+1)+d;
         d_ef(j,k)=d_ef(k,j);
     elseif i==9
         d_ef(k,j)=l(j-1)+d;
         d_ef(j,k)=d_ef(k,j);
     elseif i==10
         d_ef(k,j)=3*d;
         d_ef(j,k)=d_ef(k,j);
     elseif i==11
         d_ef(k,j)=l(j-1)+3*d;
         d_ef(j,k)=d_ef(k,j);
     else
         d_ef(k,j)=5*d;
         d_ef(j,k)=d_ef(k,j);
```

```
    end
    i=i+1;
 end
end

for k=7:8
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)= l(j-1)+2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==3
            d_ef(k,j)=4*d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==4||i==10
            d_ef(k,j)=3*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==5
            d_ef(k,j)=l(k+1)+3*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==6||i==8
            d_ef(k,j)=d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==7
            d_ef(k,j)=l(j+1)+d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==9
            d_ef(k,j)=l(j-1)+d;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)=l(j-1)+3*d;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
     end
  end

  for k=9:10
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)=l(j-1)+2*d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==3
            d_ef(k,j)= l(k-1)+3*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==4||10
            d_ef(k,j)=3*d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==5
            d_ef(k,j)=l(k-1)+d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==6||i==8
            d_ef(k,j)= d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==7
            d_ef(k,j)=l(k+1)+d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==9
            d_ef(k,j)=l(j-1)+d ;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
     end
  end

 for k=11:12
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)=5*d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==3
            d_ef(k,j)= l(k-1)+3*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==4
            d_ef(k,j)=3*d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==5
            d_ef(k,j)=l(k-1)+d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==6||i==8
            d_ef(k,j)= d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==7
            d_ef(k,j)=l(j+1)+d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==9
            d_ef(k,j)=l(j-1)+d ;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
     end
  end

  for k=13:14
    i=0;
    for j=k+2:2:Zp+Zs
        if i==0
             d_ef(k,j)=l(j-1);
             d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= l(k-1)+5*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)=5*d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==3
            d_ef(k,j)= l(k-1)+3*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==4
            d_ef(k,j)=3*d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==5
            d_ef(k,j)=l(k-1)+d ;
            d_ef(j,k)=d_ef(k,j);
        elseif i==6||i==8
            d_ef(k,j)= d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==7
            d_ef(k,j)=l(j+1)+d ;
            d_ef(j,k)=d_ef(k,j);
        end
    i=i+1;
```

```
   end
end

  for k=15:16
   i=0;
   for j=k+2:2:Zp+Zs
       if i==0
            d_ef(k,j)=7*d;
            d_ef(j,k)=d_ef(k,j);
       elseif i==1
           d_ef(k,j)= l(k-1)+5*d;
           d_ef(j,k)=d_ef(k,j);
       elseif i==2
           d_ef(k,j)=5*d ;
           d_ef(j,k)=d_ef(k,j);
       elseif i==3
           d_ef(k,j)= l(k-1)+3*d;
           d_ef(j,k)=d_ef(k,j);
       elseif i==4
           d_ef(k,j)=3*d ;
           d_ef(j,k)=d_ef(k,j);
       elseif i==5
           d_ef(k,j)=l(k-1)+d ;
           d_ef(j,k)=d_ef(k,j);
       elseif i==6
           d_ef(k,j)= d;
           d_ef(j,k)=d_ef(k,j);
       else
           d_ef(k,j)=l(k-1)-d ;
           d_ef(j,k)=d_ef(k,j);
       end
       i=i+1;
   end
  end

 for k=17:18
    i=0;
    for j=k+2:2:Zp+Zs
       if i==0
           d_ef(k,j)=l(k+1);
           d_ef(j,k)=d_ef(k,j);
       elseif i==1
           d_ef(k,j)= 2*d;
           d_ef(j,k)=d_ef(k,j);
       elseif (i==2)
           d_ef(k,j)=l(j-1)+2*d;
           d_ef(j,k)=d_ef(k,j);
       elseif (i==3)
           d_ef(k,j)=4*d;
           d_ef(j,k)=d_ef(k,j);
       elseif (i==4)
           d_ef(k,j)=l(j-1)+4*d;
           d_ef(j,k)=d_ef(k,j);
        elseif (i==5)
           d_ef(k,j)=6*d;
           d_ef(j,k)=d_ef(k,j);
        else
           d_ef(k,j)=l(j-1)+6*d;
           d_ef(j,k)=d_ef(k,j);
       end
       i=i+1;
    end
end

 for k=19:20
    i=0;
    for j=k+2:2:Zp+Zs
       if i==0
           d_ef(k,j)=l(k+1);
           d_ef(j,k)=d_ef(k,j);
       elseif i==1
           d_ef(k,j)= 2*d;
           d_ef(j,k)=d_ef(k,j);
       elseif i==2
           d_ef(k,j)=l(j-1)+2*d;
           d_ef(j,k)=d_ef(k,j);
       elseif i==3
           d_ef(k,j)=4*d;
           d_ef(j,k)=d_ef(k,j);
       elseif i==4
           d_ef(k,j)=l(j-1)+4*d;
           d_ef(j,k)=d_ef(k,j);
       else
           d_ef(k,j)=6*d;
           d_ef(j,k)=d_ef(k,j);
       end
       i=i+1;
    end
  end

for k=21:22
 i=0;
 for j=k+2:2:Zp+Zs
    if i==0
        d_ef(k,j)=l(k+1);
        d_ef(j,k)=d_ef(k,j);
    elseif i==1
        d_ef(k,j)=2*d;
        d_ef(j,k)=d_ef(k,j);
    elseif i==2
        d_ef(k,j)=l(j-1)+2*d;
        d_ef(j,k)=d_ef(k,j);
    elseif i==3
        d_ef(k,j)=4*d;
        d_ef(j,k)=d_ef(k,j);
    else
        d_ef(k,j)=l(j-1)+4*d;
        d_ef(j,k)=d_ef(k,j);
    end
    i=i+1;
 end
end

  for k=23:24
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        elseif i==2
            d_ef(k,j)= l(j-1)+2*d;
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)=4*d ;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
     end
  end

  for k=25:26
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        elseif i==1
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
```

```
        else
            d_ef(k,j)=l(j-1)+2*d ;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
     end
   end

 for k=27:28
     i=0;
     for j=k+2:2:Zp+Zs
        if i==0
            d_ef(k,j)=l(k+1);
            d_ef(j,k)=d_ef(k,j);
        else
            d_ef(k,j)= 2*d;
            d_ef(j,k)=d_ef(k,j);
        end
        i=i+1;
     end
   end

  for k=29:30
    for j=k+2:2:Zp+Zs
        d_ef(k,j)=l(j-1);
        d_ef(j,k)=d_ef(k,j);
     end
  end
end

%   Calculo da indutancia mutua entre todos
os segmentos

for k=1:Zp+Zs
    for j=k+2:2:Zp+Zs
        if l(k)<=l(j)

M(k,j)=0.0002*l(k)*log(l(k)/d_ef(k,j) +
sqrt(1+(l(k)/d_ef(k,j))^2)+d_ef(k,j)/l(k)-
sqrt(1+(l(k)/d_ef(k,j))^2));
        else

M(k,j)=0.0002*l(j)*log(l(j)/d_ef(k,j) +
sqrt(1+(l(j)/d_ef(k,j))^2)+d_ef(k,j)/l(j)-
sqrt(1+(l(j)/d_ef(k,j))^2));
        end
    end
end

%   Cálculo da indutância total

if np==1     %   Transformador com 1 espira

    %   Primário
    Mneg=0;
    Mpos=0;
    for k=1:Zp
        for j=(k+2):2:Zp
           Mneg=Mneg+M(k,j);
        end
    end

    Mo = 2*(Mpos-Mneg)
    Lprim = Lp0 + Mo

    %   Secundário
    Mneg=0;
    Mpos=0;
    for k=Zp+1:Zp+Zs
        for j=(k+2):2:Zp+Zs
           Mneg=Mneg+M(k,j);
        end
    end

    Mo = 2*(Mpos-Mneg)
    Lsec = Ls0 + Mo

    %   Indutancia Mutua entre Primario e
Secundario

    Mneg=0;
    for k=1:(Zp+Zs)/4
        for j=(k+2):2:Zp+Zs-2
           Mneg=Mneg+M(k,j);
        end
    end

    Mpos=0;
    for k=1:(Zp+Zs)/4
        for j=k+6:6:Zp+Zs
           Mpos=Mpos+M(k,j);
        end
    end

    for k=(Zp+Zs)/4+1:(Zp+Zs)/2
        for j=k+2:4:Zp+Zs
           Mpos=Mpos+M(k,j);
        end
    end

    for k=(Zp+Zs)/4+1:(Zp+Zs)/2
        for j=k+4:4:Zp+Zs
           Mneg=Mneg+M(k,j);

        end
    end

    for k=(Zp+Zs)/2+1:Zp+Zs
        for j=k+2:2:Zp+Zs
           Mneg=Mneg+M(k,j);
        end
    end

    LM = Mpos-Mneg

elseif np==2     %   Transformador com 2
espiras

    %   Primário
    Mneg=0;
    Mpos=0;

    for k=1:Zp
        for j=(k+4):4:Zp
           Mpos=Mpos+M(k,j);
        end
    end

    for k=1:Zp
       i=0;
        for j=(k+2):2:Zp
           if i==0||i==2
            Mneg=Mneg+M(k,j);
           end
           i=i+1;
        end
    end

    Mo = 2*(Mpos-Mneg)
    Lprim = Lp0 + Mo

        %   Secundário
```

```
    Mneg=0;
    Mpos=0;

    for k=Zp+1:Zp+Zs
        for j=(k+4):4:Zp+Zs
           Mpos=Mpos+M(k,j);
        end
    end

    for k=Zp+1:Zp+Zs
    i=0;
        for j=(k+2):2:Zp+Zs
           if i==0||i==2
            Mneg=Mneg+M(k,j);
           end
           i=i+1;
        end
    end

    Mo = 2*(Mpos-Mneg)
    Lsec = Lp0 + Mo

    %  Indutancia Mutua entre Primario e
Secundario
    Mneg=0;
    Mpos=0;

    for k=1:2
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0||i==2||i==3||i==5
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=3:4
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0||i==3||i==5
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=5:6
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0||i==1||i==3
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=7:8
        i=0;
        for j=k+2:2:Zp+Zs
           if i==1||i==3
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=9:10
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0||i==2
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=11:12
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=13:14
        for j=k+2:2:Zp+Zs
           Mneg=Mneg+M(k,j);
        end
    end

    LM = Mpos - Mneg

elseif np==3   %  Transformador com 3
espiras

    %  Primário
    Mneg=0;
    Mpos=0;

    for k=1:Zp
        for j=(k+4):4:Zp
           Mpos=Mpos+M(k,j);
        end
    end

    for k=1:Zp
        i=0;
        for j=(k+2):2:Zp
           if i==0||i==2||i==4
            Mneg=Mneg+M(k,j);
           end
           i=i+1;
        end
    end

    Mo = 2*(Mpos-Mneg)
    Lprim = Lp0 + Mo

        %  Secundário
    Mneg=0;
    Mpos=0;

    for k=Zp+1:Zp+Zs
        for j=(k+4):4:Zp+Zs
           Mpos=Mpos+M(k,j);
        end
    end
```

```
    for k=Zp+1:Zp+Zs
        i=0;
        for j=(k+2):2:Zp+Zs
           if i==0||i==2||i==4
            Mneg=Mneg+M(k,j);
           end
           i=i+1;
        end
    end

    Mo = 2*(Mpos-Mneg)
    Lsec = Lp0 + Mo

    %   Indutancia Mutua entre Primario e
Secundario

    Mneg=0;
    Mpos=0;

    for k=1:2
        i=0;
        for j=k+2:2:Zp+Zs
           if
i==0||i==2||i==4||i==5||i==7||i==9
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=3:4
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0||i==2||i==5||i==7||i==9
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=5:6
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0||i==2||i==3||i==5||i==7
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=7:8
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0||i==3||i==5||i==7
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=9:10
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0||i==1||i==3||i==5
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=11:12
        i=0;
        for j=k+2:2:Zp+Zs
           if i==1||i==3||i==5
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=13:14
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0||i==2||i==4
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=15:16
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0||i==2
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=17:18
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0||i==2
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=19:20
        i=0;
        for j=k+2:2:Zp+Zs
           if i==0
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=21:22
```

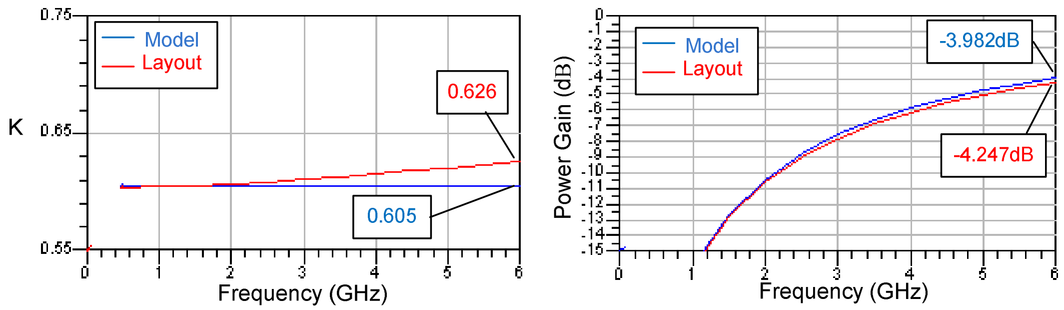
```
        for j=k+2:2:Zp+Zs
           Mneg=Mneg+M(k,j);
        end
    end

    LM = Mpos - Mneg


elseif np==4    %   Transformador com 4
espiras

    %   Primário
    Mneg=0;
    Mpos=0;

    for k=1:Zp
        for j=(k+4):4:Zp
           Mpos=Mpos+M(k,j);
        end
    end

    for k=1:Zp
        i=0;
        for j=(k+2):2:Zp
           if i==0||i==2||i==4||i==6
            Mneg=Mneg+M(k,j);
           end
           i=i+1;
        end
    end

    Mo = 2*(Mpos-Mneg)
    Lprim = Lp0 + Mo

        %   Secundário
    Mneg=0;
    Mpos=0;

    for k=Zp+1:Zp+Zs
        for j=(k+4):4:Zp+Zs
           Mpos=Mpos+M(k,j);
        end
    end

    for k=Zp+1:Zp+Zs
        i=0;
        for j=(k+2):2:Zp+Zs
           if i==0||i==2||i==4||i==6
            Mneg=Mneg+M(k,j);
           end
           i=i+1;
        end
    end

    Mo = 2*(Mpos-Mneg)
    Lsec = Lp0 + Mo

     %   Indutancia Mutua entre Primario e
Secundario

    Mneg=0;
    Mpos=0;

    for k=1:2
        i=0;
        for j=k+2:2:Zp+Zs
           if
i==0||i==2||i==4||i==6||i==7||i==9||i==11||i=
=13
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=3:4
        i=0;
        for j=k+2:2:Zp+Zs
           if
i==0||i==2||i==4||i==7||i==9||i==11||i==13
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=5:6
        i=0;
        for j=k+2:2:Zp+Zs
           if
i==0||i==2||i==4||i==5||i==7||i==9||i==11
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=7:8
        i=0;
        for j=k+2:2:Zp+Zs
           if
i==0||i==2||i==5||i==7||i==9||i==11
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=9:10
        i=0;
        for j=k+2:2:Zp+Zs
           if
i==0||i==2||i==3||i==5||i==7||i==9
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=11:12
        i=0;
        for j=k+2:2:Zp+Zs
           if i==1||i==3||i==5||i==7||i==9
             Mneg=Mneg+M(k,j);
           else
             Mpos=Mpos+M(k,j);
           end
        i=i+1;
        end
    end

    for k=13:14
```

```
      i=0;
      for j=k+2:2:Zp+Zs
         if i==0||i==1||i==3||i==5||i==7
           Mneg=Mneg+M(k,j);
         else
           Mpos=Mpos+M(k,j);
         end
      i=i+1;
      end
 end

for k=15:16
      i=0;
      for j=k+2:2:Zp+Zs
         if i==1||i==3||i==5||i==7
           Mneg=Mneg+M(k,j);
         else
           Mpos=Mpos+M(k,j);
         end
      i=i+1;
      end
end

for k=17:18
      i=0;
      for j=k+2:2:Zp+Zs
         if i==0||i==2||i==4||i==6
           Mneg=Mneg+M(k,j);
         else
           Mpos=Mpos+M(k,j);
         end
      i=i+1;
      end
 end

for k=19:20
      i=0;
      for j=k+2:2:Zp+Zs
         if i==0||i==2||i==4
           Mneg=Mneg+M(k,j);
         else
           Mpos=Mpos+M(k,j);
         end
      i=i+1;
      end
end

for k=21:22
     i=0;
      for j=k+2:2:Zp+Zs
         if i==0||i==2||i==4
           Mneg=Mneg+M(k,j);
         else
           Mpos=Mpos+M(k,j);
         end
      i=i+1;
      end
end

for k=23:24
     i=0;
      for j=k+2:2:Zp+Zs
         if i==0||i==2
           Mneg=Mneg+M(k,j);
         else
           Mpos=Mpos+M(k,j);
         end
      i=i+1;
      end
end

   for k=25:26
       i=0;
       for j=k+2:2:Zp+Zs
          if i==0||i==2
            Mneg=Mneg+M(k,j);
          else
            Mpos=Mpos+M(k,j);
          end
       i=i+1;
       end
   end

   for k=27:28
       i=0;
       for j=k+2:2:Zp+Zs
          if i==0
            Mneg=Mneg+M(k,j);
          else
            Mpos=Mpos+M(k,j);
          end
       i=i+1;
       end
   end

  for k=29:30
     for j=k+2:2:Zp+Zs
       Mneg=Mneg+M(k,j);
     end
  end

   LM = Mpos - Mneg

end
```

Appendix B

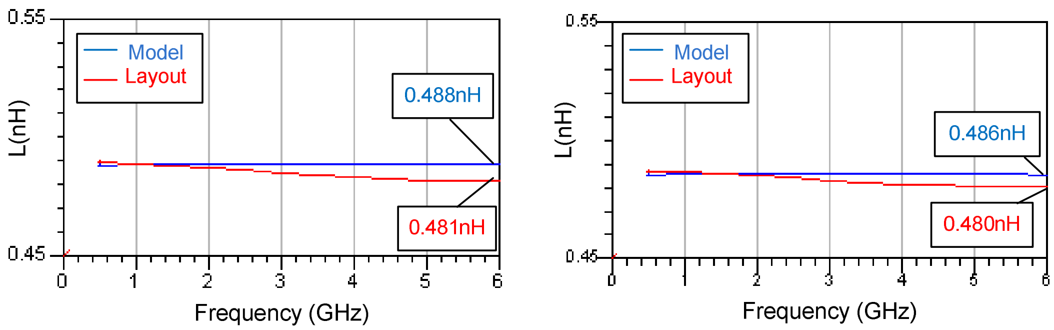
This section presents the behavior of some components from the models discussed in Chapter 4, from one to six gigahertz, comparing it to the simulations results of the Planar Transformers with a Metal 5 spiral underneath. The highlighted values in the figures emphasize the point of maximum difference between the proposed model and the simulated layout.

- **One-turn Transformer**



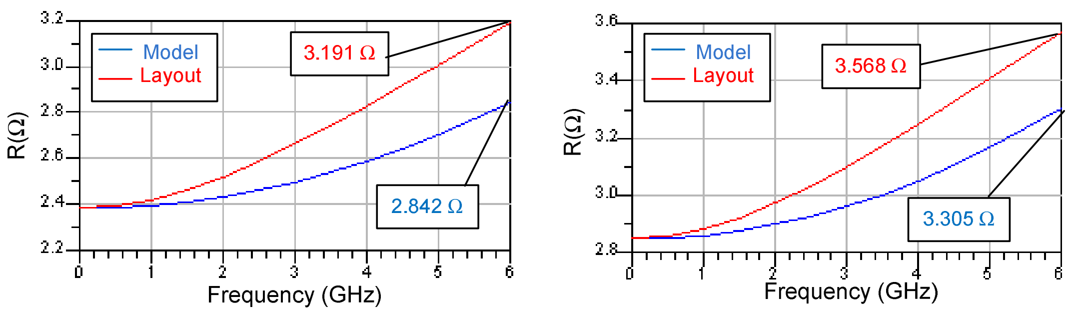
(a) (b)

Figure B-1: (a) Coupling Factor; (b) Power Gain



(a) (b)

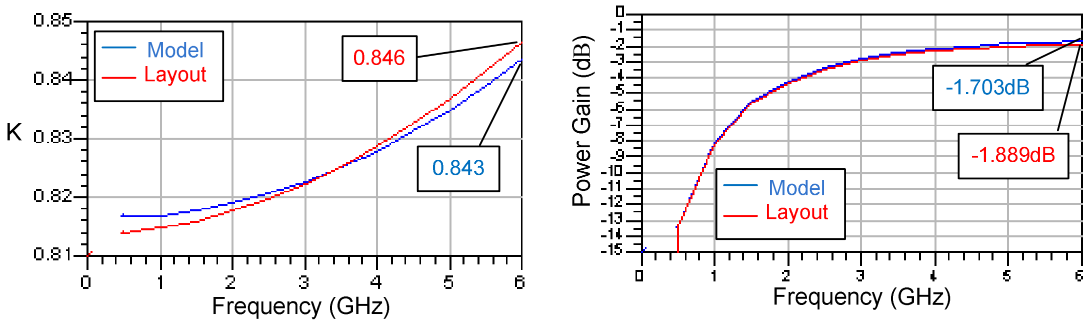
Figure B-2: (a) Primary Self-Inductance; (b) Secondary Self-Inductance



(a) (b)

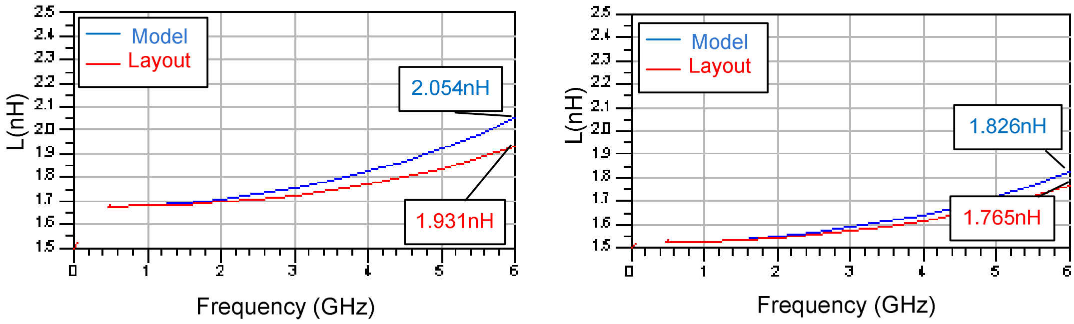
Figure B-3: (a) Primary Resistance; (b) Secondary Resistance

- **Two-turns Transformer**



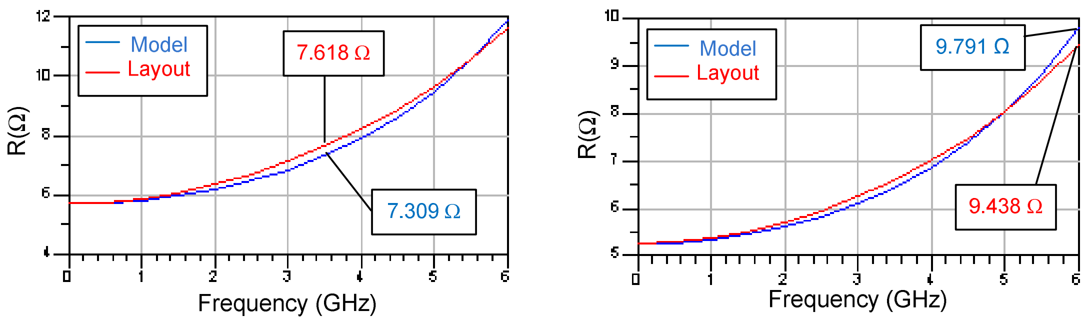
(a) (b)

Figure B-4: (a) Coupling Factor; (b) Power Gain



(a) (b)

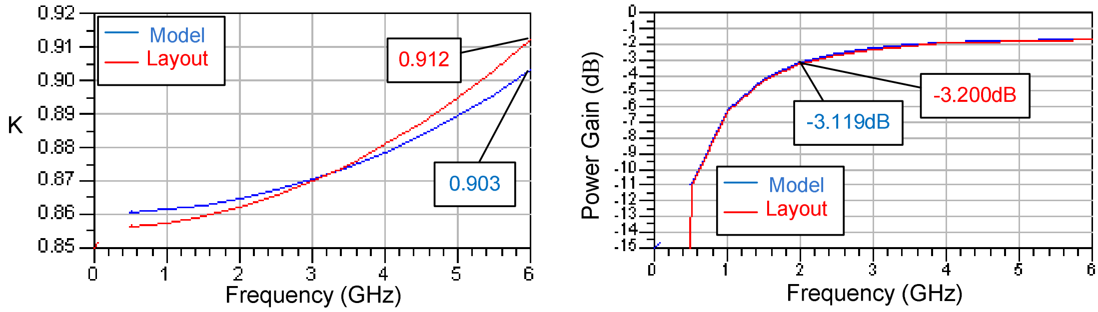
Figure B-5: (a) Primary Self-Inductance; (b) Secondary Self-Inductance



(a) (b)

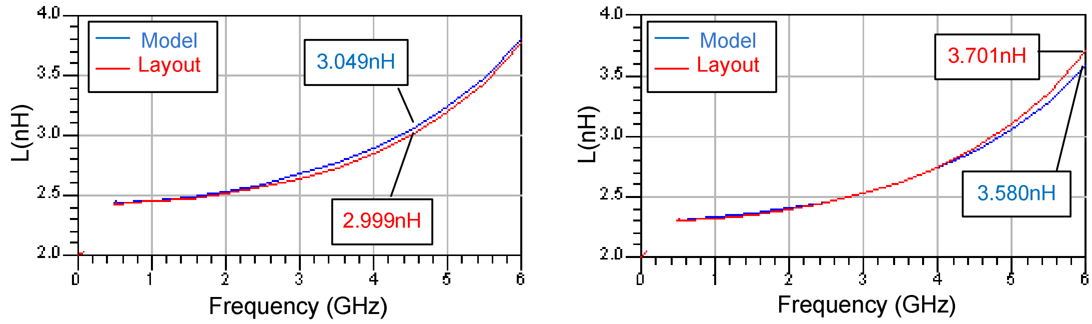
Figure B-6: (a) Primary Resistance; (b) Secondary Resistance

- **Three-turns Transformer**



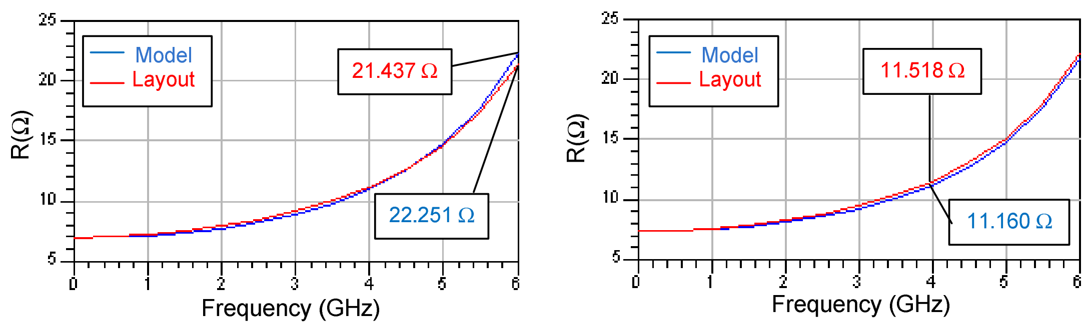
(a) (b)

Figure B-7: (a) Coupling Factor; (b) Power Gain



(a) (b)

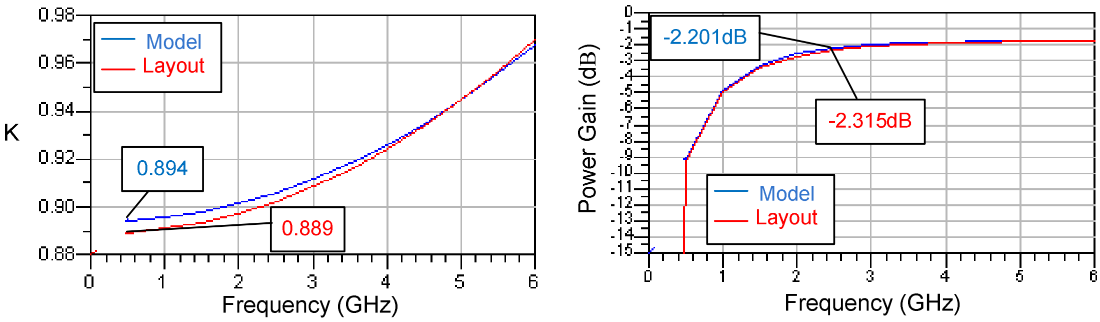
Figure B-8: (a) Primary Self-Inductance; (b) Secondary Self-Inductance



(a) (b)

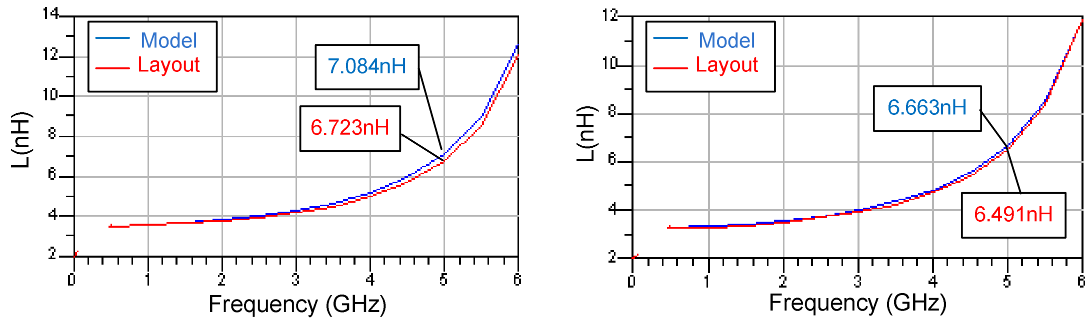
Figure B-9: (a) Primary Resistance; (b) Secondary Resistance

- **Four-turns Transformer**



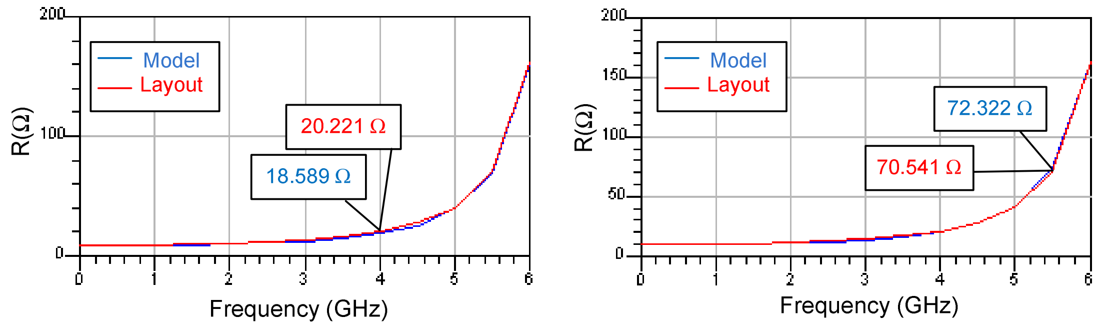
(a) (b)

Figure B-10: (a) Coupling Factor; (b) Power Gain



(a) (b)

Figure B-11: (a) Primary Self-Inductance; (b) Secondary Self-Inductance



(a) (b)

Figure B-12: (a) Primary Resistance; (b) Secondary Resistance