

## **URBAN ENERGY SUPPLY CHAIN**

A DISTRICT HEATING CASE

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#### Abstract

Energy has become one of the most crucial problems in the modern society which must be studied and solved. In this work, we present a model that optimizes an urban energy supply chain. This study will not focus on a specific area of the supply chain such as production, distribution or storage but aims to develop quantitative tools that can support strategic decisions in heating supply chain design and operation. The suggested mathematical model delineates the optimal configuration of a heating supply chain to fulfil the demand within one region (e. g. city). The decisions to be determined, by this model, were: the location, number and capacity of the boilers to be installed; the required production of each boiler and the network configuration (location of pipelines); flow rate of energy throughout the network and its direction as well as the total cost of the heating supply chain network. Moreover, the developed model is mainly driven by heating demand, which is assumed to be previously known. Illustrative examples were solved.

Keywords: optimisation model, natural gas, urban energy supply chain, design, consumption.

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## **Chapter 1**

"Houston, we have a problem ... "

James A. Lovell

## **1** Introduction

Nowadays we can say 'World, we have a problem'. Energy becomes one of the most crucial problems which must be studied and solved. It is not only a social or economical problem, or a personal or country problem, energy is a vital problem for everybody, in every community, in every business. Energy is everywhere in our lives, and as a human being, man is not prepare to live without it. Energy is present in transport, in industry, in our houses, in farming, so we can say that energy is the centre of life and activity in all societies, not from yesterday, but from a long time ago. So it could be strange why it is necessary to worry about this subject only now that we are on 21<sup>st</sup> century where everything is possible and almost nothing will be certain. Even knowing that change is constant and inevitable, sometimes what is used today can not exist tomorrow and in my opinion, energy has become a challenge, instead of a catastrophe.

Below are the reasons that explain the importance of energy decision makers.

- The threat of a disruptive climate change: along with its ecological and economic impacts, there are two things that everybody should be concerned with. Continuous increase of industrial greenhouse gases in the atmosphere is one subject to think about, which contributes to the greenhouse effect. The other point to focus on is the gradual increase of global temperatures, also called human-caused climate change. This increase can cause other changes, including sea level rise, extreme weather events, species extinctions and glacier retreat;
- The erosion of energy security: The ability to access resources for natural power and to pursue foreign policy without coercion, and avoidance of economic and environmental injury is become a threat for all nations, due to coalitions fracturing, loss of global influence, erosion of oil market and collective energy security system;
- The growing energy needs of the developing world: following the United Nations projections, the worlds' urban population is growing about three times the increase of rural population. According to estimates, in urban areas population will grow around 90% in the current century, which means that direct energy use will also increase (around 70%).

Changing the present, thinking about the future, is needed to ensure the continuity of healthy life on earth. Avoiding this bad news is crucial and two things can be done: innovation and adoption of new energy technologies or better use of existing technologies.

## 1.1 Motivation and objectives

Now that the problem is identified, it is time to face it and try to find optimal solutions for it. As explained before, there are two possible ways where researches should concentrate on:

- Innovation and adoption of new technologies;
- Better use of existing technologies.

The first option requires the discovery of a new technology and its adoption. Sometimes there is a lag between these two fields due to the uncertainty about the nature of technological change. Several studies have been done in this area, and the use of renewable energies has been promoted, not only in Europe, but all over the world. Nowadays renewable energy technologies such as wind turbines, solar photovoltaic panels, biomass power plants and solar thermal collectors contribute to 3% of the global power production and could by 2030 potentially grow to 16% or more. Due to decades of technological progress the use of renewable energies is not a dream for the future, it is real. Sustainable energy systems produce less carbon emissions, are cheaper and involve less dependence on imported fuel. So make the transition to renewable energy is urgent and it will permit a clean and sustainable growth. Although the benefice of this first option this work motivation is related to the other option - better use of existing technologies.

Better use of existing technologies is the second point and the one that we will focus on. Fossil fuels, including petroleum, natural gas and coal, are the only energy sources of sufficient scale to fulfil the worlds need. Natural gas is the chosen fuel to satisfy the heat demand in this study.

The aim of this work is to define an urban energy supply chain network. The objective is to optimize the network design by reducing the total costs. Most of the reviews in the area of fossil fuels emphasise the production method, trying to improve the efficiency of the process, and the decrease of gases emissions, such as carbon dioxide, methane and other greenhouse gases. Other studies focus on studying individual components of the supply chain, like production, transportation and storage.

This project will focus on all components of the supply chain, and a mathematical model to determine the design of an urban energy supply chain network is formulated. The optimal solutions tend to be a representation of a real solution although virtual data is used.

## **1.2 Thesis structure**

This thesis involves seven chapters. Chapter 1 is the introduction where the problem is introduced. Chapter 2 contains the literature review, where a brief background on energy, natural gas history and supply chain is presented. It also explains the heat network design and what has been already done on this subject. Here some papers and previous work that were useful for the development of this project are mentioned.

The problem is described in Chapter 3. Is also introduced in this chapter the energy supply chain network formulation, which is a mixed-integer linear programming (MILP) model. The model structure and formulation, with nomenclature, constraints and objective function is presented and explained.

Chapter 4 extends the model presented in Chapter 3 to three different single scale cases. Different population distributions are studied and results are analysed and compared. The population distribution is modified to see the energy network changes within different situations. These distributions: uniform, 'normal' and random, are study to approximate the case to real situations.

In Chapter 5 the multi-scale network design is studied and reviewed. The optimization model explained in Chapter 3 is applied to a specific cell, using multi-scale method. Results are again analysed and discussed.

The last case study is discussed in Chapter 6, and once again a MILP model is used to minimize the objective function. Here the aim is to show the importance of capital and operational costs and its influence in the objective function. Differences between this case and cases above are mentioned.

Finally, Chapter 7 is the last but not the least part of the work. Chapter 7 concludes this thesis by summarising the work that has been done and outlining some possible future research directions.

## **Chapter 2**

## 2 Related Work

## 2.1 Energy

Energy is one of the most complex problems in the modern society that have to be faced nowadays. Energy can be seen as a challenge, not a community or country challenge, but a "world challenge". The European Union and the world in general are at a cross-road concerning the future of energy. Some factors, like climate change, growing imports, increasing dependence on oil and other fossil fuels, and rising energy costs, are making our societies and economies vulnerable. As human beings we all depend of energy, it is essential to live, and the minimization of the consumption of energy will be the "challenge" of this work. It is important to understand the real meaning of energy and how it will be study. The development of this subject is necessary to improve the quality of life and opportunities in developed and developing nations.

Every concept, scientific word or term in history, had to have an origin, and 'energy' is not an exception, it was born in 1807 by Thomas Young. The concept was simple: every object, like tables, iron, robber, even volumes of water and wind, has energy, or "work-stored-within", in one form or another. Nevertheless energy existed before 1800, it is important to remember that nature does not change, only our ideas about it. Energy was not discovered or invented; it has been on earth since earth was born.

Currently the high demand for energy made by rich nations is causing concern. The present and future demands of industrial societies cannot be sustained for more than a decade or two. That is why the development of new technologies is so important. To enable resources that are now of little importance and not in use to replace those that people are using up, like non-renewable resources, is one goal to achieve as soon as possible. The contribution of renewable energies should increase in absolute energy terms. Since 1997 the European Union is working towards a target of a 12% share of renewable energy in gross inland consumption by 2010. Reaching the target will decrease greenhouse emissions, reduce annual fossil fuel consumption and encourage new technologies in European industries. Unfortunately the current projections indicate that this target will not be met.

In some cases fossil fuels compared to renewable sources have an economically unjustified advantaged due to the failure of systemically include external costs in market prices, otherwise renewable energy sources would be more able to compete with conventional fuels. There are all factors to analyse the cost of energies: the resource base and the technologies concerned;

even though the decline of the cost of most renewable energy sources over the last 20 years, the cost of renewable energies still exceeds that of conventional energy source at present. Nonrenewable energy source continues to be the 'best' economical choice according to the energy market price signals. There are other important reasons why the European Union looks unlikely to reach the goal of 12% share, like the numerous administrative problems resulting from the complexity, novelty and decentralised nature of most renewable energy applications, the low energy efficiency that has not been as high as expected and the high energy consumption.

## 2.2 Natural Gas History

At the same time that new technologies are in development, improvement of the use of nonrenewable energies is another possibility to achieve the target instead of increase the use of renewable energies. Natural gas, a gaseous fossil fuel, will be the key word of this project, a non-renewable resource. In a short period of time, approximately one and a half centuries, the energy system has moved gradually from solid fuels to liquid fuels then to gas fuels. The most used fuel nowadays, oil, is being challenged by natural gas and hydrogen.

Until the mid 19<sup>th</sup> century wood was the most common source of energy, however the sudden increase in demand for energy brings the rise of an alternative fuel, coal. Bulkiness, inefficiency and difficulty to transport are some of the reasons for this change. By the middle of the 20<sup>th</sup> century another fuel appears oil, a liquid fuel with higher energy density, a lower carbon-to-hydrogen ratio and the ability to flow through pipelines and into tanks. All of these reasons make it the world's leading energy source. In 1973 another alteration, after the oil crisis oil slowly started becoming less dominant and the market for natural gas kicked off.

Natural gas, also named Marsh gas and Swamp gas, is a very clean, safe, light and one of the most useful energy sources for many of our day-to-day activity needs and activities. Natural gas is used in our homes, for cooking and heating, and to generate electricity. It is also used for commercial businesses or as a raw material, in energy intense industries. Natural gas is formed primarily of methane but could include significant quantities of ethane, propane and other hydrocarbons. When burned it gives a great deal of energy and it is the cleanest of fossil fuels: fewer emissions of sulphur, carbon and nitrogen and almost no ash particles left after burning. This high efficiency and characteristics justify why the use of this source has grown so much and it is expected to grow even more in the future.

Natural gas was formed millions of years ago, from organic material (remains of plants and animals) that was decayed and built up in thick layers. Over time, it was covered and trapped it beneath the rock. Some factors, like pressure and heat, changed it into coal, some into oil, and some into natural gas. Unfortunately, global gas reserves are abundant, but not uniformly distributed and countries that are more dependent of natural gas, normally the industrialized countries, are not the one with the reserves. The expanded use of natural gas by Europe, Japan and USA is gradually increasing but the major gas supplier is located in the Former Soviet

Union, which represents one third of the world's original gas endowment, and the Middle East, which is the second largest gas resource, as Figure 1 shows.

Country	Reserves (Trillion Cubic Feet)	Percent of World Total
World	6,183	100.0
Top 20 Countries	5,602	90.6
Russia	1,680	27.2
Iran	974	15.8
Qatar	911	14.7
Saudi Arabia	240	3.9
United Arab Emirates	214	3.5
United States	204	3.3
Nigeria	182	2.9
Algeria	162	2.6
Venezuela	152	2.5
Iraq	112	1.8
Turkmenistan	100	1.6
Kazakhstan	100	1.6
Indonesia	98	1.6
Norway	82	1.3
China	80	1.3
Malaysia	75	1.2
Uzbekistan	65	1.1
Egypt	59	0.9
Canada	58	0.9
Kuwait	55	0.9
Rest of World	581	9.4

Source: "Worldwide Look at Reserves and Production," Oil & Gas Journal, Vol. 104, No. 47 (December 18, 2006), pp. 22-23.

#### Figure 1: World Natural Gas reserves by country, 2007

Iran and Qatar are responsible for these high values in Middle East, however, the Middle East does not dominate world gas production, and Russian Federation is the world's leading gas producer, since 1981 Russia registered the largest incremental growth. North America represents a good 'surprise'; it can produce a large amount of gas from a relatively small reserve. About consumption, North America and Russia change positions and North America is the leading consumer and Europe ranks third. As mention before, Europe and North America are the biggest importers of natural gas due to Germany and USA. More statistical data can be found in Appendix A.

### 2.3 Natural Gas Supply Chain

As a result of non uniform distribution, transportation and storage of natural gas will represent a crucial role in the whole operations process. Understanding the supply chain of natural gas network will be the starting point. In general, supply chain can be viewed as a network comprising the sequence of steps or events in a goods flow that adds to the value of a specific good. Supply chains start at the suppliers with raw materials and end at the consumers with the final product. In the middle we can find a lot of different services, organizations, like manufacturers, wholesalers, and retailers. The existence of supply chains is essential in all sides; however the dimension and complexity of the chains may vary from one industry to another.

According to this, a natural gas supply chain is defined as a network of integrated facilities that work together in a specific way. These facilities include: sources of energy, exploration and extraction, production technologies, transportation, storage facilities and distribution sector.

Finding a natural gas and petroleum deposit is the first part and has been transformed dramatically in the last 20 years with the advent of technology. It is rife with uncertainty and trialand-error, due to the complexity of the process. The exploration starts with geologists examining the surface structure of the earth, and finding out areas were natural gas reservoir might exist, using a geological survey. To obtain more detailed data about the potential reservoir area further tests can be performed by a geophysicist. Some techniques are developed and used to improve the phase of extraction, like seismic exploration, onshore and offshore seismology, and some properties of underground formation are measured, like magnetic properties, gravitational field or drill an exploratory well, to gain a full understanding of subsurface geology and the potential for natural gas deposits to exist in a given area. To analyse and join the entire sources of data available is the next step for the geophysics, where they can create a model to the structure of the surface. Despite the effort and continual evolution of technology and exploration techniques, these models are not infallible. Improving this process is essential to avoid errors and to obtain more realistic guesses.

Once a potential natural gas deposit is located, extraction is needed. Drilling natural gas depends on a variety of factors, like the nature of the formation to be drilled, the depth and size of the deposit and the characteristics of the surface. The economic factors also influence the drilling. Search and drill the natural gas is very expensive for exploration and production companies, and there is always the inherent risk that natural gas will not be found, which means that companies will not achieve the goal. Two thinks can happen when drilling has started, the well does in fact come in contact with natural gas, so the well may be completed to facilitate it's production, or the estimation was incorrect, does not exist marketable quantity of natural gas at a well site, and production does not proceed.

Lifting the natural gas out of the ground and prepare it for the transportation is the next step of the supply chain. If there are enough commercial quantities of natural gas it is needed to process it into pipeline quality gas and purity specifications must be met. Underground natural gas is not the same that comes through pipelines to homes or businesses; sometimes it has other compounds or gases that must be removed. Natural gas used by end users is composed almost exclusively of methane. Production of natural gas can be divided into two distinct parts: well completion and processing of natural gas. These two parts are extremely necessary before its use by consumers to acquire the required characteristics.

As mentioned before, due to the unique distribution of gas reserves, in many instances, natural gas produced in a particular well need to travel a large distance to reach its point of use. So the transportation system requires a lot of attention and work. It consists of a complex network of pipelines, which tend to be as quick and efficient as possible. To transport natural from its origin to its end users will be the function of pipelines. It can be transported to areas with high natural gas demand or it can be put into storage facilities, if the transport is not required at a certain time. So transport and storage are linked. Pipelines for natural gas transport can be separated in three major types: the gathering system, which transports raw natural gas to the processing plant; the interstate pipeline, which transports it into areas with high demand, normally high density areas; and the distribution system, which is the safest way of transporting energy, due to the fact that it is underground.

All the tasks that were explained before require some time and sometimes when natural gas reaches its destination it is not needed, so it is stored into a reconditioned aquifer, reservoir or cavern. Subsurface area is located 'some' meters below the ground surface but it depends on the geology and geological structures. A gas field must be safe and developers must make sure that stored gas cannot leak. Storage will always ensure the demand, even if there is an unforeseen supply disruption, and it can be used for commercial reasons by industry: to buy and store natural gas when prices are low and to sell it if prices increase. Storing the natural gas allows the country, city or area, to import a quantity of fuel and ensure that there is available gas whenever it is needed, even if the production is not enough.

Distribution is the final step in terms of providing natural gas to end users. While some consumers, the ones with high consumption of natural gas, receive natural gas directly from the pipelines, most consumers receive it from a local distribution company. Dispersion of small consumers' leads local distributions companies to have extensive small-diameter pipeline networks. Distribution costs are the responsible for the high cost of transport for natural gas, normally for individual consumers it is the majority of natural gas cost. Due to this importance, natural gas distribution deserves a particular attention.



Figure 2: Natural Gas Supply Chain

Figure 2 shows the natural gas supply chain and in this work the focal point will be the last part of the chain: gas delivery and customers.

### 2.4 Heat network design

As said before natural gas is the fuel that this study will focus on. This fuel can be used for a diversity of activities and operations. In this work natural gas will be used as a fuel at certain facilities and provides heat production. The connection between production facilities and customers is the under grounding pipelines, which transport the heat from one area to another. In the end of the natural gas supply chain can be found the customers separated in several sectors, like commercial, industrial, residential, and transport sectors.

This work is incorporate in BP Urban Energy Systems project and the aim of it is to reduce the energy intensity of cities. It is easily perceived that in cities some energy optimisation can be done in terms of efficiency, the first phase of this project is concerned with the identification of eventual improvements, the benefits and impacts for the cities and what must be done to achieve it. To review the situation of each city some reasons can be pointed: projections that predict around 90% of population growth will be in urban areas, scaling laws for urban services that span across the spectrum of cities, radical changes can happen within cities from integration of services due to this pressure, and if these changes take place, it will turn into an innovative city which can improve the process efficiency and reduce its resource flows.

The starting point will be the actual gas network that supplies cities and understanding how the city efficiency can be improved through the introduction of some changes, like building in environmental strategies, decentralised electricity generation and storage or more radical approaches to the management of waste among others. The first part of Figure 3 shows a

typical electricity network as it is today with the areas of responsibility of the transmission and distribution system operators, the centralised large-scale generation and the consumers (commercial, industrial and domestic). However, it is known that the electrical network will be required to adapt to the change in technology, scale and location of generating plants. In principle, gas networks are similar. The future network could look more like that illustrated in the second part of Figure 3 with building-integrated renewable and co-generation playing an important role in urban areas and large scale renewable generation being added outside urban areas. This brings us to the distribution of system operators a more complicated control and coordination problem.



Figure 3: From typical to possible future urban power network

In this project a virtual city will be studied, which has similar characteristics to a "normal" city of the United Kingdom, that could be, for example, London. Within different sectors: commercial, industrial, transport and residential, according to UK energy statistics, domestic buildings are the ones with higher consumption of energy, and space heating is the responsible for it. Heating represents more than half of residential energy consumption and almost one third of the consumption in commercial buildings, so this is where we will focus our efforts.

The natural gas chain, from the delivery point to the customer, must be reviewed to make it clear and easy to understand. A boiler will be the connection between raw material, natural gas, and the product that is expected at the end, heating; pipelines will be the link between boilers and final customers, as shown in Figure 4 below. Green lines represent the area in study.



#### Figure 4: Community Energy Diagram

A model will be developed to indicate different energy networks when submitted to different patterns of use. Different inputs will generate different networks designs and it will be explored how optimisation might be used. Decrease of the total cost is the goal of this optimisation; however the consumer's heat demand must be satisfied.

In optimisation it is necessary to consider two elements: the objective function, the one that influences all system behaviour, and constraints, which set the limit of the system and define if a solution is feasible or unfeasible.

Building blocks of optimisation problems can be defined as parameters and decision variables. Decision variables can be divided into: continuous, integer and binary variables, according to what is necessary to represent: if it is a decision such as produce or not, exists or not, a binary variable should be used; if it is to define the number of boilers or the quantity of exported heat, an integer variable is the best option.

Defining the objective function and constraints influences the methods of the optimisation problems. For example, a mixed-integer nonlinear programme can be used if the problem contains both integer and binary variables. On the other hand, a mixed-integer linear programming formulation is adopted if the study exhibits a linear objective function and constraints with binary and integer variables.

### 2.5 Previous work

On the type of problem in study several works have been published.

Sugihara, Komoto and Tsuji (2002) propose a new model for determining urban energy systems in a specific area. It shows how important it is to adopt new highly efficient energy technologies. This model considers 7 customers types separated in 2 sectors: residential sector is separated from business and commercial sector, and different sets of energy system alternatives are designed for each sector. It takes into account District Heating and Cooling (DHC) system as a centralized-type energy system. DHC consists of a system comprising a set of energy suppliers and consumers, district heating (DH) pipelines, heat storage facilities and power transmission lines. A DHC can produce steam, hot water or chilled water at a central plant and then pipe that energy out to buildings in the district, even if it is a commercial or residential building, for space heating, domestic hot water heating or air conditioning. It can use a variety of conventional fuel, like coal, oil or natural gas, or it could use as raw material renewable fuels, such as biomass or geothermal. Three indices are evaluated, cost, CO<sub>2</sub> emission and primary energy consumption and this model determines not only the capacity and operational strategy of DHC, but also the sharing of each energy as a system alternative. As constraints are defined the energy demand for operation, the equipment capacity in DHC plant, the peak demand purchased from outside utilities, annual purchased energy from outside utilities and the definition on energy systems shares and as a conclusion a trade-off relation among the three indices is illustrated.

Soderman & Pettersson (2006) formulated a problem with the aim of reducing the total cost of a distributed energy system (DES), considering the production and consumption of electrical power and heat, power transmissions, transport of fuels to the production plants, transport of water in the DH pipelines and storage of heat. This research developed a mixed integer linear programming which includes the running and investment costs. The first cost is calculated with cumulative energy (heat production) multiplied by a unit operational cost coefficient. For the second one the annuity factors can change, according to the type of production unit or pipeline part. So pipeline investment costs, the costs for heat losses along the pipeline and exchanger equipment costs are analysed in this project, which also contains heat losses from the storage but losses from the pipelines are not taken into account.

Çomaklı, Yüksel and Çomaklı (2004) study the energy and exergy losses occurring in the DH network system. It is a useful work due to the emphasis it gives to losses during heat distribution and the energy balance analysed on the pipeline network.

Some studies, like Dzenajavičienė, Kveselis, McNaught, and Tamonis (2007) remind the importance of the economic situation on a particular area or city, existing heat-generating sources, and the state of the heat supply and distribution pipelines, fuel prices and many other factors. It is necessary to evaluate this item if some changes are planned to occur.

Related to economic aspects is the total costs portion that is the aim of several projects. Morais & Marangon Lima (2007) is a good example of it. This works gives importance to transmission fare, plant investment and fuel cost. The last one can be split into two parts: the production and the transportation costs. Some methodologies to charge gas pipeline and transmission network are proposed: MW-mile, gas-mile, invested related asset cost and the Aumman-Shapley (AS) Method. The first two options are used when there is available information about locations and when the circuit flow is known; the other method, AS allocation is used only if it is not possible to identify a point-to-point transaction, which is not the case in study.

In Ajah, Patil, Herder and Grievink studies (2006) the costs are analysed in a different way and divided into four categories. The purchase cost of the system components, the construction cost of the system, the installation cost and the operation and maintenance cost. To conclude, cost can be analysed in different ways, but it should include as much components as possible to make a realistic model. Any energy model is a simplified representation of the real system, so it is necessary to considerate all the possible parts to obtain good results.

## 2.6 Conclusions

In this chapter it was explained what has already been done in the area in study and why these studies are important. It starts with the general idea of energy, and after it specifies natural gas as a fuel able to produce energy. Natural gas can be used to produce hydrogen or automobile fuel, in manufacture of fabrics, glass, paint, etc, as a source of electricity generation, but in this work we will focus only on natural gas used for heating homes or other buildings. A brief explanation of the natural gas supply chain is given and what could help later to develop the optimization model.

Some methodologies and techniques that could be useful and contribute to define the supply chain configuration are explained. According to this review, some ideas are taken and will be part of the model developed. The next chapter presents an optimization approach for the structure design and operation of the future energy supply chain network. The network is described as a static MILP model which is used to determine the supply chain of heat production, transportation and deliverer to customers in a virtual city.

## **3 Conceptual Model**

## 3.1 Problem description

This chapter presents a model-based method to determine the design of an urban energy supply chain network. The proposed network consists of a set of production facilities and the distribution system associated. The network is represented by a mathematical model which is formulated as an MILP optimisation problem. The construction of this network will be driven by heating demand. The aim is to minimize the total cost of the network with the guarantee of satisfying all the customers' heat demand. Costs associated with the heat production source and position, and distribution network are the main aspects that influence the final solution.

The application of this model is for a virtual city which is divided into cells; however, the program can be applied to any real city, as long as required information is available.

The proposed model is used to investigate a number of strategic decisions. For the production facilities, the decisions are about the number of boilers to satisfy all the demand, the geographic location, capacity and amount of heat to produce in each boiler. The distribution system also involves decisions like the definition of the required pipelines and the way they are distributed in the system. Another important decision is to quantify the heat that is necessary to transport from one cell to another in order to satisfy the demand. Each cell can export or import. It should be always present that customers demand must be fulfilled in each cell of the city, even if it represents an increase in costs.

## 3.2 Model Structure

The structure of the proposed model is shown in Figure 5. It is assumed that there is no structure in the city when the optimization starts; it is like a new city without any production or transportation facilities. In Figure 5 it is shown not only inside grids (q) which divide the city into equal cells but also outside grids (w), which represent the outside area of the city. Each cell represents a different region or area within the city. In some grid positions (p) it is clear the existence of a heating unit, boiler (b) that can produce hot water from natural gas. Boilers can be in different positions and there are three different types of boilers. Hot water produced from boilers flows to consumers though pipelines and will satisfy the heat demand in each cell.

As shown in Figure 5 this model contains one possible configuration of the energy supply chain within a city. The optimisation model will find the best solution for the design of this chain, trying to obtain the most economical solution. The model will eliminate all the expensive options. Analysing the answer it is possible to determine the optimal distribution for each heating unit and draw the distribution map. In the following subsections, each component of the model will be explained.



Figure 5: Proposed Model Structure

#### 3.2.1 Grid squares

The identification of the positions of each component in the systems is an important step in this model, so it is necessary to divide the city into grid squares of equal sizes. The virtual city will be divided into 16 cells and each grid has a length of 2 kilometres (km), which means that a virtual city covers an area of 64 km<sup>2</sup>. The yellow area in Figure 5 represents the city.

Outside cells are represented by green squares. For a better representation of a real city and a more detailed result, it was necessary to introduce surrounding squares with the same size that inside squares: 2 x 2 km. These cells are the ones that can contain boilers with bigger capacity. Due to its size, it was decided that internal cells only contains small and medium boilers, while outside cells can have all the three options.

Each square is represented by an index p in a set of indices from  $p_1$  to  $p_n$ . There is also an index for internal positions, q, and an index w to represents outside positions. This definition

allows the user to define different characteristics to each area, such as heat demand, possible production facilities, population, etc.

Inside each cell it is possible to have commercial, industrial or residential buildings as it is shown. Multiple buildings in one cell could also occur due to its size. The same ideology is used for boilers: one cell can contain one boiler or more, with different capacities. Cell demand is determined by the population density and its consumption. The aim, as said before, is to satisfy the total heating demand of each cell, avowing high cost.

### 3.2.2 Distance between cells

Distance between cells is an important part of the final solution. The total travel distance will represent the cost of the pipelines, which is proportional to its length. It is important to define how the distance between the boiler and the customer is measured.

To estimate the distance of any two cells, the reference point will be the centre of the cell. So if a distance from point A to point B is needed, it must be measured from the distance at the midpoint of point A to the midpoint of point B.

Three different approaches for measuring distance can be used: Euclidean, Manhattan and Road network. Figure 6 illustrates the three network options when hot water is transfer from one cell to another.





Manhattan is represented by the orange line and is calculated as the shortest distance between the boiler (centre of supplier cell) and the midpoint of end cell, with distance being constrained to horizontal or vertical directions. Diagonal distances are not accepted according to this method.

The road network method is represented by the blue line and calculates the shortest path distance following the road network. It assumes that there is already a road network before the simulation.

The green line represents the Euclidean method, which refers to the straight-line between two points: the midpoints of each cell. It is easily calculated if coordinates from cells are given. This is the chosen method for this model to calculate distance between cells. Pythagorean Theorem will be used to find the Euclidean distance between midpoints of all cells, even internal or outside positions of the city. All the distance will be given in kilometres.

dist<sub>p,pp</sub> = 
$$\sqrt{(x_p - x_{pp})^2 + (y_p - y_{pp})^2}$$
 (3.1)

The equation above shows how the distance between cells can be calculated and a table with all the distances is given in Appendix B.



### 3.2.3 Distance of one cell

Figure 7: Distance within one cell

Although the importance of the distance between cells, internal distance must be also considered. It is shown in Figure 7 above and can be defined as the distance within the cell to transport the heat to each individual consumer. Heat is transported from the supply cell to the midpoint of the end cell, and later on is transported to the final client through pipelines. To a better approximation of this value, it was necessary to calculate the internal average distance times the total number of customers in each cell. Distance in one cell can be represented by equation (3.2). Other calculations and needed explanations are described in Appendix B.

distance<sub>average</sub> 
$$\geq$$
 size<sub>p</sub>  $\times \int_0^1 \int_0^1 \frac{1}{6} \left[\sqrt{2} + \ln(1 + \sqrt{2})\right] \frac{1}{\sqrt{y}} d_x d_y$  (3.2)

#### 3.2.4 Production facilities - boilers

Natural gas is the resource that will be used in the boiler as a fuel, to generate the required heat. A boiler can be defined has a sealed vessel that can produce hot water by the conversion of the chemical energy of external combustion source. The ratio of the output, hot water, to the input, natural gas, of the boiler defines the efficiency, which has a specific value for each boiler. It is assumed that the efficiency of the boilers is constant throughout the working range, and start/stop costs are neglected.

According to its capacities, different types of boilers are considered in this model. Boilers are represented with an index b in a set of indices from  $b_1$  to  $b_3$ . In agreement with the capacity, boiler prices also change from one boiler to another and the one with higher capacity is more expensive then others. Two types of boilers cost will be analysed, capital and operational costs, as explained later. It should be mention that there is not a direct relation between the capacity of the boiler and its capital cost. Boilers with large capacity are cheaper then what was expected.

Another different characteristic of each boiler is the production rate. Different rates are defined for each boiler size. This rate is a comparison of production of each boiler, in watts, with the natural gas cost, in euros ( $\in$ ).

#### 3.2.5 Transport facilities- pipelines

Hot water circulates from the boiler thought pipelines to customers. As shown in Figure 7, pipes are in the channel which is under the ground. In the system, the warm water circulates in a network of heat-insulated pipes. A heat exchanger is installed at each user. The cold water coming out from the exchangers returns to the plant in separate insulated pipes.

The common characteristics of each pipe are the diameter, the supply and the return temperature.

### 3.2.6 Heat losses

Heat losses correspond to thermal losses occurring during the hot water transfer through pipelines. According to some works, heat losses per square meter can be calculated by the formula:

$$hI = \alpha^{*} (\Delta t)^{*} (diam^{*} \pi)^{*} 10$$
(3.3)

Where

 $\alpha$  represents the heat constant, previously defined;

Δt represents the temperature difference between inside and outside pipeline;

diam represents the diameter of the pipeline, in meters.

Heat losses depend of the thermal properties of the pipeline and its characteristics.

Two types of heat losses will be study and considered: external and internal losses. External losses refer to losses verified during heat transfer, while internal losses consist of losses occurring inside the cell.

### 3.2.7 Types of consumers

As mention before, each cell will have a specific demand according to its population. Each square is composed of two different types of consumers, which is represented with an index t. Consumers can be defined as commercial and residential, to a better representation of a real city. Residential consumers,  $t_2$ , have a high heat demand during the evening, while commercial consumers,  $t_1$ , need more heat during the morning and afternoon.

#### 3.2.8 Periods of the day

To define the characteristics of each consumer, commercial or residential, different parts of the day were defined with the index d. Three periods are considered: morning, afternoon and evening, all with the same duration. Morning,  $d_1$ , cover the period between 0h and 8h; afternoon,  $d_2$ , between 8h and 16h; and evening,  $d_3$ , is defined as the period between 16h and 24h. Each period has a specific demand according to the type of consumer, as explained before.

#### 3.2.9 Demand

Heat demand is the factor that influences the heat transfer from one cell to another. It is previously determined for each cell, according to its population and individual consumption. At the optimal solution the defined demand must be satisfied in every cell. Heat demand is

expressed through an indicator demand and is calculated by multiplying the total cell population by the daily consumption. The demand equation shows it.

demand<sub>p</sub> = 
$$\sum_{t} pop_{p,t} \times \sum_{d} (\sum_{t} cons_{d,t})$$
 (3.4)

Only internal cells have heat demand. External cells represent the outside part of the city, so they will not have any population, and consequently heat demand.

## 3.3 Model Formulation

The mathematical formulation is composed by the nomenclature, constraints and objective function terms. Indices, parameters and decision variables are included in the nomenclature. The variables are further categorised into: continuous, integer and binary design variables. As said before the model proposed for this optimisation problem is an MILP.

### 3.3.1 Nomenclature

p/pp	grid position/cell
b	boiler type
d	periods of the day
t	types of consumers
n	average distance constraint index

#### 3.3.1.2 Sets

q <sub>p</sub> = {p, if p is an internal cell}	internal cell position
w <sub>p</sub> = {p, if p is an external cell}	outside cell position

#### 3.3.1.3 Parameters

α	1-5 constant heat losses W / $m^2  ^{\circ}C$
b_cap <sub>b</sub>	boiler capacity in W
c_ce <sub>b</sub>	capital cost – equipment cost in €

c_etransp	capital cost - pipelines for external distribution in $\in$ /W
c_itransp	capital cost - pipelines for internal distribution in €/W
c_om <sub>b</sub>	operational cost – operation & maintenance cost in €/W
$dist_{p,pp}$	distance of cell p to midpoint of cell pp in Km
d <sub>p</sub>	total demand of cell p in W
hl	heat losses W in p per Km
in_t	pipeline inside temperature in $^{\circ}C$
maxht	maximum heat transfer
maxn	maximum number of boilers per cell
minhf	minimum distributed heat flow
minp	minimum production
op_gas	operational cost - natural gas in €/W
op_pumping	operation cost - pumping in €/W
out_t	pipeline outside temperature in °C
pi	pi number
pr_rate	production rate
size <sub>p</sub>	length of each cell p in Km
Xp	abscissa of cell p
Уp	ordinate of cell p
8	boiler efficiency
θ	pipeline diameter in m
3.3.1.4 Continuous	s Variables

cc_prod	capital cost: production in €
cc_prodq <sub>p</sub>	capital cost: production (boilers) in cell p in $\ensuremath{\in}$
cc_transp	capital cost: transport in €

cc_transpp <sub>p</sub>	capital cost: pipelines used to transfer heat for cell p in $\in$
cc_transpq <sub>p</sub>	capital cost: pipelines in cell p in €
daveep	average distance for external losses in p in Km
davei <sub>p</sub>	average distance for internal losses in p in Km
eloss <sub>p</sub>	external thermal losses in W
exp <sub>p</sub>	total exported heat in p in W
ht <sub>pp,p</sub>	imported heat of cell p - distributed heat from pp to p in W
iloss <sub>p</sub>	internal thermal losses in W
imp <sub>p</sub>	total imported heat in p in W
opc_om	total operational cost: operation and maintenance cost in $\in$
opc_omq <sub>p</sub>	operational cost: operation and maintenance cost in cell $p$ in ${\boldsymbol{\varepsilon}}$
opc_prod	total operational cost: production in €
opc_prodq <sub>p</sub>	operational cost: production in cell p in $\in$
opc_transp	total operational cost: transport in €
opc_transpp <sub>p</sub>	operational cost: transport within cell p in $\in$
opc_transpq <sub>p</sub>	operational cost: transport in cell p in €
$pr_{b,p}$	production of boiler b in position p in W
pref <sub>p</sub>	real production in p in W
Z	objective function - total cost in €
3.3.1.5 Binary Variables	
ie <sub>p</sub>	= 1, if the cell p imports heat; 0 otherwise
$yht_{p,pp}$	changing heat from p to pp: = 1 if heat is transferred from p to pp; 0 otherwise
3.3.1.6 Integer Variables	
yb <sub>b,p</sub>	number of boilers type b chosen in cell p

#### 3.3.2 Constraints

#### 3.3.2.1 Heat balance

The heat balance of a specific cell includes all the heat transferred, which could be importation or exportation, individual heat demand and production, and internal and external losses. It must be stipulated by the following constraint:

$$\operatorname{imp}_{p} + \operatorname{pref}_{p} = d_{p} + \operatorname{eloss}_{p} + \operatorname{iloss}_{p} + \operatorname{exp}_{p}, \forall_{p}$$
(3.5)

#### 3.3.2.2 Boilers production

As said before, each cell has its own demand. This demand must be satisfied eventually by local production, if a boiler is located inside the cell, or by importing heat from nearby grids.

The presence of a boiler within a particular grid will confirm this demand satisfaction. However, the boiler production is limited by its capacity through the equation:

$$pr_{b,p} \le yb_{b,p} \times b_{cap_{b}}, \forall_{b,p}$$

$$(3.6)$$

In order to avoid unnecessary and inefficiency use of boilers, production must be higher than an establish percentage of boiler capacity, minp:

$$pr_{b,p} \ge yb_{b,p} \times b_{cap_{b}} \times \min p, \forall_{p,b \in SETp}$$
(3.7)

As it was mentioned before, the heat balance includes the boiler production, although it is not perfect and some losses occur during production, so efficiency of the boiler must be considered. Due to boiler inefficiency, real production is the value that is necessary to satisfy the demand and can be calculate by:

$$\operatorname{pref}_{p} = \sum_{b} (\operatorname{pr}_{b,p} \times \varepsilon), \forall_{p}$$
(3.8)

#### 3.3.2.3 Heat transfer

In this model it is possible to export heat thought pipelines or, in spite of having boilers inside the cell to satisfy the heat demand, as explained above, heat importation can also be done. Importation to cell p is defined as the sum of importations from cells pp to p:

$$\operatorname{imp}_{p} = \sum_{pp} \operatorname{ht}_{pp,p}, \forall_{p}$$
(3.9)

For exportation, the definition is similar. It can be defined as the sum of all exportations from origin cell p to external cells pp.

$$\exp_{p} = \sum_{pp} ht_{p,pp}, \forall_{p}$$
(3.10)

The heat transfer is permitted, although there is a minimum and maximum flow rate of transfer needed to justify the establishment of a transport system between two grids in the network.

The high limit is defined by the end cell demand and the maximum heat transfer, previously defined by the user.

$$ht_{p,pp} \le yht_{p,pp} \times max \, ht \times d_{pp}, \forall_{p,pp}$$
(3.11)

Minimum level depends of the demand of the end cell and the minimum heat transfer, MHT, as shown in next equation:

$$ht_{p,pp} \ge yht_{p,pp} \times d_{pp} \times \min hf, \forall_{p,pp}$$
(3.12)

Export to one cell and import from that cell is not allowed in this model. A particular grid can only import heat from neighbouring grids or export heat to other grids, or neither, but not both:

$$yht_{p,pp} + yht_{pp,p} \le 1, \forall_{p,pp}$$
(3.13)

Heat transfer from any cell to external cells is also not authorized. This constraints is a consequence of the fact that no demand is allowed in cells located outside the city.

$$\mathsf{yht}_{\mathsf{p},\mathsf{w}_{\mathsf{p}}} = \mathbf{0}, \forall_{\mathsf{p},\mathsf{w}_{\mathsf{p}}} \tag{3.14}$$

#### 3.3.2.4 Heat losses

External and internal losses will occur during heat transfer. Due to the fact that the transport system consists on two separated pipelines, the total heat losses in cell p must be the double of its value for one pipeline.

External losses in cell p can be defined as the thermal losses during the transport of heat from one cell to the midpoint of another. These losses are calculated by the multiplication of the distance from midpoints cells by heat losses per unit explained above.

$$eloss_{p} = \sum_{pp} (2 \times dist_{p,pp} \times yht_{p,pp} \times hl), \forall_{p}$$
(3.15)

Internal losses refer to losses that occur within the cell, during the heat transfer from the midpoint of the cell to individual consumers. In this case, the number of boilers inside the cell and allowed exportations or importations influences the result. An additional function is used to calculate the variable davei<sub>p</sub>. This function is explained in Appendix B. The size of each grid is also considered to find the final value for internal losses.

iloss <sub>q</sub> = 2 × davei <sub>q</sub> × hl × size <sub>q</sub> × 
$$\sum_{q}$$
 pop <sub>q,t</sub>,  $\forall_{q}$  (3.16)

#### 3.3.3 Objective function

The objective of the proposed model is to minimise the overall cost of a possible natural gas network chain. To find the optimal solution, costs will be analysed and divided in two sectors: capital and operational costs.

Capital costs, defined as the total investment required to complete a project and bring it to a commercially operable status, are divided in two different parts: facility and transport. These are the costs associated with the establishment of all facilities and normally are one-time costs.

Operational costs are the costs resulting from the day-to-day of an operation, which can be fixed or variable. In this case, three variable operational costs are analysed: production, maintenance and transport costs. Figure 8 shows the operation and capital costs and its subdivisions. It is also in evidence what can influence the values of each cost.





#### 3.3.3.1 Capital Cost

#### 3.3.3.1.1 Facility capital cost

Transport and facility costs are the two aspects which will be analysed. Concerning the production, boilers are the necessary equipment. Facility capital cost can be easily calculated by multiplying the number of boilers by their capital costs as follows:

$$cc\_prod = \sum_{b} (\sum_{p} (yb_{b,p} \times c\_ce_{b}))$$
(3.17)

#### 3.3.3.1.2 Transport capital cost

Between the production point and customers, a transport system is necessary to transfer the heat from one point to another. Transport capital costs include all the pipeline costs and its installation. As it was explained in the heat losses section, the total cost is the double of the transport cost for one pipeline, due to the fact that the transport system contains 2 pipelines.

$$cc_transp = \sum_{q} (2 \times dave_{q} \times size_{q} \times \sum_{t} (pop_{q,t})) \times c_transp + c_etransp \times \sum_{p} (2 \times dave_{p})$$
(3.18)

The equation above is the one that is used to calculate this cost.

#### 3.3.3.2 Operational Costs

#### 3.3.3.2.1 Production operational cost

Production costs are related to the required fuel to operate the boilers. Natural gas is the required resource for heating the water, and its production costs can be easily calculated. Production rate is considered for this operational cost, due to the fact that bigger boilers can produce the same amount of energy using much less resources.

$$opc \_prod = \sum (\sum (pr_{b,p} \times pr \_rate_b \times op \_gas))$$
(3.19)

#### 3.3.3.2.2 Transport operational cost

The heat transport will be through pipelines and is divided into two distinct parts: transport to midpoint cell and transport within the cell.

The first part, transport to midpoint cell, can be summarize as the distance between cells, which was explained before, times the pumping costs and the quantity of imported heat. Transport costs are then imputed to end cell:

opc transpp 
$$_{P} = \sum_{p \in P} (ht_{PP,P} \times dist_{pp,p}) \times 2 \times op pumping , \forall_{p}$$
 (3.20)

Cost associated with the transport within the cell also depends of the heat demand, size of the cell and internal distance, previously estimated, and can be calculated with the equation:

$$opc\_transpq_p = 2 \times davei_p \times demand_p \times op\_pumping, \forall_p$$
 (3.21)

The sum of these two capital costs gives the total transport cost in cell p, which is calculated by the following expression:

$$opc\_transp = \sum_{p} (opc\_transpq + opc\_transpp_{p})$$
(3.22)

#### 3.3.3.2.3 Operation and Maintenance operational cost

To continually assure the production that satisfies the heat demand it is important to maintain all the equipment in good conditions. Boilers and pipelines must be in constant overhaul to avoid some faulty equipment or inappropriate work conditions, which result in low boiler efficiency. In this model it is considered that O&M costs remain stable during the simulation period. This cost is only attributed to cells with boilers, as shown in next equation:

$$opc\_om = \sum_{b}\sum_{p} (yb_{b,p} \times c\_om_{b})$$
(3.23)

#### 3.3.3.2.4 General Objective Function

Combining the cost terms derived in section (3.3), the total heat supply chain cost can be represented as follow:

$$z = opc \_prod + opc \_transp + opc \_om + cc \_transp + cc \_prod$$
 (3.24)

The first three terms represents the operational cost, while capital costs are represented by the others two terms.

Minimizing z corresponds to the aim of this work, which can be easily and shortly defined as the minimisation of the supply chain total cost.
## Chapter 4

# 4 Single Scale Results

The model described before will be applied to a virtual city with different population distributions to a better approximation of a real case study. Uniform, normal and random distributions will be analysed and differences between input data are explained. In a single scale application a city defined by a regular 4 by 4 grid is studied and all results will be presented and compared. In this study case, the outside area of the city is also taken into account. The single scale application looks to the city and outside area as a whole and shows one possible network design. This model is applied to a virtual city, although it is representative of real cities. The use of virtual city models is the easiest and simple way, due to the fact that no documentation or historical information is required, only representative data.

## 4.1 Case Study 1: Uniform distribution

The uniform population distribution is the first case study that is analysed. A city is divided into sixteen cells, all with the same size. Each grid cell has 2 km in length and is identified with an index p. The area surrounding the city is also divided into square cells of equal size. Reducing the total costs while satisfying the demand is the aim of this case study. Some decisions will be made, on whether boilers should be located or how should be the distribution network. Boiler capacity and quantity of heat produced are also part of the decisions.

In a uniform distribution, the population is the same within every cell and is equality distributed according to the types of consumers. Demand depends only on the population and type of consumers in each cell, and the optimisation will assume these values as constant. Production can be different from one grid to another; boilers can be located either inside or outside the city, although the same demand must be satisfied in every cell. It is expected, as a result, that boilers will be also uniformly distributed in this network.

## 4.1.1 Input Data

Understanding the data requirements for the application of a natural gas supply chain is an important step of this model which requires lots of data. Since no real values were available some research was made. The aim is to use representative data which could be valid for real cases. The following subsections describe the input data required for the optimisation model. This involves: demand data, heat transfer losses, distances, boilers and transport characteristics and costs.

### 4.1.1.1 Demand

Population and consumption per person, according to the types of consumers and periods of the day, characterise and determine the total demand in each cell of the city. This will be the same due to the consumption of uniform population distribution.

A city of 40000 inhabitants, uniformly distributed in each square, as shown in Table 1 is considered. A descriptive population was assumed.

Cell index	Population Commercial Residential		Demand (W)
p1	1250	1250	2500000
p2	1250	1250	2500000
р3	1250	1250	2500000
p4	1250	1250	2500000
р5	1250	1250	2500000
p6	1250	1250	2500000
р7	1250	1250	2500000
p8	1250	1250	2500000
p9	1250	1250	2500000
p10	1250	1250	2500000
p11	1250	1250	2500000
p12	1250	1250	2500000
p13	1250	1250	2500000
p14	1250	1250	2500000
p15	1250	1250	2500000
p16	1250	1250	2500000

Table 1: Population and Heat Demand - case 1

It is considered that each cell has the same number of residential and commercial consumers. The consumption per person, according to the period of the day and types of consumers is shown in Table 2.

Deviada of the day	Type of consumers		
Periods of the day	Commercial	Residential	
Morning	300	300	
Afternoon	500	200	
Evening	200	500	

Table 2: Consumption in W per person according periods of the day and type of consumer - case 1

Using the data in Table 2 the model is able to calculate the total demand per cell, as summarised in (3.4).

As explained before, demand per cell will be the same. It is defined that cells in outside positions will not have any heat demand. So heat transfer for outside positions is not possible to occur.

### 4.1.1.2 Heat transfer and losses

From boilers to consumers, heat is transferred through pipelines. This transfer is possible:

- From one cell to another, if one cell does not have any boiler or need more heat than what is available in the cell;
- Within the cell, from the middle point to customers.

Heat transfers have minimum and maximum flow rate boundaries. To justify the network systems between boilers and consumers, the flow must be 10% higher of the demand of end cell and can not exceed 5 times the value of the estimated demand of end cell.

Although some heat losses will occur during the transfer. The first step to calculate the total heat losses is to define heat losses per unit, which can be calculated by equation (3.3). The required data is shown in Table 3.

After calculating the value for heat losses per unit, the total heat losses per cell it is easily calculated if the distance is a known parameter.

Scalar	Value
α (W/m <sup>2</sup> °C)	5
int_t (°C)	80
out_t (°C)	10
<i>θ</i> (m)	0,1
π	3,14

Table 3 Assumed values to calculate heat losses per unit

#### 4.1.1.3 Distance

Once again two distinct distances must be defined: distance from one cell to another and distance within one cell. So in Appendix B all the distances can be verified. The virtual city is divided into 16 inside squares, and each square will have the same representative length, 2x2 km is the assumption made by the user.

#### 4.1.1.4 Production facilities - boilers

Three types of different boilers are analysed. The main difference is the capacity of each boiler. Small boilers are characterized by the index  $b_1$  and have the capacity of 1 Megawatts (MW),  $b_2$  represents the medium boiler with 5 MW of capacity and  $b_3$  is used for the biggest boiler with the capacity of 15 MW.

Capacity differences influences, not only the capacity availability, but also the size and the equipment capital price. In equation (3.17) the equipment cost is directly proportional to the total capital cost. As shown in Figure 9 equipment costs are not linear. Therefore, capacity increases and price drops.



Figure 9: Equipment Costs

As said before, not only the price is influenced by the capacity of boilers, but also its size. In agreement with the boilers size, positions for the boilers have some restrictions. Boilers type 3, corresponding to the bigger boilers, can be located only in cells located outside the city. It is prohibited to have boilers type 3 inside the cell, while the small and medium boilers can be installed whether inside or outside. This is defined by the equation:

$$yb_{b_3,q} = 0, \forall_q \tag{4.1}$$

Another important aspect is the production rate defined for each boiler. This rate compares the capacity with the price of the boiler. For small boilers a high rate is expected. Production rate for  $b_1$  is 1 W per  $\in$ , for  $b_2$  is 0.7 and 0.5 for  $b_3$ . This means that production will be expensive in small boilers, as usual.

Efficiency is the last point which needs to be mentioned. Like happens in other devices, sometimes it is not possible to work with 100% of thermal efficiency. This means that some losses occur during the production process. So it is proposed an efficiency coefficient to all boilers type. In this case, it is assume that boilers convert around 90% of the fuel they use into useful heat.

A summary of the boilers characteristics can be seen in Table 4.

Boiler	-	Capacity (MW)	Production Rate	Efficiency (%)	Costs (€)
	b <sub>1</sub>	1	1		15000
	b <sub>2</sub>	5	0,7	90	30000
	b <sub>3</sub>	15	0,5		35000

Table 4: Boilers characteristics and costs

#### 4.1.1.5 Transport facilities- pipelines

Pipelines permit the heat transfer from boilers to consumers. Its characteristics are diameter, supply temperature and return temperature. The length of a total pipeline network is influenced by the positions of boilers in the city so it can only be determined after the optimisation. The assumptions for the pipelines are described in Table 5.

Diameter (m)	0.1
Supply temperature (°C)	80
Return temperature (°C)	10

Table 5: Pipeline Characteristics

## 4.1.1.6 Costs

Costs are one of the most difficult aspects do define due to the existence of various facilities in supply chain, that involve diverse funds. Sometimes there is a lack of detailed information, or it changes day-to-day. It also happens that costs are not the same from one country to another. So it is decided that representative costs will be used. Capital and operational costs can be consulted in Table 6.

Costs type		Index	Costs value (€/W)
Capital: pipelines for ext. distribution		c_etransp	5
Capital: pipeline for int. distribution		c_itransp	5
			15000
Capital: equipment	b <sub>2</sub>	c_ce	30000
			35000
Operational: natural gas		op_gas	0.001
Operational: pumping		op_pump	0.001
Operational: operation & maintenance			1500
		c_om	3000
			3500

Table 6: Costs definition – case 1

Costs where capacity of equipment is not refer are assumed to be fixed for all boiler types.

## 4.1.2 Results and discussion

The aim of the proposed model was to satisfy the heat demand listed in Table 1 by mapping out all possible energy supply chain configurations. As explained, the model was tested with the supply of hot water via pipelines to residential or industrial consumers, so as to satisfy the heat demand. Boilers are the mechanism responsible to produce hot water using natural gas as a resource. The optimal solution led to the network structure shown in Figure 11.

Cell index is enclosed within a circle in the middle of each cell. Boilers symbols mean that exist, at least, one boiler according to the size of the symbol. The total number of boilers is indicated near each symbol. Heat transfers are represented by an arrow, which indicates the heat transfer direction. The number in the middle of each transportation link, i.e. arrow, denotes the corresponding transfer heat from one cell to another one. In Figure 10 it is possible to see the used symbols in all network structures.



Figure 10: Legend for network structure



Figure 11: Network structure for optimal solution - case 1

Figure 11 shows that, for uniform population distribution, boilers appear almost uniformly distributed. Outside positions are not occupied and the most common situation is one medium and one small boiler installed in internal positions. It is also used just one or two medium boilers in internal positions and there is a production facility in every cell, which means that every cell will produce its own heat. Although, some cells satisfy their local heating demand with heat transfer from neighbouring cells. Heat exportation from one cell to another is a more economical alternative than establishing a new production facility in those areas where there is just a small portion of demand to fulfil. It should also be mentioned that heat importations minimize the total cost and avoid heat losses within the cell, as explained in section (3.3). Pipelines are used to deliver hot water from the point of production to consumers, and short distances are preferable as visible in the Figure 11.

The cost of the examined network configuration is 1 121 055 €. A breakdown of these costs is listed in Table 7.

Costs for Possible Network Configuration – Case 1 in €		
Capital Costs		
Equipment Cost	690 000.000	
Transportation modes	213 985.984	
Total Capital Cost	903 985.984	
Operational Costs		
Fuel Cost	71 204.577	
Pumping Cost	76 864.563	
Maintenance Cost	69 000,000	
Total Operational Cost	217 069.140	
Total Costs	1 121 055	

Table 7: Costs for Possible Network Configuration – case 1

It can be seen from Table 7 that equipment costs is the most expensive section in this network. All the 29 productions facilities used in this network represent more than 60% of total costs. And the total capital costs represent 80% against 20% of operational costs.

The MILP model was solved by an Intel Pentium M, 1.73 GHz Acer machine via a modelling tool referred to as GAMS using Cplex v9.0 code. The corresponding computational statistics are summarised in Table 8. As can be seen from the above results, to solve the optimal configuration a low gap is attained (0,45%) and the short CPU time is satisfactory (1000 seconds). This promising is a good indication of having a model that is capable of mapping out the whole heating network for a real city, like London or Lisbon, in a short time with a minimal optimality gap.

Table 8: Summary of computational results for case 1

Number of single equations	5878
Number of single variables	2694
Number of continuous variables	608
Optimality gap (%)	0,45
CPU time (sec)	1000,06

## 4.2 Case Study 2: normal distribution

Some initial conditions of this case study are equal to what is explained in case 1, like the characteristics of the cells. A city is again divided into sixteen grids and the size of each is the same. However, the population distribution in this case is different from one cell to another. 'Normal' population distribution is now studied since it is known that population will grow in urban areas, but not uniformly in all parts of the city. Urbanization is explained by several global and local factors. As global factors can be mentioned the national and international market growth and economic globalization. Local factors include the populations' socioeconomic situation, housing available and land-use policies.

According to a 'normal' distribution, there will be a high population concentration and density in the middle of the city. While in the neighbourhood area the tendency is to have low population concentration. Consequently energy demand will be higher in cells located at the mid-point of the city and it is expected that boilers will be installed in a bigger quantity in central cells and will not be uniformly distributed all over the city.

## 4.2.1 Input Data

### 4.2.1.1 Demand

In Table 9 the population is shown, according with the types of consumers, and consequently demands. This example has the same population as in case 1 (40 000 inhabitants), and it is assumed that residential and commercial users are located essentially in the centre of the city. Cells in the corners, such as cell 1, 4, 13 and 16, will be almost desert, and in the middle, cells 6, 7, 10 and 11, will include more than 50% of total population (see Figure 12). Due to this 'normal' distribution, there will be a high demand in the centre of the city. Demand is calculated with the values given in Table 9.

Call index	Population		
Cell Index	Commercial	Residential	Demand (W)
р1	300	300	600000
p2	1000	1000	2000000
р3	1000	1000	2000000
p4	300	300	600000
р5	1000	1000	2000000
p6	2700	2700	5400000
р7	2700	2700	5400000
р8	1000	1000	2000000
р9	1000	1000	2000000
p10	2700	2700	5400000
p11	2700	2700	5400000
p12	1000	1000	2000000
p13	300	300	600000
p14	1000	1000	2000000
p15	1000	1000	2000000
p16	300	300	600000

Table 9: Population and Heat Demand - case 2

As in case 1, it is considered that the number of residential and commercial consumers is the same. Table 2 shows more detailed information about consumption per consumer.

It is not indicated, but cells outside the city will not have any heat demand. It is considered that outside the city there is no population and consequently no demand.

#### 4.2.1.2 Other input data

All the remaining data concerning heat transfer and losses, distance, transport and production facilities and costs are the same as defined for case 1 (section 4.1).

## 4.2.2 Results and discussion

Figure 12 shows the optimal network solution for a normal population distribution, where most of the population is situated in the middle of the city. Commercial and residential consumers occupy namely cells 6, 7, 10 and 11. These cells are the ones with higher energy demand. This energy demand will be satisfied, once again, by hot water coming through pipelines. In Figure 10 it can be seen a brief description of the symbols used in the network structure.



Figure 12: Network structure for optimal solution – case 2

In Figure 12 above the contrast of colours between internal cells is visible. Internal cells with a dark colour represent places with higher energy demand, while cells with bright colours are used to regions with lower demand, and normally with a small number of inhabitants. Boilers are uniformly distributed but a larger number is installed in the middle of the city, where there is more energy demand to satisfy. Boilers of type 3 are not necessary to assure all the heat transfer to customers, so there will not be any boiler in positions outside the city. In some cases, like cells 1, 4, 13 and 16 it is cheaper to transfer the heat from neighbourhood cells instead of

setting up a new production facility. As in case 1, some cells have two types of boilers installed and also import heat, these two aspects will avoid high internal heat transfer within the cell.

Compared with case 1 a 'normal' population distribution results in a cheaper network option, although, the number of consumers is equal and they all have the same consumption. The total cost of this network configuration is 1 014 129€ and a description of those can be consulted in Table 10. This higher cost can be explained by the decreasing number of installed boilers within the city (represents lower equipment costs), these changes will increase the operational costs, in particular the heat pump operation. However, the costs difference in capital costs has a bigger impact on the final results.

Costs for Possible Network Configuration – Case 2 in €		
Capital Costs		
Equipment Cost	585 000.000	
Transportation modes	203 499.977	
Total Capital Cost	788 500	
Operational Costs		
Fuel Cost	66 200.563	
Pumping Cost	100 928.439	
Maintenance Cost	58 500.000	
Total Operational Cost	225 629	
Total Costs	1 014 129	

Table 10: Costs for Possible Network Configuration – case 2

Once again, boilers costs represent two thirds of the total costs and the capital costs represent more than 75%, while operational costs represent only 25% of the total. To solve this model was used an Intel Pentium M, 1.73 GHz Acer machine via GAMS using Cplex v9.0 code. The case statistic results can be found in Table 11.

Number of single equations	5878
Number of single variables	2694
Number of continuous variables	608
Optimality gap (%)	0,0009
CPU time (sec)	17,52

Table 11: Summary of computational results for case 2

This model was solved faster than in case 1 and presents a realistic network configuration mode, which is very close to the 'best possible' solution due to its small gap. The number of equations and variables are equal to case one.

## 4.3 Case Study 3: random distribution

This case is another example of the application of the developed model. In this case a random population distribution is applied and analysed. In real cases, sometimes there are some areas with non-uniform population distribution due to some physical and human factors that can affect population density. Physical factors include resources, climate and relief (shape and height of land); while human factors are of political, social and economic reasons. Due to what is mentioned above, it is important to analyse a random population distribution.

According to a random distribution, some unpopulated areas will exist in the city or sparsely populated areas, which means areas with few people and where normally it is difficult to live. However, there will be also some areas with high concentration of population, places that are densely populated.

## 4.3.1 Input Data

## 4.3.1.1 Demand

In this case the population will be different from cases 1 and 2, due to the lack of predictability of population distribution. Although there are 40 000 inhabitants in this city, located in internal cells of the city, there is a high variety of consumers and it is not possible to predict the result of this case.

To calculate the energy heat demand in each cell it is necessary to define the population and consumption per person. These data is expressed in Table 12 and in Table 2.

Collindox	Population		Domand (M)
Cell Index	Commercial	Residential	Demand (W)
p1	200	530	730000
p2	800	1200	2000000
р3	1200	2800	4000000
p4	2900	1300	4200000
р5	650	120	770000
р6	70	110	180000
р7	1500	2800	4300000
p8	1800	2100	3900000
р9	2200	2000	4200000
p10	2400	2700	5100000
p11	700	500	1200000
p12	10	300	310000
p13	3100	2800	5900000
p14	900	1400	2300000
p15	220	50	270000
p16	10	30	40000

Table 12: Population and Heat Demand – case 3

#### 4.3.1.2 Other input data

All the remaining data concerning heat transfer and losses, distance, transport and production facilities and costs are the same as defined for case 1 (section 4.1).

## 4.3.2 Results and discussion

Although it is a random population distribution, it was possible to predict areas with higher probability of having installed boilers. These are essentially top right and bottom left corners. The model confirms this idea (see Figure 13).



Figure 13: Network structure for optimal solution - case 3

Figure above shows areas with two types of boilers and areas with no production facility, due to the random energy demand. This heat transfer configuration seems a little confused, although this energy network represents the most economical option for this population distribution. The used random distribution does not need any boiler type 3 installed in outside cells of the city. In order to satisfy all the city demand 21 boilers type 1 and 2 are required. This is a higher number when compared to case 2. This higher number of boilers implies a lower capital cost for the network, as shown in Table 13.

Costs for Possible Network Configuration – Case 3 in €		
Capital Costs		
Equipment Cost	585 000.000	
Transportation modes	198 586.761	
Total Capital Cost	783 587	
Operational Costs		
Fuel Cost	65 943.745	
Pumping Cost	89 598.482	
Maintenance Cost	58 500.000	
Total Operational Cost	214 042	
Total Costs	997 629	

#### Table 13: Costs for Possible Network Configuration – case 3

Curiously the total costs of the optimized network in this case is 997 629€, which is the cheapest option when compared with 2 cases above, and all options have the same numbers of inhabitants and energy demand within the city, only the type of distribution is different. This lower cost can be explained by the high number of installed boilers in cells with a huge heat demand that will avoid transportation modes and reduce the pumping costs when compared to the other two cases.

Table 14 resumes all the computational statistical data of this model that was solved in the Intel Pentium M, 1.73 GHz Acer machine via GAMS using Cplex v9.0 code. The time required to solve this network configuration is very short and the solution presents a null optimality gap, which means that this network configuration correspond to the 'best possible' network.

Table 14: Summary of computational results for case 3

Number of single equations	5878
Number of single variables	2694
Number of continuous variables	608
Optimality gap (%)	0
CPU time (sec)	5,34

## **Chapter 5**

# 5 Multiscale network design

In chapter 4, the virtual city is represented by a grid structure but it is treated as a whole block, using a single scale method. This optimization can be used in another perspective where the solution from single scale is captured and again overall costs are minimized but this time in an individual cell. The aim is to provide an optimal or near-optimal solution that the single scale methodology was unable to obtain in an efficient way. Figure 14 shows this multiscale concept.



Figure 14: Multiscale concept

The first step is to solve the energy single network problem. The identification of the macroscale variables and the macroscale structure of the problem, such as heat balance and losses, heat transfer constraints, operational and capital costs is done. Then the micro scale model is solved where each cell is divided into several parts as previously done for the global region. This is found on the model described in chapter 4 with some changes. Some variables are added to maintain the same model ideology. One random cell will be studied, but surrounding cells will be also used to represent the heat exportations and importations. Both models are connected and the results are analysed simultaneously.

## 5.1 Case Study 4: multi scale in one cell

The methodology explained before will be applied to case 3 of chapter 4 where the chosen cell is number 10. The previous optimisation solution (section 4.3) will be compared with the results

of the multiscale technique and discussed. It is expected that the total costs will be minimized and it is possible to see the energy network configuration within one cell.

## 5.1.1 Input Data

The input data needed for this case study is taken from the results of case 3 of chapter 4, where a random population distribution was analysed. The objective is to improve the optimization network calculated before, using the same model, but now focus on a small cell of the city.

## 5.1.1.1 Demand

In this case, the demand of cell 10 from case 3 will be randomly distributed in the new cell. So there will be 5100 inhabitants in this cell which is the sum of the population earlier defined in cell 10: 2400 commercial and 2700 residential consumers, and the total demand will be 5,1 MW, like in cell 10. In Table 15 more detailed information is shown.

Collinday	Population			
Cell Index	Commercial Residential			
р1	50	50	100000	
p2	475	475	950000	
р3	250	250	500000	
p4	225	225	450000	
р5	450	450	900000	
р6	50	50	100000	
р7	200	200	400000	
р8	400	400	800000	
p9	450	450	900000	

#### Table 15: Population and Heat Demand – case 4

According to the type of consumers and periods of the day, consumption per person can change. Although it is the same as used in cases of chapter 4 so it can be consulted in Table 2.

Outside grids will be treated in a different way but, as for the other cases; they will not have any heat demand.

#### 5.1.1.2 Heat transfer and losses

Heat transfer and losses are calculated in the same way as before. I

Table 3 is defined all the required data.

#### 5.1.1.3 Distance

Distances in this case will be different from the cases solved previously, but the same methodology is used, Euclidean method, and Pythagoras theorem to calculate it. This new city, represented by cell 10 from case 3, is divided into 9 cells, and each cell has the same size, that is  $\frac{2}{3} \times \frac{2}{3}$  km.

Equation (3.1) will be used again to calculate external distances and it can be seen in Appendix 3. Internal losses will be calculated as it has been done for previously cases.

### 5.1.1.4 Production facilities - boilers

As it is an internal cell, only two types of boilers will be analysed. The initial assumption that boilers with bigger size can not be installed at internal positions is kept, as shown in equation (5.1) and in this case the optimisation is applied only to internal positions of the city.

$$yb_{b_{3},p} = 0$$
 (5.1)

Only heating boilers with the capacity of 1 MW and 5 MW will be considered. Production rate for each boiler, thermal efficiency and capital and operating costs are the same used before but Table 16 shows all the information.

Boile	er	Capacity (MW)	Production Rate	Efficiency (%)	Costs (€)
÷.	b <sub>1</sub>	1	1		15000
	b <sub>2</sub>	5	0,7	90	30000

Table 16: Characteristics of boilers 1 and 2

### 5.1.1.5 Transport facilities and Costs

Pipelines definition remain the same, see Table 5. In Table 6 are defined all the costs to solve this case.

#### 5.1.1.6 Additional Information

In this case study, it is necessary to define and implement some new variables in order to apply the multiscale technique.

*input*<sub>q</sub> - represents the heat importation of each square. In the case in study it corresponds to the heat imported from cells 6, 9, 11 or 14 (surrounding cells). So each side of the 'small city' will have an importation grid where total heat transfer will be signed. This value will influence the heat balance, because the grid demand can be satisfied by this transferred heat. In this case only cell 6 is considered, because is the one that exports heat to cell 10 in previously case 3.

 $output_q$  - represents the total exported heat from the cell in study (cell 10 of case study 3). This variable will also change the heat balance. We assume that the exportation will be done by square number 5, which is the one that represents the middle of the cell.

Due to addition of these new variables, a new heat balance equation is defined in this case:

$$input_{p} + imp_{p} + pref_{p} = demand_{p} + eloss_{p} + iloss_{p} + exp_{p} + output_{p}, \forall_{p}$$
(5.2)

It is defined that the costs from the heat transfer are attributed to the cell that receives the heat and not to the cell that exports it. Cell that imports will have the heat transfer costs, while the cell that exports will not have any associated cost. Due to Input variable, a new cost is defined as indicated below:

$$cost_in = \sum_{p} (cost_inp_p + cost_inq_p)$$
(5.3)

Where:

cost\_inp<sub>p</sub> represents the cost of heat transfer from cell p, in the present case is from cell 9 (of case 3) to each division of the cell in analysed (cell 10);

 $cost_inq_p$  represents the cost of the heat transfer within the nine divisions of the cell (e. g cell 10).

The objective function also needs some adaptation. The new cost is included in objective function, as shown below.

$$z = opc \_ prod + opc \_ transp + opc \_ om + cc \_ transp + cc \_ prod + cos t \_ in$$
 (5.4)

The result of this optimization model will be compared will the initial costs given by case study 3.

## 5.1.2 Results and discussion

The aim of this case was to apply the model explained in previous chapters, in one cell of the city, using a multiscale technique, to reduce the total costs within the cell. The objective is to find an optimal network configuration within one cell, looking at the cell as a whole and divided in grids, like what was done before with cities. In this way more detailed results will be obtained. The initial conditions must be the same as the ones defined at the macro scale such as energy demand, boilers characteristics and so on. In order to assure it, some modifications in the model were made, some parameters were added and some equations changed.

The final results applied to cell 10 of case 3 (see section 4.3) are shown in Figure 15.



Figure 15: Network structure for optimal solution – case 4

The first big difference in this network is the reduced number of production facilities. While in case 3, cell number 10 has 3 boilers, in this case it requires only 2 medium boilers installed in internal cells. In this case, population is randomly distributed and, as in case 3, boilers are set up in cells with higher energy demand. This is only possible due to a better location of each

production facility when applied the multiscale concept. Boilers are located in cells that have more heat demand, instead of being in the middle.

It is also visible that every square of cell 10 will import heat from the external position instead of only one heat transfer to the middle of the analysed cell; however this is the most economical option for this network.

Costs for Possible Network Configuration – Case 3 and 4 in €				
	Cell 10 from case 3	Case 4		
Capital Costs				
Equipment Cost	75000	60000		
Transportation modes	19533	10200		
Total Capital Cost	94533	70200		
Operational Costs				
Fuel Cost	8000	6108		
Pumping Cost	5943	9359		
Maintenance Cost	7500	6000		
Total Operational Cost	21443	21467		
Extra Cost (imported heat)		2377		
Total Costs	115976	94044		

Table 17: Costs for possible network configuration - case 4

The cost comparison shown in Table 17 illustrates the decrease of the total costs when applying the multiscale method. In cell 10 the total costs is now 94 044€, while it was 115 976€ in case study 3. Remembering that the energy demand and all the conditions are the same, this possible network configuration represents an excellent improvement, with a costs reduction by almost 20%.

It is also proved that applying this multiscale method it is possible to satisfy all the energy demand of this part of the city in an efficient way, reducing the total production (real production decreases from 9.9 to 7.9 MW) and decreasing the total costs of the network.

In this case, there is another important point which should be mentioned: the heat losses within the cell are reduced in almost 50%, from case study 3 to case study 4. Decrease of heat losses during transfers contributes to a better solution of the optimization model. It permits to satisfy

the heat demand of the cell with a low production quantity, which has a positive effect in objective function.

As with previous models, the network configuration was solved using GAMS with Cplex v9.0 code. An Intel Pentium M, 1.73 GHz Acer machine was utilized to execute the run. The corresponding computational statistics are visible in Table 18. This model presents an excellent central processor usage time and in this case the MIP solution is coincidence with the best possible solution (0% optimality gap).

Number of single equations	1974
Number of single variables	1014
Number of continuous variables	204
Optimality gap (%)	0
CPU time (sec)	1,080

Table 18: Summary of computational results for case 4

# **Chapter 6**

# 6 Influence of costs

Case study 5 will be the last one and the aim of it is to analyse and understand the influence of costs in the network configuration. As explained before, capital and operational costs are determined according to values previously defined, and the minimization of the overall cost is the objective of this optimisation. These costs are just representative of a possible real situation. In this case study costs are changed to see the difference in the given solution network. Two options are studied: in option 1 capital costs are doubled and operational costs remain its initial values is the. In option 2 capital costs preserve its value while operational costs doubled.

## 6.1 Case study 5: influence of capital and operational costs

Some changes are expected in the network configuration, in one case is expected a larger number of production facilities installed in the city and less heat transfers, while in the other option is expected a small number of boilers and a high number of heat transfers.

## 6.1.1 Input Data

This case will be similar to case study one, with changes in the costs definition. Population will be uniformly distributed within the sixteen divisions of the city, so the total demand will be the same.

## 6.1.1.1 Demand

As already mentioned, in this case is used a uniform population distribution, like in case one, so again there will be 2 500 inhabitants in each cell, equally divided in the two types of consumers. The city will have 40 000 inhabitants and its distribution can be checked in Table 1 and in Table 2 where is expressed the diary consumption per person, according the type of consumer.

### 6.1.1.2 Costs

As explained before, two options will be studied. Table 19 shows the differences of each option.

Option 1: Double capital costs

Option 2: Double operational costs

Costs type		Index	Costs value (€)	Costs value (€)
		Index	Option 1	Option 2
Capital: pipelines for ext. distribution		c_etransp	10	5
Capital: pipeline for int. distribution		c_itransp	10	5
			30000	15000
Capital: equipment	b <sub>2</sub>	c_ce	60000	30000
			70000	35000
Operational: natural gas		op_gas	0.001	0.002
Operational: pumping		op_pump	0.001	0.002
			1500	3000
Operational: operation & maintenance	b <sub>2</sub>	c_om	3000	6000
			3500	7000

Table 19: Costs definition - case 5

## 6.1.1.3 Other input data

All the remain data concerning heat transfer and losses, distance, transport and production facilities are the same as defined for case 1 (section 4.1).

## 6.1.2 Results and discussion

In this case is evident the influence of capital and operational costs while defining an energy network configuration within one city.

Figure 16 shows the results of the model applied to option one.



Figure 16: Network structure for optimal solution - case 5 option 1: increase capital costs

In option 1 (see Figure 16) the result showed a tendency to implement boilers of type 3 in outside positions, to satisfy all the heat demand. It is visible that the majority of the boilers are not set up inside the city, there are only 5 boilers installed in central cells (four boilers of type 2 and one boiler of type 1), and these boilers are responsible for only 23% of the real production (production that already considers the efficiency of each boiler). Boilers in surrounding cells are in larger number and represent 77% of real production. Heat transfers are mainly from boilers of type 3, that are located in outside cells, to internal parts of the city. If analysed in detail, in Figure 16 is visible that there are only 4 heat transfers between internal cells. According to this input data, the possible network structure contains 11 boilers in the model to ensure the heat demand.

Figure 17 defines the optimal energy network configuration that represents the most economical alternative that guarantees the production of all the heat demand and its transfer to consumers, using the input data of option 2.



Figure 17: Network structure for optimal solution - case 5 option 2: increase operational costs

In option 2, for capital costs are used the same values that were defined in previously cases, but operational costs have doubled. This case satisfies the heat demand with 32 boilers, almost three times the number of boilers in option 1. Every cell has its own boilers and, in this case, the total heat travelled distance is shorter than in the first option. In option two the heat passes through 36 km of underground pipelines while in option one this distance increases to almost 60 km. Only boilers of type one and two are chosen to produce all the necessary heat and, in contrast with case one, it does not occur any heat transfer from outside to internal cells. Boilers located in internal cells represent 100% of total heat production.

Next, Table 20 illustrates the costs difference between these two options. It should be remembered that the total costs of case one (that has the same population distribution and heat demand) is 1 121 055€.

Costs for Possible Network Configuration – Case 5 in €				
	Option 1	Option 2		
Capital Costs				
Equipment Cost	750000,000	720000,000		
Transportation modes	574928,634	204372,800		
Total Capital Cost	1324929	924373		
Operational Costs				
Fuel Cost	62824,809	140713,648		
Pumping Cost	456331,404	113621,120		
Maintenance Cost	37500,000	144000,000		
Total Operational Cost	556656	398335		
Total Costs	1881585	1322708		

Table 20: Costs for possible network configuration - case 5

This case study shows the importance of the total costs of capital, operation and maintenance of any system, in the beginning phase of any project and its implications.

With an increase of capital costs the tendency is to implement a few numbers of boilers in the city and avoid internal cells to install it. Normally this case has a longer heat travel distance that is almost the double of the value in option two. Option one is the most expensive option with a total cost of 1 881 585€ which represents an increase of 70% when compared with case one.

When the difference is in operational costs (that double its value), the number of production facilities are expected to increase so as to decrease the heat travel distance and the majority of the boilers tends to be installed within the city, in internal cells. However the total costs of the network present in this case is not so different from case one: 1 322 708€ which is equivalent to an increase of 20%.

In both cases, the capital cost of the optimal network includes the equipment cost and transportations modes, and corresponds to 70% of total costs.

The MILP models were solved in an Intel Pentium M, 1.73 GHz Acer machine via a modelling tool referred to as GAMS using Cplex v9.0 code. Table 18 shows the computational statistic for each option. The optimal gap was assumed to be 0.01%, decreasing this percentage would have increased the accuracy slightly and decrease speed of computation. It appears from Table 21 that the solution time is reasonable for a problem of this size.

	Option 1	Option 2
Number of single equations	5881	5879
Number of single variables	2697	2695
Number of continuous variables	608	608
Optimality gap (%)	0,01	0,01
CPU time (sec)	519,830	106,66

Table 21: Summary of computational results for case 5

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# **Chapter 7**

# 7 Conclusions and further work

This chapter provides a summary of the most important conclusions of this thesis. It also outlines some of the potential directions for further work in the area of energy supply chain design.

## 7.1 Contribution of this thesis

The main problem of this work was to define a model that optimizes an urban energy supply chain able to satisfy the heat demand. Reducing the total cost of the supply chain network is the aim of this project which satisfies all the heat demand within a city.

The design of an urban supply chain network is not an easy task due to several factors. The first reason is the complexity of the energy supply chain network which involves many components within the design framework, like production plants and transportation modes. This involvement increases the level of complexity of the energy supply chain network. The degree of this complexity is increased by the considerable uncertainty in energy demand. The rigorous existing environmental regulation is the second reason that affects the design of a future energy supply chain. Government is pressing in order to reduce the greenhouse gases emissions, essentially CO<sub>2</sub> emissions, by increase the efficiency of the process and decrease the production losses with the guarantee of satisfying the heat demand with less production, is vital in this study. Another important factor is the concern of building a viable and sustainable energy supply chain network. In this case the chosen fuel to produce the required heat for the city is natural gas, but it could be produced from a variety of feedstock, like coil or oil at a wide range of sizes from country scale to household scale. It is important that the optimization model presented in these cases can represent a real situation or can be easily adapted to other inputs. And collecting the needed and correct data marks the fourth reason for the difficulty in designing an energy supply chain. All the above explained reasons will significantly influence the strategic decisions involved in choosing the optimal configuration of a future energy supply chain.

This project has not focused on a specific area of the supply chain such as production, distribution or storage but aimed at developing quantitative tools that can support strategic decisions in heating supply chain design and operation. The suggested mathematical model delineates the optimal configuration of heating supply chain to fulfil the demand within one region (e.g. city). This model comprises: grid squares, production facilities, transportation modes, heat demand that is affected by the number of customers in each cell, the consumption per person and the type of the day. Distance between cells or within one cell and heat losses

are two factors that were also analysed in order to build this model as realistic as possible. The decisions to be determined, by this model, were:

- > The location, number and capacity of the boilers to be installed;
- The required production of each boiler and the associated network configuration (location of pipelines);
- > Flow rate of energy throughout the network and its direction;
- > The total cost of this heating supply chain network.

Moreover, the developed model is mainly driven by heating demand, which is assumed to be previously known.

Within this thesis, the suggested model was extended to three different cases: single scale, multi scale methodology and influence of costs. The versatility of the model was examined using different population distributions within the city, changing the operational and capital costs and applying the multiscale concept.

The model was firstly explored using a single scale technique was applied to three situations. Different population distributions were applied to a virtual city, although all the situations have the same number of inhabitants. The input data was alike in terms of the characteristics of boilers, heat transfers and losses and costs definition. They also utilised the same transportation modes. Uniform, 'normal' and random distributions were defined and analysed. The key difference between these three configurations lied in the heat demand in each cell due to different population density. The reason behind this variation was to study the effect of possible population distribution in the design of a heating network structure as well as to compare the differences in costs. Based on assumptions and taking costs as a performance measure, it is found that the optimal configuration of the future heating supply chain is the installation of boilers and production in cells where there is high demand levels. If there is a small heat demand in one cell to fulfil, or the production is not enough to satisfy all the heat demand, normally the optimal solution imports the needed heat from a neighbour cell instead of installing a new boiler. It was also demonstrated the tendency to import by every cell of the city. In all study cases, there is always imported heat from internal cells, what is justified by the heat losses decrease.

The second part considered a multiscale concept that was applied to the results from case study 3. One cell was chosen and the aim was to optimize and understand the difference between the results. The network configuration within the cell was defined and the total cost decrease when rerun the optimization model. It means that it is possible to optimize the final solution using the multiscale technique, due to appropriate boilers positions according to population distribution and decrease of heat losses.

The third part studied the influence of capital and operational costs. This model was applied in a city with the same population distribution that case one: uniform distribution. The results were obvious and conclusive. When capital costs are expensive the tendency is to invest in boilers with maximum capacity so as to avoid high fixed costs with equipment. Heat transfers tend to increase from external cells to internal positions. In contrast with this example, when operational costs are expensive the optimal configuration presents a large number of production facilities installed within the city. The aim in this case is to avoid costs between the heat transfer and distribution to customers, so heat travel distance tends to be minimized and number of boilers within the city tends to increase.

Both cases show how variations in demand distribution (case one, two and three), alterations in model perspective (case four) and changes in the numerical values of various problem parameters (case five) may lead to significant changes in the structure of the optimal network configuration. This confirms the impact of input data and demand variability on the design and operation of the future energy supply chain network.

As a global conclusion it can be said that the greatest importance of this project lies in a model that is able to map some of the future energy supply chain configurations. However, this is just representative of a real situation. In real-life problems may be much larger and more complicated, with more decision factors and uncertainty. This model is just the first step in what can be a project with great potential of expansion and many upgrading lines. It has to be remembered that this project is like the 'first stone' in a great structure, which is the BP Urban Energy Systems project.

## 7.2 Future Work

As said earlier this work has a great potential for improvements. There are still several tasks that can be more deeply investigated.

The first suggestion is to widen the range of applications, using more than one type of fuel. In this case it is only considered the burning of natural gas, although there are more possibilities to guarantee the heat production and should be mentioned that natural gas is a non-renewable resource. So extending this model to another conventional or renewable fuel will enrich this project. Following this idea at the initial point of the supply chain, also the end point can be improved, adding new variables to the model. Instead of satisfying only the heat demand, electricity and cooling demand should be considered, for example, and consumers can be divided, not only in residential and commercial, but in two more categories: industrial and transport consumers.

In these models the production, transportation and distribution aspects were defined, but it could also be analysed and added heat storage facilities and the fatigue and lifetime of the installations.

Another important point is the temperature levels at which the heat is produced. In this model temperature is previously defined, but some models can determine the optimal supply temperature in the network. Benonysson, et al., 1994 presents a model that minimizes the costs of the district heating with the optimal selection of supply temperature.

Although natural gas is a very clean, light and efficient fuel, it emits some  $CO_2$  when burned. Given the overwhelming and urgent need to reduce emissions, emissions of greenhouse gases should be included in this model. The aim would be, not only to reduce costs, but to reduce gas emissions.

# 8 Bibliography

Ajah, Augustine N.; Patil, Anish C.; Herder, Paulien M. and Grievink, Johan (2006) Integrated conceptual design of a robust and reliable waste-heat district heating system, Issue 7 (27), Pages 1158 - 1164.

Aspelund, Audun and Jordal, Kristin (2007) Gas conditioning - the interface between CO2 capture and transport, Issue 3 (1), Pages 343 - 354.

Bejan A. (1995) Theory of heat transfer-irreversible power plants--II. The optimal allocation of heat exchange equipment, Issue 3 (38), Pages 433 - 444.

Benonysson, Atli; Bøhm, Benny and Ravn, Hans F. (1995) Operational optimization in a district heating system, Issue 5 (36), Pages 297 - 314.

BP – Imperial College Urban Energy System Project, First Annual Report, December 2006.

Chang, Yoon-Suk; Jung, Sung-Wook; Lee, Sang-Min; Choi, Jae-Boong and Kim, Young-Jin (2007) Fatigue data acquisition, evaluation and optimization of district heating pipes, Issues 14-15 (27), Pages 2524 - 2535.

Çomaklı, Kemal; Yüksel, Bedri and Çomaklı, Ömer (2004) Evaluation of energy and exergy losses in district heating network, Issue 7 (24), Pages 1009 - 1017.

Curti, V.; Favrat, Daniel and von Spakovsky, M.R. (2000) An environomic approach for the modeling and optimization of a district heating network based on centralized and decentralized heat pumps, cogeneration and/or gas furnace. Part II: Application, Issue 7 (39), Pages 731 - 741.

Dahm, J. (2001) District Heating Pipelines in the Ground - Simulation Model, Version 1.

Davis, P.; Burn, S.; Moglia, M. and Gould, S. (2007) A physical probabilistic model to predict failure rates in buried PVC pipelines, Issue 9 (92), Pages 1258 - 1266.

Dzenajavičienė, E.F.; Kveselis, V.; McNaught, C. and Tamonis, M. (2007) Economic analysis of the renovation of small-scale district heating systems - 4 Lithuanian case studies, Issue 4 (35), Pages 2569 - 2578.

Garrard A.; Fraga E.S. (1998) Mass exchange network synthesis using genetic algorithms, Issue 12 (22), Pages 1837 - 1850.

Larsen, Helge V.; Bøhm, Benny; Wigbels, Michael (2004) A comparison of aggregated models for simulation and operational optimisation of district heating networks, Issues 7-8 (45), Pages 1119 - 1139.

Lazzarin, R.; Noro, M. (2006) District heating and gas engine heat pump: Economic analysis based on a case study, Issues 2-3 (46), Pages 193 - 199.

Li, Lianzhong and Zaheeruddin, M. (2004) A control strategy for energy optimal operation of a direct district heating system, Issue 7 (28), Pages 597 - 612.

Luo, Lingai; Fan, Yilin; Toundeur, Daniel (2006) Heat exchanger: From micro- to multi-scale design optimization, Issue 13 (31), Pages 1266 - 1274

Morais, M. S. and Marangon Lima, J.W. (2007) Combined natural gas and electricity network pricing, Issues 5-6 (77), Pages 712 - 719.

Nystedt , A .; Shemeikka, J.; Klobut, K. (2006) Case analyses of heat trading between buildings connected by a district heating network, Issue 20 (47), Pages 3652 - 3658.

Soderman, J. and Pettersson, F. (2006) Structural and operational optimisation of distributed energy systems, Issue 13 (26), Pages 1400 - 1408.

Sugihara, H.; Komoto, J. and Tsuji, K. (2002) A Multi-Objective Optimization Model for Urban Energy Systems in a Specific Area, Issues 6-9 (5), Pages 37 - 42.
### Appendix A

# **Natural Gas History**

In this appendix, production and consumption of natural gas by areas are shown and explained.



#### Figure 18: Natural gas production by area

World natural gas production rose by 3% in 2006, above the 10-years average of 2.5%, driven by above-average growth in Russia and North America. Russia registered the largest incremental growth followed by the US, Egypt and China. UK production continued to decline. Figure 18 shows that Europe and Eurasia have the largest portion of natural gas production, which is around 35% of the total production, while North America is in second place and comprises more than 26%.

Figure 19 show that Europe and Eurasia are the leader, in terms of consumption. Although the USA is the country with the highest natural gas consumption, which represents more than 20% of total world consumption, while Russian Federation presents only 15%. The tendency of consumption in US is projected to peak in 2020, followed by a decline in 2030. In Europe natural gas is expected to be the growing fuel source, with a gradual demand increase until 2030.



Figure 19: Natural gas consumption by area

#### **Appendix B**

## **B.** Distances

#### **B.1 Distance between cells**

As explained before, heat will be transferred from boilers to consumers' through underground pipelines. The travel distance is an important and required value to determine some variables, like heat losses and operational costs. Each cell is defined by its coordinates, which will be used to estimate the distance.

So between cells, distance can be calculated with the following formula:

dist<sub>p,pp</sub> = 
$$\sqrt{(x_p - x_{pp})^2 + (y_p - y_{pp})^2}$$
 (B.1)

Table 22 shows the distance between internal cells and in Table 23 are visible the distances between external and internal cells. It is not define the distance between external cells, because it is assumed that is not possible any heat transfer from internal to external positions or between external positions.

	р <sub>16</sub>	p <sub>15</sub>	p <sub>14</sub>	р <sub>13</sub>	p <sub>12</sub>	p <sub>11</sub>	p <sub>10</sub>	p <sub>9</sub>	p <sub>8</sub>	p <sub>7</sub>	p <sub>6</sub>	p <sub>5</sub>	p <sub>4</sub>	p <sub>3</sub>	p <sub>2</sub>	<b>p</b> 1
р <sub>1</sub>	8,485	7,211	6,325	6	7,211	5,657	4,472	4	6,325	4,472	2,282	2	6	4	2	0,765
<b>p</b> <sub>2</sub>	7,211	6,325	6	6,325	5,657	4,472	4	4,472	4,472	2,282	2	2,828	4	2	0,765	<b>n</b>
p <sub>3</sub>	6,325	6	6,325	7,211	4,472	4	4,472	5,657	2,282	2	2,282	4,472	2	0,765		
p <sub>4</sub>	6	6,325	7,211	8,485	4	4,472	5,657	7,211	2	2,282	4,472	6,325	0,765			
p <sub>5</sub>	7,211	5,657	4,472	4	6,325	4,472	2,282	2	6	4	2	0,765				
p <sub>6</sub>	5,657	4,472	4	4,472	4,472	2,282	2	2,282	4	2	0,765	•				
p7	4,472	4	4,472	5,657	2,282	2	2,282	4,472	2	0,765	-1					
p <sub>8</sub>	4	4,472	5,657	7,211	2	2,282	4,472	6,325	0,765							
p <sub>9</sub>	6,325	4,472	2,282	2	6	4	2	0,765	-							
p <sub>10</sub>	4,472	2,282	2	2,282	4	2	0,765									
p <sub>11</sub>	2,282	2	2,282	4,472	2	0,765										
p <sub>12</sub>	2	2,282	4,472	6,325	0,765											
р <sub>13</sub>	6	4	2	0,765												
p <sub>14</sub>	4	2	0,765													
p <sub>15</sub>	2	0,765														
p <sub>16</sub>	0,765															

Table 1: Distance between internal cells

	p <sub>32</sub>	<b>p</b> <sub>31</sub>	<b>p</b> 30	<b>p</b> 29	p <sub>28</sub>	p <sub>27</sub>	p <sub>26</sub>	p <sub>25</sub>	p <sub>24</sub>	p <sub>23</sub>	p <sub>22</sub>	p <sub>21</sub>	p <sub>20</sub>	<b>P</b> 19	p <sub>18</sub>	p <sub>17</sub>
р <sub>1</sub>	8	8,246	8,944	10	10	8,944	8,246	8	6,325	4,472	2,282	2	2	2,282	4,472	6,325
p <sub>2</sub>	8,246	8	8,246	8,944	8,485	7,211	6,325	6	4,472	2,282	2	2,282	4	4,472	5,657	
p <sub>3</sub>	8,944	8,246	8	8,246	7,211	5,657	4,472	4	2,282	2	2,282	4,472	6	6,325		
p <sub>4</sub>	10	8,944	8,245	8	6,325	4,472	2,282	2	2	2,282	4,472	6,325	8			
p <sub>5</sub>	6	6,325	7,211	8,485	8,944	8,246	8	8,246	7,211	5,657	4,472	4				
p <sub>6</sub>	6,325	6	6,325	7,211	7,211	6,325	6	6,325	5,657	4,472	4					
p <sub>7</sub>	7,211	6,325	6	6,325	5,657	4,472	4	4,472	4,472	4						
p <sub>8</sub>	8,485	7,211	6,325	6	4,472	2,282	2	2,282	4							
p <sub>9</sub>	4	4,472	5,657	7,211	8,246	8	8,246	8,944		-						
p <sub>10</sub>	4,472	4	4,472	5,657	6,325	6	6,325									
p <sub>11</sub>	5,657	4,472	4	4,472	4,472	4										
p <sub>12</sub>	7,211	5,657	4,472	4	2,282		•									
р <sub>13</sub>	2	2,282	4,472	6,325		1										
p <sub>14</sub>	2,282	2	2,282													
р <sub>15</sub>	4,472	2,282														
р <sub>16</sub>	6,325															

Table 2: Distance between external and internal cells

#### **B.2 Distance within cells**

Distance within each cell is calculated in a different way. It will depend on:

- Size of the cell;
- Number of boilers within the cell;
- Existence of heat importation.

The first point will be the same for every cell. It is considered that the size of each cell is 2x2 km. And the parameter for heat transfer is 1 if there are one or more heat transfer in the cell and 0 otherwise. Figure 20 shows two possible methods to calculate the internal heat transfer distance:

Option 1 - considering all the boilers in the middle of the cell, and the heat transfer for the midpoint



Option 2 - considering a distinct position for each boiler and also for the heat transfer

Figure 20: Methodology for internal distances

In this case study, option 2 will be developed and used. Internal distance is a function of the number of boilers and the heat transfer defined by:

distance<sub>average</sub> = 
$$a + b \times y^{\alpha}$$
 (B.2)

Where

$$v = number of boilers + heat transferm$$
 (B.3)

Internal average distance for a square of 1x1 km can be described as:

distance<sub>average</sub> 
$$\geq \int_{0}^{1} \int_{0}^{1} \frac{1}{6} \left[ \sqrt{2} + \ln(1 + \sqrt{2}) \right] \frac{1}{\sqrt{y}} d_{x} d_{y}^{\alpha}$$
 (B.4)

In Microsoft excel, the line for internal distance was drawn and a linear approximation was applied to define the parameters a and b of the equation above. The line for internal distance was defined for more than one third of all possibilities: in the limit, it is one boiler per consumer and heat transfers for the cell. To a better and real representation of this line, it was decided that seven linear functions are enough to define the general function. Figure 21 above show the general function represented by linear functions.



Figure 21: Linear approximation for y between 1 and 10



Figure 22: Linear approximation for y between 10 and 50



Figure 23: Linear approximation for y between 50 and 3500

It was decided that these seven functions are enough to define the general function. The calculated coefficients are shown in Table 24.

	Coefficient a	Coefficient b
Line 1	0,446364	-637,66
Line 2	0,242312	-127,53
Line 3	0,168525	-45,56
Line 4	0,121156	-15,94
Line 5	0,080771	-4,72
Line 6	0,053205	-1,32
Line 7	0,034092	-0,35

Table 24: Coefficient a and b for linear functions

To simplify the calculations, every function was defined for a square of 1x1 km. So it is needed to double the value to obtain the real internal average distance in this case study.