UNIVERSIDADE DE LISBOA
INSTITUTO SUPERIOR TÉCNICO

Downstream Petroleum Supply Chain
Strategic and Tactical Planning under Uncertainty

Leão José Fernandes

Supervisor: Doctor Susana Isabel Carvalho Relvas

Co-Supervisor: Doctor Ana Paula Ferreira Dias Barbosa Póvoa

Thesis approved in public session to obtain the PhD Degree in
Engineering and Management
Jury final classification: Pass with Distinction

Jury

Chairperson: Chairman of the IST Scientific Board

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Doctor Henrique Aníbal Santos de Matos
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Resumo

O objetivo deste trabalho é implementar modelos de programação matemática que permitam o planeamento estratégico e tático de Cadeias de Abastecimento de Petróleo (CAP), no seu segmento *downstream*, visando maximizar os lucros, e minimizar os custos e riscos, considerando às incertezas nos preços de crude, produtos derivados e procura dos clientes. O problema considera um conjunto de empresas de petróleo que produzem ou importam produtos petrolíferos e comercializam-nos a várias regiões de clientes, através de infraestruturas participadas: refinarias, centros de armazenagem, transportadores e retalho. Consideram-se redes multinível, multiproduto e multimodais, uni e multiempresas que podem ser reestruturadas partindo de redes de CAP existentes, ou planeadas integralmente.

Ao longo da tese, são desenvolvidos quatro modelos de programação linear inteira mista (PLIM), concretamente os modelos individualista, colaborativo, estocástico multiobjectivo, e otimização robusta. Foi desenvolvida uma estrutura generalizada e detalhada da formação de preços e de custos para a CAP, e uma função de linearização de preços para determinar os custos, tarifas e preços de serviços multiempresa com economia de escala por localização ou rota, entidade, produto e tempo. Foi desenvolvido um modelo de risco neutro utilizando a programação estocástica (PE) de dois estágios, e cenários de incerteza na procura dos clientes a fim de maximizar o valor atual líquido esperado (VALE). Foi implementado um modelo da otimização estocástica multiobjectivo (MO), recorrendo ao algoritmo *augmented ε-constraint*, para maximizar o lucro esperado e minimizar uma das várias medidas de risco: a variância, o índice de variabilidade, o risco de incumprimento e o valor-em-risco condicional (CVaR). Dado que a definição de múltiplos cenários de incerteza faz aumentar a dimensão do problema, são desenvolvidas formulações de otimização robusta (OR) que maximizam os lucros totais da CAP para os sucessivos aumentos dos níveis de incerteza nos custos de crude, nos preços de produto, e nas procuras de clientes. Finalmente, é construído para a CAP Português uma base de dados real com localizações, coordenadas geográficas, distâncias, custos, tarifas, preços e procuras, entre 1999 e 2013, que é utilizada para validar os modelos, e apresentados os resultados. Os métodos desenvolvidos sendo generalizados, podem ser utilizados para planear redes semelhantes noutras indústrias.

Palavras-chave: Cadeias de abastecimento de petróleo, Planeamento estratégico e tático, Gestão de incerteza e risco, PLIM estocástico, Otimização robusta.

Abstract

Petroleum Supply Chains (PSC) are a strategic sector made up of complex infrastructure networks, which are strongly influenced by globalization, lean thinking and uncertainty. As costs escalate, margins flatten and product demands stagnate due to competition and environmental regulation, PSC managers are forced to build efficient and flexible networks while integrating multi-entity installations. Here, the strategic and tactical planning delivers an important competitive edge given today's uncertainty. The use of mathematical programming tools for advanced decision support is crucial to improve profits and reduce the PSC costs and risks.

This thesis follows along this need and develops strategic and tactical planning models for the multi-entity, multi-echelon, multi-product, and multi-transportation PSC, under the context of crude costs, product prices, and customer demand uncertainties. The strategic decisions concern the installation, expansion, operation and closure of depots, resource capacities and transport routes. The tactical decisions involve the time-dependent crude oil and derived products' production, storage, primary and secondary transportation, importation and exportation volumes and multi-stage inventories. The models developed determine the network topologies for the current, grassroots and retrofit designs where the total PSC profits are maximized by incorporating the revenues of refineries, bulk storage, transporters and retailers.

This research developed four mixed integer linear programming (MILP) models, specifically the individualistic, collaborative, stochastic, multi-objective stochastic, and robust optimization MILPs. In these works a generalized and detailed price formation and cost structure is proposed for the PSC, and a piecewise linearization function is developed to determine the collaborative economy of scale multi-entity costs, tariffs and prices per localization, route and product. Considering demand uncertainty, the stochastic programming MILP maximizes the expected net present value (ENPV), whereas the multi-objective model implements the augmented ε-constraint algorithm in order to maximize the expected profits and minimize one of many risk mesasures: the variance, variability index, downside risk, or the conditional value-at-risk (CVaR). As the PSC planning under uncertainty is a large scale problem, numerical complexity are observed in the stochastic models, hence, it is proposed a robust optimization approach in order to reduce the problem size when considered the crude oil costs, product prices, and customer demand uncertainty. A real world database is constructed, based on the Portuguese downstream PSC, including locations, geographical coordinates, distances, costs, tariffs, prices, and customer demand between 1999 to 2013, which is used to validate the models, and presented the results. These methods are generalized and may be used to plan similar networks for other industries.

Keywords: *P*etroleum supply chain, Strategic and tactical planning, Uncertainty and risk management, Stochastic MILP, Robust optimization.

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My first thoughts as I begin to write this page is a sense of gratitude and realization, knowing that along with the PhD, I have also lived the biggest challenge of my life. The last 7 years were by far the most strenuous, and at the end I feel much joy and happiness as I come to the concluding moments of the PhD. My supervisors told me right on the onset that my Masters was a 100-meter lap compared to the PhD which is a marathon, and so it was.

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THANK YOU ALL.

With fond affection,

Leão José Fernandes

Table of Contents

List of Figures

List of Tables

CHAPTER 1.

CHAPTER 2.

Acronyms

A1 – Jet fuel

AARC – Affine adjustable robust counterpart

AdC – Autoridade de Concorrência

APETRO – Associação Portuguesa de Empresas Petrolíferas

API – American Petroleum Institute

ARA – Antwerp-Rotterdam-Amsterdam

B5 – Biodiesel

BMU – Billions of monetary units

BTS – Bureau of Transportation Statistics

C3 – Propane

C4 – Butane

CIF – Cost, insurance and freight

CLH – Compañía Logística de Hidrocarburos

COCO – Company owned company operated

CODO – Company owned dealer operated

CVaR – Conditional value-at-risk

DGEG – Direcção Geral de Energia e Geologia

DODO – Dealer owned dealer operated

DR – Diário de República

DSS – Decision support system

EEV – Expectation of the expected value problem

EIA – U.S. Energy Information Administration

ENPV – Expected net present value

Eurostat – European Commission's Directorate-General for Energy

EVPI -– Expected value of perfect information

FOB – free on board prices

FU – Fuel oil

GO – Diesel

GPS – Global pointing system coordinates

HPP – Hours per period

INE – Instituto Nacional de Estatística

INGAA – Interstate Natural Gas Association of America

KKT – Karush–Kuhn–Tucker

LHS – Left hand side

LP – Linear programming

LPG – Liquefied petroleum gases

MAD – Mean absolute deviation

MAPE – Mean absolute percent error

MILP – Mixed integer linear programming model

MINLP – Mixed integer non-linear program

MM3 – Million cubic meters

MMU – Millions of monetary units

MSE – Mean square error

NLP – Non-linear program

NPV – Net present value

NWE – North-West Europe

OECD – Organization for Economic Co-operation and Development

PSC – Petroleum supply chain

RHS – RightHand Side

RO – Robust Optimization

RP – Expected recourse problem solution value

SAA – A sample average approximation method

SC – Supply Chain

SEN – State Equipment Network

SP – Stochastic program

STN – State–task network

TPP – Times per period

U5 – Gasoline 95

U8 – Gasoline 98

VaR – Value-at-risk

VSS -– Value of stochastic solution

WACC – Working average cost of capital

WS – Expected value of the wait-and see problem

CHAPTER 1. Research Objectives and Contributions

1.1 Introduction

The Petroleum Supply Chain (PSC) is a highly competitive sector, which influences the countries' economies and hence is constantly under pressure to make available oil products to markets, timely, cost efficiently, with minimum resources and the lowest risk. The PSC is a sequence of activities that source, transform and deliver petroleum products. This involves huge volumes of product, and therefore implies substantial investment and complex infrastructures, in order to guarantee profitable and stable operation in spite of uncertainty. As shown in figure 1.1, the PSC has two distinct supply chains: one that comprises the activities dealing with the crude oil, which is known as Upstream; and the second that encompasses the activities related to the derived petroleum products, which is known as Downstream. The Upstream consists of the prospection, exploration, production, marine transportation and trading of the crude oil, whereas the downstream comprises of the refining, primary and second transportation, storage terminals, and retail commercialization of the petroleum products. There is a formation though of a new sector called midstream, due to the convergence of natural gas and shale oil, and the importance of trading, which involves the long distance transportation and wholesale storage of crude and petroleum products. This activity is not undertaken by the petroleum companies but by pipeline players such as OAO Transneft, Plains All American, TransCanada, Kinder Morgan. It should be clarified though that some authors refer to refining as midstream (Leiras, Lima), which actually is part of the downstream (API, APETRO, EIA). In this thesis, we use the classic definition from EIA "Upstream refers to crude oil exploration, production, and other operations prior to refining. Downstream refers to refinery operations, product sales, and marketing".

In the upstream, prospection involves identifying world reserves and legal access to drilling sites. Exploration involves prospection, digging and connecting oil wells to production platforms. Production involves startup, shutdown and operating of oil fields and production facilities with safety and profitably. Environmental health and safety regulations, and drilling and recovery technology, and crude oil prices play a major role here. Trading involves buying and selling crude oil and petroleum products, in order to overcome the imbalances between their availability, production, capacities, specifications, demand, and prices.

In the downstream, the refineries buy crude oil, produce petroleum products, and either sell the product to petroleum companies for national consumption or export the product at wholesale market prices, thereby collecting a refining margin. Refineries support the cost, insurance and freight (CIF) charges in case of exportation. The petroleum companies then transport the product up to a storage depot of convenience using the primary distribution which involves the refinery to depot and inter-depot transportation, and is generally undertaken by pipeline, maritime or railway. Companies may import products, where they pay the wholesale prices and support the CIF charges up to their depots. The products are held at storage depots on behalf of the companies, until their transportation using the secondary distribution to retail clients. The latter comprise of petrol stations or direct clients in the transportation, heating, residential, industrial, commercial, electrical utilities, factories, airports and hospitals wherefrom is recovered a retail profit margin.

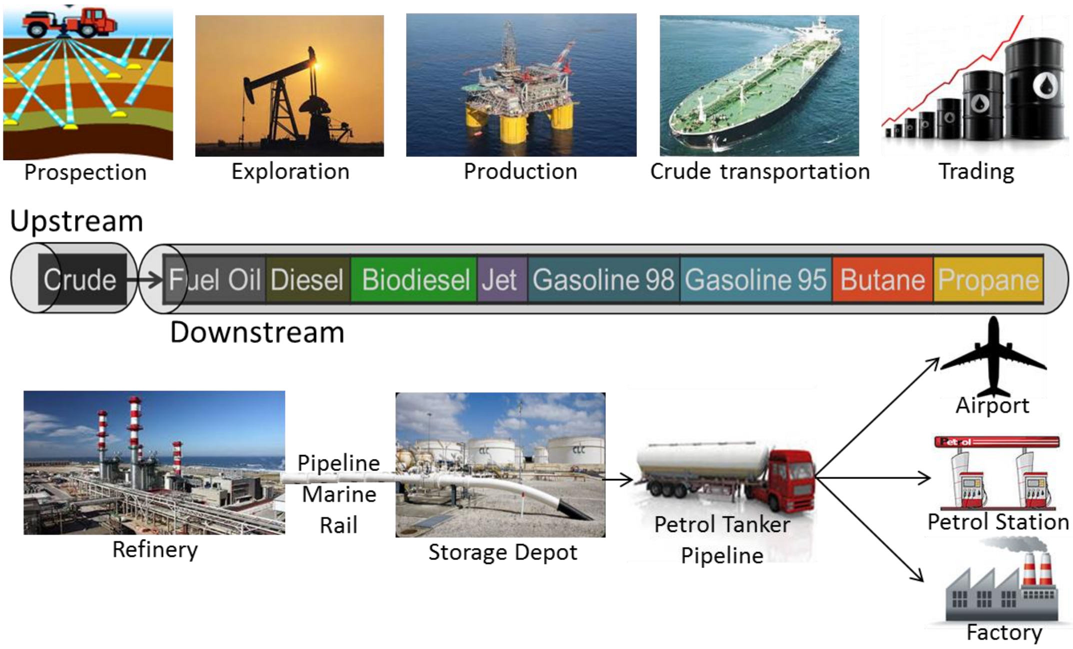


Figure 1.1. The Upstream and Downstream Petroleum Supply Chains

This is an investment intensive industry where specifically the downstream PSC needs to build or rely on physical infrastructures such as wholesale hubs, refineries, bulk or primary transport, storage depots, secondary transport and retail stations. These infrastructures may be owned by a single company or multi-company consortiums and may be designed to manage various products of a single company, products of various companies or a single product for various companies. Although the installations may be owned by various companies and frequently competitors, product is handled as batches belonging to a single petroleum company until it is sold to the retail client. Transportation routes are strategic to the PSC and are normally maritime to link with the wholesale markets, could be pipeline, maritime or railway between refineries and storage depots as well as inter-depot transportation, and finally pipeline, railway or road between the depots and the retail stations. Recently, logistics operators have emerged to provide one or more of these activities as external services. However, it is common that their capital is totally or partially owned by the petroleum companies. Finally the retail stations may be company or dealer owned and/or operated, and will normally but not compulsorily receive products from an exclusive company. However, this involves a truly collaborative network, and for all entities to benefit, it is crucial to have the right locations, sizes, prices, quantities and participations in the different infrastructures. Consequently, a fundamental activity of the petroleum companies is to design, construct and manage collaborative networks that maximize the profits and reduce risks for the entire PSC, which constitutes the scope of the developments presented in this thesis.

1.2 The PSC Planning Paradigm

The strategic positioning of this sector creates significant dynamics for the PSC whose underlying fundamentals must be understood. We synthesize, in figure 1.2, the main opportunities and challenges, which drive the complex task related with the PSC planning activities. The challenges come from the increased competition, unbundling of services, huge investments, reduced profits, and alternative energy competition; and the crude costs, product prices and demand uncertainties; as well as the regulatory changes, increased taxation, emission reductions, political, financial instabilities, law reforms, and fair price controls. Nevertheless, significant opportunities result from the rise in the oil products' demand due to fleet renewals, increased trucking and aviation, improved operating conditions, new investments in inter-modal facilities, and the availability of geographic information systems. With respect to uncertainties, the product demand is significantly influenced by price levels, economic growth trends, and weather conditions. Demand is directly proportional to the economic expansion and inversely to the product prices. Price variations result from the seasonal and short-term high-demands and not from tax changes, manufacturing and marketing costs, which although significant are fairly stable.

Figure 1.2. The PSC Planning Paradigm

Concerns relating to the issues identified above and consequent impacts on the PSC demand, production and profit margins have already been highlighted by academic and sectorial publications (Relvas *et al.*, 2013 APETRO, API, DR, CLH, EIA). This competitive environment creates significant opportunity for the improvement of the PSC efficiencies, through the collaboration between entities, the integration of planning levels and multi-echelons, efficient use of inventories, timely procurement, and risk management methodologies. However, this results in very large scale problems, which provide an important incentive for the optimization theory and advanced PSC planning models, in order to efficiently design and plan flexible multi-entity PSC networks, integrating the refining, trading and distribution activities and respective uncertainties.

This thesis focuses in the development of integrated models for the planning activities in for the PSC, as well as in solution methods for tackling complexity. Notwithstanding, both the upstream and downstream PSCs are long supply chains, and although they are complementary, both have a whole range of differences in their localizations, equipment, legalities and business environment. Considering the main concern of this research, from this point forward, we close up only on the downstream PSC.

1.3 State of the Art

The fabric of the PSC industry changed dramatically in recent years with the hike in crude oil prices in 2009, which created opportunities to develop pre-salt crude oil and shale oil production, but then the recent oil price meltdown upturned their viability and has resulted in costs, prices and demand volatility. Consequently, companies and academia are again trying to improve the PSC efficiencies as seen in the recent developments synthesized in figure 1.3. Notice that our attention has converged to the focus of our research, as we look at the main areas explored in the downstream PSC. Notice also the identification of niche research areas, wherein the Downstream PSC Strategic and Tactical Planning, which had been forgotten for some time, is now in the limelight with renewed interest. Moreover, the focus is on integrating various PSC activities, decision levels and modelling approaches, and hence some of these developments are presented so as to support the research objectives. Each topic is analyzed further in this section, providing an historical review of achievements collected from the scientific literature.

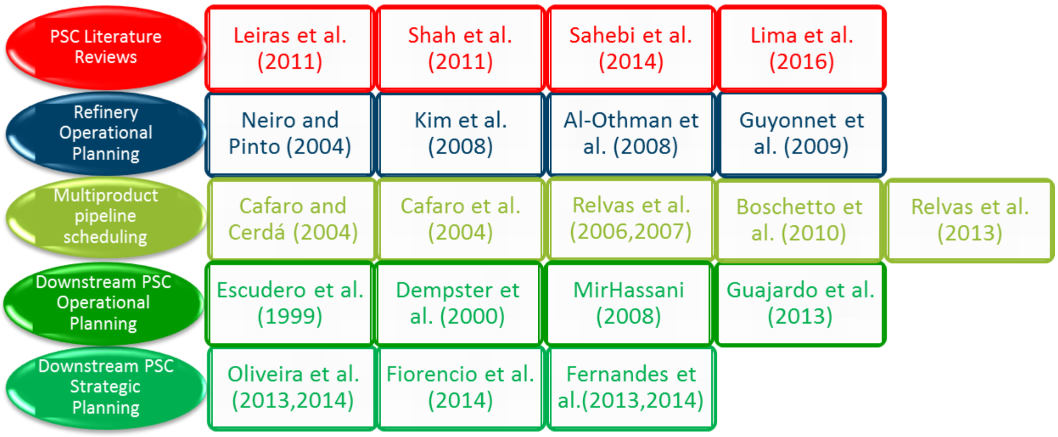


Figure 1.3. Developments in the Downstream PSC research

The PSC design and planning models have been recently reviewed in Leiras *et al*. (2011) with respect to upstream, midstream, and downstream PSC planning while Shah *et al.* (2011) addressed oil refinery scheduling and planning. In a recent review, Sahebi *et al.* (2014) refer to crude oil supply chain management as a flourishing research area. They identify some unaddressed avenues of research: full vertical integration of strategic and tactical decisions; full horizontal integration of the complex crude oil supply chain; dealing with nonlinear models essential for the formulation of refinery operations; development of efficient solution techniques for multi objective modelling as a result of global factors including international taxation, transfer prices and environmental impacts. Multi-stage stochastic modelling to study uncertainty problems has been identified as a key area for investigation. Lately, Lima *et al.* (2016) reviewed the strategic, tactical, operational downstream PSC planning literature, identifying the decision levels, uncertainty features, formulation approach, sustainability and objective functions, entity collaboration. They identify opportunities for developing more robust optimization decision-support tools to integrate decision levels and the PSC characteristics in order to simultaneously deal with uncertainty, risk, resilience, sustainability and collaboration between entities.

As we study the downstream literature, a concentrated focus is observed for the refinery operational planning, multiproduct pipeline scheduling, the downstream strategic planning and the downstream operational planning. Together these areas constitute the downstream PSC, which is the focus of this thesis. Hence, we review the main developments in these areas with a view to grasp the big picture and comprehend the underlying fundamentals of the downstream PSC that will better allow us to define the main research objectives.

Refinery operational planning is modelled in Neiro and Pinto (2004) using a mixed integer non-linear program (MINLP) to determine stream flow rates, properties, operational variables, inventory and facility assignments, considering aggregate nodes of production units, tanks and pipelines, interconnecting refineries, terminals and pipelines. Decomposition methods allow increased time periods and scenarios. Kim *et al*. (2008) proposed a supply network and market planning MILP to determine the refinery production rates, and transportation routes from distribution centers to markets. The maximized profits are the differences between revenue and depot and transportation fixed and variable costs. Al-Othman *et al*. (2008) later proposed a two-stage stochastic model for operational planning of crude oil production, processing and distribution to maximize resource utilization and profits. Crude production plans are the first-stage decisions while refinery and petrochemicals production and shipment, storage, lost demand and backlog quantities are the second-stage decisions. Guyonnet *et al*. (2009) integrate the crude oil unloading, refinery production planning, and product distribution models using penalties for unsatisfied demand, optimizing the overall PSC profits.

Multiproduct pipeline scheduling is modelled in Cafaro and Cerdá (2004) using a MILP for transportation between an oil refinery and depots, adopting a continuous time and volume representation. This reduces the model size and processing time, thereby permitting longer time horizons and increased slug pumping flexibility compared to the discrete representations. Later, Cafaro *et al.* (2004) improved the continuous-time MILP to reduce energy consumption and pump maintenance costs by incorporating heuristic rules to select the receiving terminal and minimize the total flow restart volumes and the number of single-delivery pumping runs. Relvas *et al*. (2006,2007) propose a MILP to integrate multiproduct pipeline scheduling and rescheduling with depot inventory management of petroleum products. Product sequences, pumping schedules and batch volumes are determined with customer demand, quality control and operating constraints. Boschetto *et al*. (2010) studied through an optimization model a real-world pipeline network, by addressing the associated network scheduling activities. A hierarchical approach is developed which explores the integration between a MILP model and a set of heuristic modules. Later, Relvas *et al.* (2013) propose an alternative and holistic modelling concept to obtain faster solutions within reduced time frames through a single efficient model so as to avoid decomposition approaches.

Operational planning of the downstream PSC was first modelled in Escudero *et al*. (1999), using a 2-stage stochastic scenario analysis and a deterministic equivalent model. Later, Dempster *et al*. (2000) developed deterministic and stochastic models for the logistics tactical and operational planning to minimize overall costs. MirHassani (2008) proposed a MILP for the operational planning of a capacitated transportation network of existing refineries and depots, to satisfy customer demand. MirHassani and Noori (2011) extend the MILP to a two-stage stochastic model for capacity expansion to minimize transportation and investment costs. Total inventory and transportation costs are minimized, determining inventory shortages and excesses. More recently, Guajardo *et al*. (2013) proposed a tactical planning model to minimize the production, transportation and inventory planning, hubs and depots costs. Vendors' contributions are maximized with distribution planning and decentralized sales.

Strategic planning of downstream PSC is modelled in Oliveira *et al*. (2013, 2014) using a two-stage stochastic MILP initially using Lagrangian and later the Benders (L-Shaped) decomposition methods. Investment and logistics costs are minimized while the portfolio and timings of network capacity expansion projects and inventory decisions are determined for varying demand. They identify that few works have analyzed the presence of uncertainty in oil supply chains when dealing with important characteristics of the PSC. Later, Fiorencio *et al*. (2014) develop a deterministic MILP decision support system (DSS) that maximizes the NPV while determining an investment portfolio and installation times of vessel and transportation capacity expansions. Fernandes *et al*. (2013, 2014) presented two strategic and tactical planning MILPs (one individualistic and the other collaborative) to design current, grassroots and retrofit uni and multi-entity PSC networks while maximizing the network profits. Depot locations, resource capacities, transport modes and routes are installed and operated and the multi-stage crude oil and petroleum products inventories and the import, export, production, storage and transportation volumes are determined. Piecewise linearization is used to determine the economy of scale costs and tariffs.

The status quo of the literature in terms of research in planning within the PSC, namely at the downstream level, suggests the need to select adequate techniques so as to cope with the complex pool of decisions and large scale problems at hand. These techniques are presented in figure 1.4 and the specific developments reviewed thereafter.

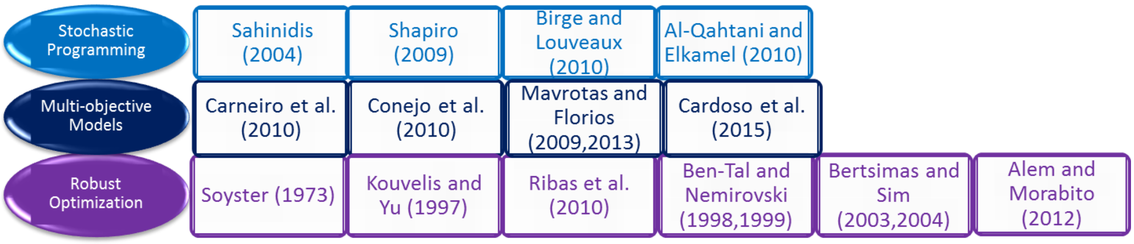


Figure 1.4. Developments in the modelling methods

Stochastic programing has been extensively used to model uncertainty in production planning, location, transportation, logistics, finance, and engineering design (Sahinidis, 2004). Despite the progress in two-stage stochastic programs, the authors identify multistage and stochastic integer programming as conceptually and computationally challenging. Shapiro *et al.* (2009) present the practical application of stochastic modelling with uncertain parameters, by presenting various modelling concepts such as recourse actions, chance probabilistic constraints, the nonanticipitivity principle, two stage and multi-stage SPs, and risk averse optimization. They propose the sample average approximation (SAA) problem to replace the expected value problem, and the Monte Carlo sampling techniques that use the complexity analysis to constrain the size of the stochastic scenarios. Birge and Louveaux (2010) have presented two-stage recourse, multi-stage stochastic and stochastic integer programming methods and Monte Carlo, approximating expectations and multistage approximation sampling methods for scenario generation. The expected value of perfect information (EVPI), value of stochastic solution (VSS), expected value of the wait-and see problem (WS), expected recourse problem solution value (RP) and expectation of the expected value problem (EEV) are solved to evaluate the quality of the stochastic solution. Al-Qahtani and Elkamel (2010) propose a two-stage stochastic MILP using robust optimization for multisite refinery network capacity expansion planning considering crude cost, product price and demand uncertainties. A sample average approximation (SAA) method with statistical bounding techniques is used to obtain economic benefits and process flexibility. A deterministic model minimizes the total installation and operating costs and the stochastic model minimizes variance, mean-absolute deviation, value-at-risk or conditional value-at-risk.

Multi-objective models have been proposed to maximize the expected profits and minimize the risk measures either in the objective function or in constraints to provide tradeoff solutions. Carneiro *et al*. (2010) propose a two-stage stochastic model to maximize the expected net present value ENPV and reduce the conditional value-at-risk (CVaR) for the PSC network design with oil supply, cost, final price and demand uncertainty. Conejo *et al*. (2010) presented stochastic programming models for price, cost, supply, demand and equipment uncertainties in the electrical markets. Variance, variability index, downside risk and value at risk are identified as non-coherent risk measures and CVaR as coherent. Later, Mavrotas (2009) and Mavrotas and Florios (2013) propose an augmented ε-constraint method, to determine the trade-offs between two objective functions. Using this method, Cardoso *et al.* (2015) maximize NPV and minimize risk for a closed-loop supply chain while determining the installation, sizing and location of plants, warehouses and processes, inventory levels and forward flows to retailers.

Robust Optimization (RO) was first introduced by Soyster (1973) by extending the generic (nominal) linear programming model to incorporate the worst-case deviations of the uncertain parameters. Later, Ben-Tal and Nemirovski (1998, 1999) presented a less conservative non-linear optimization problem to control the trade-off between robustness (feasibility) and performance (optimality). More recently, Bertsimas and Sim (2003, 2004) proposed a new RO methodology to ensure solution robustness by also varying the degree of conservatism of the solution, which can represent the risk attitude of the decision-maker. Several papers have employed this methodology as it maintains the computational tractability of the robust counterpart. Alem and Morabito (2012) use RO with production planning of typical furniture companies where cost uncertainties are modelled in the objective function and demand uncertainty in the constraints using polyhedral uncertainty sets. The authors show that beyond a certain level of the budget of uncertainty, there is no further deterioration of the objective function values and the probability of constraint violation tends to zero. Kouvelis and Yu (1997) presented a different RO Min-max regret approach which is modelled in Ribas *et al*. (2010) for the PSC strategic planning with crude oil production, refined product demand and market price uncertainties.

Although many areas of the downstream PSC have been researched with more related to the refinery planning and pipeline planning problems, most of the authors refer to the vertical integration of various levels of decision, the horizontal integration of the various activities of the PSC, and the modelling of uncertainty and risks as important areas of investigation. This has motivated our interest in developing design and planning models for the PSC, in order to integrate various PSC echelons and planning levels, besides new modelling approaches from the operational research community. Specifically, after developing the PSC models, these could be improved to consider uncertainty and risk management in order to build efficient and resilient PSCs. Given the multi-entity operation of the PSC, integrated and collaborative strategic, tactical and operational planning under cost, price and demand uncertainty are the main goals of this research, as will be presented in the following sections. It is noteworthy to relate that a vast majority of works invariably use the exact solution methods of MILP and MINLP as the base planning methodology.

1.4 Research Objectives and Motivation

Motivated by this investigation, and to obtain a focus on the objectives of the current research, we develop a modelling framework, as presented in Figure 1.5, that summarizes the characteristics of the PSC planning problems. The planning problem may be considered at the three strategic, tactical and operational planning levels; by considering a wide range of uncertainties that affect the PSC, which may be modelled using various solution strategies that have been identified in the literature, thereby maximizing or minimizing some solution metrics.

Using the framework, we define the scope of this thesis, before presenting the main research objectives. Specifically, the thesis will develop the strategic and tactical planning mathematical programming models, considering the costs, prices and demand uncertainties, using mixed integer linear programing (MILP), stochastic programming (SP), multi-objective optimization (MO) and robust optimization (RO), in which we propose to maximize the profits and the eNPV and minimize the costs and the risk measures variance, variability index, downside risk, VaR and CVaR with increasing levels of uncertainty. Operational planning will not be covered in this thesis and is proposed for future research.

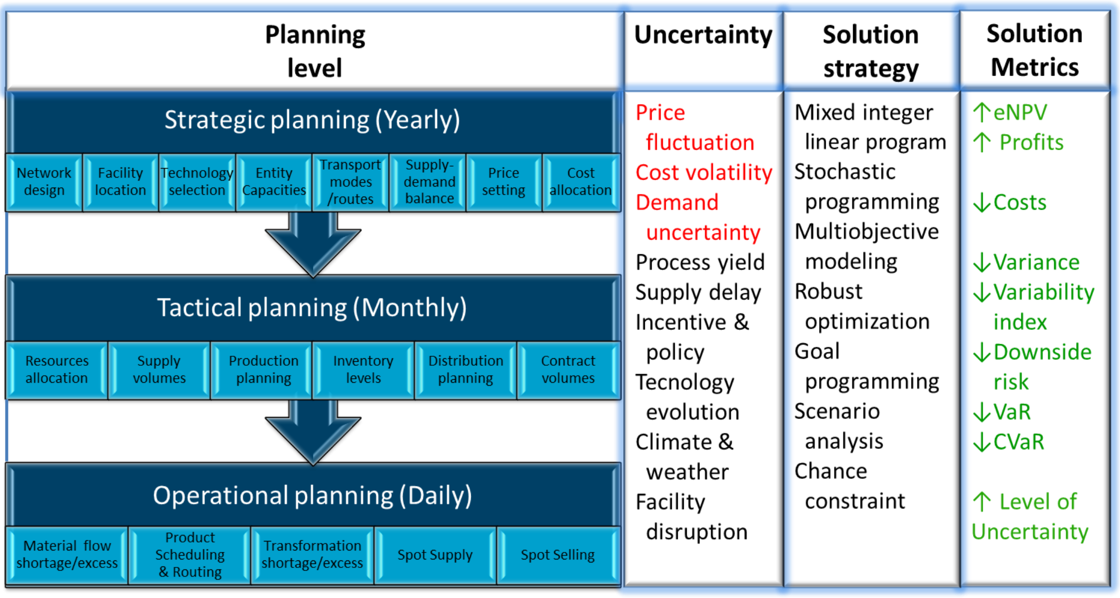


Figure 1.5. The PSC Planning modelling framework

The current research establishes five major objectives. The first is to develop a mathematical model for a individualistic multi-entity, multi-echelon, multi-product and multi-transportation PSC in order to determine the strategic planning decisions. On the strategic decisions it will be considered entities installation, operation, resource capacities, expansion and closure of storage depots and definition of multi-modal routes. On the tactical decisions of crude oil, refinery production, import, export, transportation and customer fulfilment volumes at various locations are to be studied. Current, grassroots and retrofit designs, and associated network flows are to be generated considering the customer demand, production, and depot and transportation resources capacity restrictions. The model should maximize total network profits, incorporating refinery revenues and gross retail margins while deducting the transportation, storage, importation, exportation and product costs.

A second objective of the thesis is to expand the model into a collaborative strategic and tactical planning model that will determine the earlier strategic decisions for a multi-entity PSC network where the entities may utilize competitor installations without limitation. The product and service tariffs are to be distinctly determined per entity and product using formulations that must incorporate economies of scale for calculating costs and tariffs. The strategic and network flow decisions are improved with the tactical decisions relating to the periodic crude oil and multi-stage product inventory volumes whereas the piecewise linearized entity tariffs must be determined in order to improve the grassroots, retrofit and current network designs. The model should maximize the total network profits, comprising the oil company direct profits and consolidated revenues from participated PSC member infra-structures.

The third objective is to implement a stochastic planning model and a multi-objective model to reduce risk, wherein the network designs will respond to various uncertainties including crude cost, product price and customer demand. The stochastic planning model will improve the earlier collaborative planning model, incorporate uncertain demand scenarios and maximize the expected net present value. A multi-objective model will then be developed for the PSC strategic and tactical planning, by incorporating risk measures into the stochastic model, in order to simultaneously improve the total profits and reduce risk under cost, price and demand uncertainties. Risk measures from the supply chain and financial risk management literature, such as, variance, variability index, probabilistic financial risk, downside risk, conditional value at risk and worst-case revenue will be modelled for the PSC planning problem.

The fourth objective is to implement a robust optimization approach for the collaborative planning in order to incorporate immunization against crude oil cost, product price and customer demand uncertainties. Specifically, RO is seen to have the advantage of not increasing the problem complexity when compared to its deterministic equivalent problem. Hence it will be used to model the three uncertainties simultaneously as the earlier presented stochastic and bi-objective approaches have been seen to cause numerical difficulties even with a small number of scenarios. It is aimed to solve the RO model for varying levels of uncertainty where after the decision-maker may choose among the resulting trade-off solutions which will vary from the worst-case solution to the deterministic optimal solution.

A final objective is to study in detail the real case downstream of the Portuguese PSC network in Portugal and build an information and decision support model for this chain. The developed real case database will be used to validate the models for different design settings wherein the obtained results and model responses, specifically the objective values and risk measures, will be compared for various entities, refineries, depots, customer locations and products with respect to costs, tariffs, profits and network volumes in order to evaluate the practical contributions of this research.

1.5 Problem Description

To pursue the objectives proposed for this research, we devise a general representation that characterizes the integrated downstream PSC strategic and tactical planning problem as shown in figure 1.6, followed by the problem description.

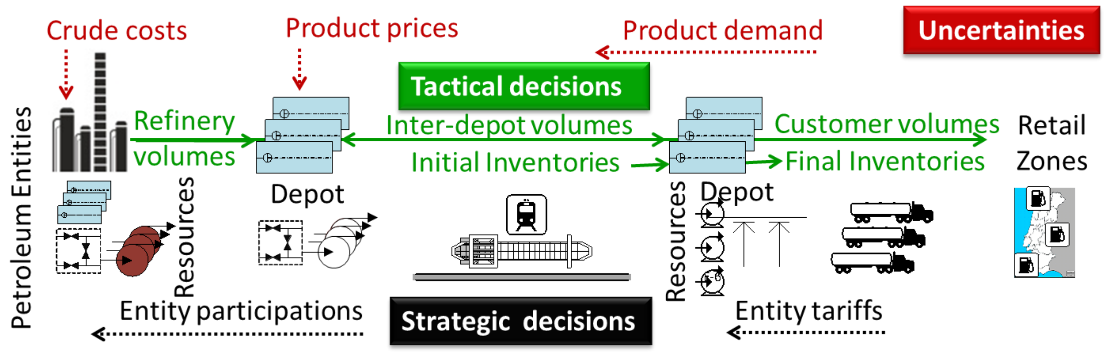


Figure 1.6. The Downstream PSC Strategic and Tactical planning problem

The aim is to develop optimization models for individualistic and collaborative strategic and tactical planning of the downstream PSC under uncertainty, which may be defined as follows:

- Given a set of petroleum companies that produce or import a set of petroleum products at a known set of refineries, and commercialize them to a set of customer regions of the Portuguese PSC;
    - Determine the optimal current, grassroots, and/or retrofit PSC network designs for the multi-echelon, multi-product, and multi-modal transportation PSC network;
    - Strategic decisions of installation, operation, expansion, or closure of storage depots, resource capacities, and multi-modal routes;
    - Strategic-tactical decisions of entity investment participations and the economy of scale costs and tariffs per location, route, entity and product;
    - Tactical decisions of time-dependent and multi-stage transfer volumes and inventories for crude oil and derived products at the production, storage, primary and secondary transportation, importation, exportation, and customer fulfillment stages;
- So as to maximize the total network profit, comprising the oil company direct profits and their consolidated revenues from participated PSC member infrastructures: refineries, depots, transporters, and retailers; and simultaneously reduce the risk of not attaining such profits under the context of uncertainty;
- Subject to the production, depot, transportation, resource capacity, inventory, and demand restrictions;
- Considering uni-entity and multi-entity networks; pipeline transportation as product flow per time period restricted to the pipeline physical capacities; and ship, truck, and wagon transportation as discrete product flow blocks, where the total transported volume for the allowed products and between origin-destination pairs, is restricted to the modal fleet capacity per time period;

Such that the solution is always feasible and near optimal within a pre-defined range of uncertainty in the crude costs, product prices, and product demand.

The development of the downstream PSC planning models has been motivated by the emerging need in Portugal in recent years, following the public awareness of the necessity to restructure the Portuguese PSC and the absence of a benchmarking and decision support tool. Hence the models will be appropriately designed so as to assist the different decision makers, specifically at the level of the PSC regulators, PSC operators as well as petroleum companies (main producer and other major companies that share the physical network), which constitute the stakeholders of the problem under study.

One important aspect is that the petroleum companies are full or partial owners of the refineries, storage depots, transportation infrastructures and retail stations, and therefore incorporate a relevant share of their profits. The representation of the problem within a model will therefore be developed and implemented so as to be configurable, in order to permit designs and analysis of the PSC network based on the profile of the decision-maker. Hence, the decision maker may analyze parts of the network, or the network in the perspective of an individual company, or as an integrated collaborative network, where the petroleum companies earn revenues by commercializing products and also aggregate profits from their participations in refineries, depots and transportation infrastructures. These plants have the autonomy to purchase and manage the crude oils and petroleum products; nevertheless, they serve the needs and the objective of improving the consolidated profits of their parent petroleum companies. In order to overcome the above difficulties and explicitly take into account the different decision makers, existing and partially owned infrastructures, the PSC is modelled as a multi-entity network, thereby determining the contract tariffs per location/route, entity, and product, which are based on the model's decisions of committed volumes and entity capacities.

The structures of the PSC are greatly influenced by the country dimensions and trade practices. In general, bigger countries have efficient networks, managed by single entities which may be public or private companies. Non-integrated installations of different companies who generally adopt individualistic strategies through self-owned installations are seen in smaller countries, which is not very efficient. However, due to free trade and environmental regulations and the EIA guidelines, many countries have created regulatory bodies and have taken the step to create collaborative infra-structures which are owned by a single entity or logistic operator, and whose installations may be used by several petroleum companies. This is the case in most member states of the American and European continents.

As the models are multi-entity, the notion of individualistic operation and entity collaboration is introduced. The former assumes that the petroleum companies individually own or participate in shared infra-structures, and may transfer product volumes that are restricted by their invested capacities per infra-structure, for which they pay non-differentiated tariffs determined by the model. The latter provides a collaborative environment wherein the companies may freely use competitor installations, and the model will calculate the economy of scale free trade costs, tariffs and profits determined per location or route, entity and product. Hence, in both cases, the physical installation and capacities are shared; however, the denominated capacities are not. In the first case, an entity may not resell its unused capacity to a competitor, so both loose an opportunity to earn more revenue. In the second case, entities allow competitors to use their unused capacity by paying a tariff, so both gain and earn a profit from this activity. Here, the collaboration is at the strategic level. The same could also be performed at the tactical level where entities could share inventories, and any excess inventory could be used by a competitor at a premium. However, collaboration at the tactical level is not modelled in this thesis. Entities could also use competitive strategies to negotiate their installation capacity and inventories on the spot market, but such competitive strategies occur mainly at the operational level and are not within the scope of this work. They are proposed for further research. The collaborative models that will be proposed increase the efficiency of the network by increasing profits, and reducing the costs and risks for the infrastructure owner as well as for the two competitor entities.

The proposed models must design the current, grassroots and retrofit network, wherein the first takes an existing network and determines the tactical decisions, the second designs a completely new network without considering the existing network, and the third takes an existing network and redesigns it considering the set objectives and parameters. The investment quotas per entity in existing installations are assumed not to change, whereas for the new installations these are assumed to be equal to their market share quotas which are known. The investment quotas will be used by the model to determine the distribution of the profits per entity. The objective function will maximize the total network profits, however, the objective terms are distinctly determined per entity. Although not explored in this thesis, the objective function has been so defined so as to allow multi-objective optimization wherein may be defined a weight per entity in the objective function which could be their market share quotas. The objective function structure can also be easily explored using a goal programming approach wherein an entity establishes a minimum profit threshold. These settings will directly impact the model determined strategic and tactical decisions, costs, tariffs and profits per location or route, entity and product. The entity tactical decisions in the individualistic model, therefore, will depend directly on the strategic decisions as the entities may only use self-owned installations up to a maximum of their participations. This is not the case in the collaborative models, where both the strategic and tactical decisions are free trade decisions, which are determined based on each entity's demand capitalization.

1.6 Costs, Tariffs, and Profits formation

Within the PSC, the costs, tariffs, and profit determination are important issues that need to be well defined. A motivational example is developed in figure 1.7 to illustrate a three echelon downstream PSC considering one refinery, one depot and four clients that are located in the same client zone. Two entities or companies operate this network, distributing two products (P1 and P2) to the final clients. The refinery uses a 95 km pipeline to supply a depot, which then supplies, using a 40 km road, a single client zone. Entity E1 owns refinery R1 and client petrol stations C1 and C2, while entity E2 owns the petrol stations C3 and C4. Using this illustrative example the detailed costs, tariffs, and profits were determined per product and entity.

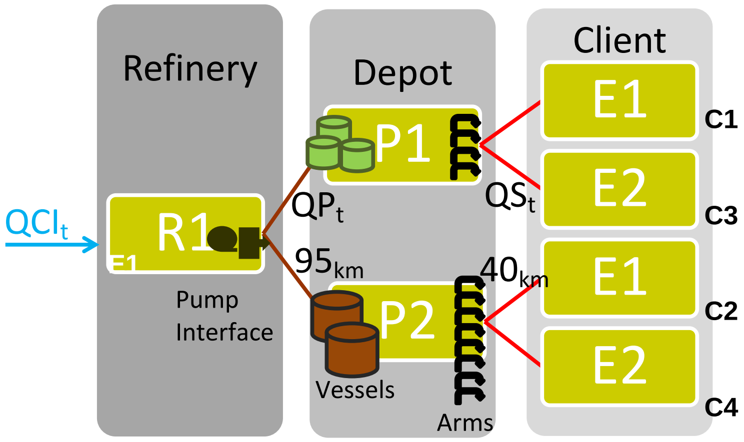


Figure 1.7. An Illustrative Downstream PSC

Some basic concepts are defined based on the assumptions in the industry. An entity is a petroleum company, which may interact with the PSC as an owner entity (infrastructure owner) or/and as a client entity (product owner). The infrastructures are independent managed generating costs related to their activity, equipment and resources. The infrastructure serves the clients and hence, these costs must be imputed to the product volumes of each entity client. By adding a price markup, are determined the infrastructure's tariffs per entity and per unit volume of product that is payable by the entity client. The infrastructure thus makes a profit which is the difference between the determined tariff and the costs incurred. This infrastructure profit is incorporated into the entity owners' profits.

The crude oil prices, ex-refinery product prices and the retail product prices are parameters, that are assumed to be the spot market prices for the different products based on the WTI Crude Oil and Brent Crude Oil indexes, which are reported by the price benchmarking agencies (Platts and Argus) and the European and American statistical agencies (EuroStat and EIA). The benchmarked prices are formed by the summation of the crude or product costs and the various activity costs; hence it is critical for companies to minimize all costs in order to stay profitable and maximize revenues.

As the multi-stage integration is proposed, the total PSC profits are maximized. This is the sum of the refinery profits, depot storage profits, transportation profits, and retail profits for all entities and their participated infrastructures. In the case of the refinery and the retail segment, are considered service tariffs which are the model parameters. The refinery margin (profit) is determined as the difference between the ex-refinery prices and the sum of crude costs and refinery tariff. In the retail segment, the gross retail margin (profit) is considered, which is determined as the difference between the retail prices and the sum of the ex-refinery prices and the retail tariff. To determine the net retail margin, it is still required to subtract, from the gross retail margin, the depot storage tariffs and the transportation tariffs that the client entities have to pay to deliver the product to a retail location. These are determined by the model as they are required to study the profitability of these infrastructures per product and per client entity.

Now, considering the storage depots, the depot fixed costs, the resource (equipment) fixed costs, the depot operating costs, and the resource operating costs are first determined, unitized and imputed per product (P1 and P2) based on their expedited volumes per client entity. Volume based costs are also obtained by multiplying the product volumes by the infrastructure unitary cost. This detailed cost breakdown is performed in order to determine the costs per product (P1 and P2) and per entity (E1 and E2). The depot tariffs are then calculated as the depot costs indexed by a depot markup percentage. Consequently, the depot profit per entity and product is the difference between the aggregate depot tariffs and the total depot costs. A similar procedure is followed for the costs, tariffs and profits computation for the transportation infrastructures, however here, are consider additionally, the distance based costs, which again may be fixed, operational or volume based (for example energy).

Note that these infrastructure profits revert to the owner entities, proportionally to their capital participation. The total PSC profits per entity are consequently the sum of their participation in the refinery, transportation, storage depot and retail profits.

Hence the infrastructure pays the costs and bills the tariffs to the client (product owner) entities, whereas their differences that determine the profits are incorporated by the owner entities based on their capital participation. Hence, if entity E2 transfers product through a depot owned by E1, then the depot operator pays the costs, E2 pays the depot tariff, while E1 collects the profits.

These are calculated per infrastructure, owner entity, client entity, and product in order to determine the cost, tariff and profits per entity and product in order to obtain their profitability. One issue here is the imputation of the costs to entities and products. Those costs that are directly related to an entity and product, as is the case of unitary costs, are directly allocated to a product and entity. When not directly related, these are allocated to entities and products, using their ownership participation, volume based piecewise cost linearization or other criterion as presented in this thesis.

1.7 Structure of the Thesis

This thesis is structured along six chapters. The first chapter proposed the main objectives, which guided our research plan, resulting in the production of four different development studies, which are the following four chapters of the PhD thesis. Two of these have been published in International Journals, one is in the review process, and the final one will be submitted after further improvement. As these articles were written independently for publication, some repetition may be perceived and the candidate apologizes for this fact. Nevertheless, wherever possible, effort has been made to eliminate parts of the published articles so as to avoid redundancy. Table 1.1 synthesizes the research that has been published in books, journals and proceedings, and presented at International and National Conferences. This has been accomplished through continuous research by the candidate, who constantly improved the quality of the work under the guidance of the supervisors.

The second chapter presents a individualistic strategic and tactical planning model that considers a multi-entity, multi-echelon, multi-product and multi-transportation downstream PSC network, involving companies having financial participation in shared refinery, transportation and storage depot installations, while considering the supply sources, production, resource capacities, and customer demand requirements. A deterministic mixed integer linear programming (MILP) model is developed to determine: the strategic planning decisions of installation, operation, closure, and locations of storage depots, their capacities and expansion, and the multi-modal transportation routes; and the multi-stage tactical decisions of crude oil, refinery production, importation, exportation, transportation and customer fulfillment volumes at various locations. The multi-echelon total profits are maximized, by incorporating the refinery revenues and gross retail margins while deducting the transportation, storage, importation, exportation, product, and retail operator costs. A bi-level transportation network flow formulation is proposed, using primary and secondary distribution. A parameter database, covering a twelve year period, is constructed for the Portuguese PSC, built from official data obtained from the Portuguese Economics Ministry, the European Commission for Energy and Google Maps. The MILP is validated for the real-case Portuguese PSC, which is supplied by local refineries or by a regional hub. Uni-entity networks as well as multi-entity networks with individualistic operation are modelled, presenting the current, grassroots and retrofit designs.

The third chapter presents a collaborative and dynamic MILP for the strategic and tactical planning of downstream PSCs, in order to allow collaborative utilization of competitor installations without limitation. This study extends the earlier model from chapter 2 by adding collaborative design, operator-profit incorporation, and generalizes the modelling of resources using transport mode and product families. The model determines the optimal design and tactical decisions relating to the periodic multi-stage crude oil and derived products inventories for a downstream PSC with multi entities, echelons, transportation modes and products, by including competitor installations, resource sharing and supply in order to fulfill customer demands. Collaborative planning permits multi-echelon profit maximization where the entities incorporate profits from partly owned refineries, storage depots and transportation and retail infrastructures. The multi-entity investment values, product and service costs and tariffs are determined individually per entity and product using piecewise linearization in order to reflect economies of scale. These may be based on total investment costs or only non-amortized costs. Current, grassroots, and retrofit network designs with respect to the real case Portuguese PSC, are studied for the strategic decisions of installation, expansion, closure and affectation of depots, transport modes, routes and resource capacities; the time-dependent tactical decisions concerning the crude oil and product importation, exportation, refining, storage and transportation volumes besides multi-stage inventories; and the fair trade product costs, tariffs and profits per PSC design, location and route.

The fourth chapter develops a stochastic MILP for the strategic and tactical planning of the PSC considering demand uncertainty. The expected net present value is maximized considering the optimistic, realistic and pessimistic scenarios of the uncertain multi-period, multi-product customer demand. As the multiple scenarios result in larger number of variables and equations, a compact stochastic model formulation is developed in order to reduce its size and improve the model performance. The stochastic model is then further enhanced to a multi-objective MILP for risk management, wherein are simultaneously maximized the expected net present value and minimized one or more risk measures including variance, variability index, downside risk and conditional value at risk, while satisfying uncertain product demand. We incorporate modelling approaches from the supply chain risk management literature, specifically techniques from Conejo *et al*. (2010) and Mavrotas and Florios (2013) who proposed the augmented ε-constraint method, to determine the trade-offs between two or more objective functions. The PSC strategic and tactical decisions are thus obtained by simultaneously minimizing the economic risk of the PSC. The stochastic and the multi-objective MILPs are tested for the real case Portuguese PSC considering the current, grassroots and retrofit designs. In the first case are obtained the stochastic solutions and the respective performance indicators whereas for the multi-objective model are obtained the Pareto-optimal solutions. Model improvements are proposed in order to improve the scenario generation, decomposition techniques, and robust optimization.

The fifth chapter extends earlier research by presenting an improved PSC strategic and tactical collaborative planning deterministic MILP and robust optimization (RO) model formulations by simultaneously considering crude oil costs, product prices and customer demand uncertainties. Recent historical data for the real case PSC in Portugal is analyzed and constructed the cost, price and demand uncertainty sets based on their minimum, maximum and standard deviations. Three robust counterparts are formulated for uncertainties in the objective function and the demand parameters, using polyhedral uncertainty sets, in order to develop an uncertainty planning model as tractable as the deterministic MILP. The RO formulations implement protection against the crude costs, product prices, and customer demand uncertainties. Considering the Portuguese PSC and the above uncertainties, are determined: the PSC infrastructures, capacities to install, operate, and close; the strategic fair price costs and tariffs per location/route, entity and product; and the multi-stage product transfer and inventory volumes per period. Trade-off solutions as well as their probabilities of constraint violation are obtained for the total profits maximization objective for varying levels of uncertainty, thereby obtaining numerous solutions between the deterministic optimal and the worst-case solutions, so the decision-maker may exercise his/her choose. The solutions are compared with respect to the changes in the network structure and the objective function values. The RO models' performance statistics are compared with respect to the deterministic, and the scenario-based stochastic and multi-objective approaches, confirming that the RO methodology obtains tractable robust and trade-off solutions, considerably faster than the stochastic models.

Finally, chapter six presents the conclusions and the main contributions of this PhD thesis, as well as some guidelines for future developments based on the work developed.

Table 1.1 shows the division of the thesis structure into chapters, each of which aggregates the related papers that were published in international peer reviewed journals and presented either in oral or in poster presentations at international and national conferences.

All the work leading to the publishing of the papers, was essentially developed by the candidate. This included the generation of the ideas, the development of the models, the construction of the test cases, the testing of the models, the documentation of the results and writing of the articles. The supervisors contributed here to the discussion of the ideas, the test results, and to the analysis and first round of review of the articles.

Table 1.1: Thesis Chapters, Journal papers, Conference papers and Presentations.

Ch.2. Strategic Network Design of Downstream Petroleum Supply chains: Single versus Multi-entity Participation

- Strategic Network Design of Downstream Petroleum Supply chains: Single versus Multi-entity Participation, Fernandes LJ, Relvas S, Barbosa-Póvoa AP, 2013, Chemical Engineering Research and Design, 91, 1557-1587.
  - Optimal Network Design of Petroleum Supply Chains, Fernandes LJ, Relvas S, Barbosa-Póvoa AP, 2011, in INESC-Coimbra (eds.) Proceedings of IO 2011, 15th Congress of APDIO, Faculty of Economics, University of Coimbra, ISBN: 978-989-95055-4-4, 129-140.
    - IO 2011, 15th APDIO Congress, University of Coimbra, 2011.
  - Strategic Planning of Petroleum Supply chains, Fernandes LJ, Relvas S, Barbosa-Póvoa AP, 2011, Computer-Aided Chemical Engineering, 29b, Proceedings of ESCAPE 21, Elsevier, 1738-1742.
    - ESCAPE 21, 21st European Symposium on Computer Aided Process Engineering, Chalkidiki, Greece, 2011.
  - EURO XXIV, 24th European Conference on Operational Research, University of Lisbon, 2010. Network Design of Petroleum Supply Chains

Ch.3. Collaborative Design and Tactical Planning of Downstream Petroleum Supply Chains

- Collaborative Design and Tactical Planning of Downstream Petroleum Supply Chains, Fernandes LJ, Relvas S, Barbosa-Póvoa AP, 2014, Industrial & Engineering Chemistry Research, 53, 44, 17155-17181.
  - Design and Planning of Downstream Petroleum Supply chains, 2012, Computer-Aided Chemical Engineering, 30a, Proceedings of ESCAPE 22, Elsevier, 432-436.
    - ESCAPE 22, 22nd European Symposium on Computer Aided Process Engineering, University College London, 2012.

Ch.4. Downstream Petroleum Supply Chain Design Planning and Risk Reduction with Uncertainty

- Downstream Petroleum Supply Chain Design Planning and Risk Reduction with Uncertainty. Fernandes LJ, Relvas S, Barbosa-Póvoa AP, Paper to be submitted to an international journal with peer review.

- Risk Management in Petroleum Supply chain, Fernandes LJ, Barbosa-Póvoa AP, Relvas S, 2009, Proceedings IO 2009, 14th Congress of APDIO, Faculty of Science & Technology, New University of Lisbon, ISBN: 978-989-20-1639-9, 59-66.
  - IO 2009, 14th APDIO Congress, New University of Lisbon, 2009.
- Risk Management Framework for the Petroleum Supply chain, Fernandes LJ, Barbosa-Póvoa AP, Relvas S, 2010, Computer-Aided Chemical Engineering, 28, Proceedings ESCAPE 20, Elsevier, 157-162.
  - ESCAPE 20, 20th European Symposium on Computer Aided Process Engineering, Ischia, Italy, 2010.
- Supply Chain Risk Management Review and a New Framework for Petroleum Supply Chains, Fernandes LJ, Barbosa-Póvoa AP, Relvas S, 2011, chapter in D.D. Wu (ed.): Quantitative Financial Risk Management, Computational Risk Management Book Series, vol 1, Springer-Verlag, 227-264.
- Downstream Petroleum Supply Chain Planning under Uncertainty, Fernandes LJ, Relvas S, Barbosa-Póvoa AP, 2015, Computer-Aided Chemical Engineering, Proceedings ESCAPE 25, Elsevier.
  - ESCAPE 25, 25th European Symposium on Computer Aided Process Engineering, Copenhagen Denmark, 2015.
- Petroleum Supply Chain Network Design and Tactical Planning with Demand Uncertainty, Fernandes LJ, Relvas S, Barbosa-Póvoa AP, 2015, Operations Research and Big Data Proceedings of IO2015, 17th Congress of APDIO, Springer, 59–66.
  - IO 2015, 17th APDIO Congress, Escola Superior de Tecnologia e Gestão, Instituto Politécnico de Portalegre, 2015.

Ch.5.Strategic and Tactical Planning Robust Optimization for Petroleum Downstream Supply Chains under Uncertainty

- Strategic and Tactical Planning Robust Optimization for Petroleum Downstream Supply Chains under Uncertainty, Fernandes LJ, Relvas S, Alem D, Barbosa-Póvoa AP, Journal paper submitted to AIChE Journal an international journal with peer review.
  - Robust Optimization for Petroleum Supply Chain Collaborative Design and Planning, Fernandes LJ, Relvas S, Alem D, Barbosa-Póvoa AP, 2016, Computer-Aided Chemical Engineering, Proceedings ESCAPE 26, Elsevier.
    - ESCAPE 26, 26th European Symposium on Computer Aided Process Engineering, Portorož, Slovenia, 2016.

CHAPTER 2. Strategic Network Design of Downstream Petroleum Supply Chains: Single versus Multi-entity Participation

2.1 Abstract

This chapter considers the multi-entity, multi-echelon, multi-product and multi-transportation downstream PSC network with shared installations, resource capacities, supply sources and demand requirements. A deterministic mixed integer linear program (MILP) is developed for strategic design and planning of the downstream PSC network that determines optimal depot locations, capacities, transportation modes, routes and network affectations for long term planning. The MILP maximizes the multi-echelon total profits for the petroleum companies along the supply, refining, distribution and retail stages. A multi-entity PSC network is considered, involving companies' financial participation in refineries, transportation and storage depots. The MILP is tested with the real-case Portuguese PSC network involving production at local refineries and supply from a regional hub. Uni-entity networks as well as multi-entity networks with individualistic operation are modelled, presenting the current, grassroots and retrofit designs.

2.2 Introduction

The downstream Petroleum Supply Chain (PSC) represents a major economic sector that surmises production and delivery of petroleum products to consumers in a cost-efficient manner. Until recent times, PSC network structures have been fairly static and scarcely investigated due to the lack of detailed information and reduced competition. However, nowadays competition has increased and is currently driving PSC strategic planning with a strong focus on cost reduction. To answer to this new challenge the optimization of the supply chain structures and resources is emerging and has been recognized recently as a topic to explore (Al-Qahtani and Elkamel 2008; Guajardo *et al.* 2013). The key issue is to determine the optimal network design using geographical information and considering long-term supply, demand, prices and costs. Few works have addressed such problem, which constitutes a research field with high potential. Nevertheless, some literature has been seen to tackle various stages of the PSC, as reviewed below.

Iyer *et al.* (1998) developed a MILP for offshore crude oilfield capacity planning and operations scheduling to decide reservoir, wellsite, drilling and platform installation schedules and capacities, besides time-dependent oil production rates. A piecewise linear approximation models the reservoir performance, resource, and surface pressure constraints. Cheng and Duran (2004) develop a dynamic programming based simulation model, that determines the crude oil inventory and transportation control strategies that minimize the expected total cost. Later, Al-Othman *et al.* (2008) propose a two-stage stochastic model for operational planning of crude oil production, processing and distribution to maximize resource utilization and profits. First-stage decisions are the crude production plans, while the second-stage decisions are refinery and petrochemicals production and shipment, storage, lost demand and backlog quantities. Sinha *et al.* (2011) use multi-agents to improve the PSC performance with co-evolutionary particle swarm optimization based on Cauchy distribution. Agents simulate processes and negotiate to share resources and reduce crude oil inventories.

Pinto and Moro (2000) use a nonlinear model to address refinery operations' scheduling. Production planning considering market demands is identified for improving profits. Later, Más and Pinto (2003) minimize refinery operating costs using two continuous-time MILP models. The first considers aggregated pipelines and intermediate tanks to schedule oil tankers to piers assignment, and tanker and pipeline loading and unloading operations. The second MILP uses the aggregate plans to schedule the loading and release of individual tanks and pipelines. Neiro and Pinto (2004) extend earlier work with a mixed integer non-linear program (MINLP) for operational planning of the midstream PSC considering aggregate nodes of production units, tanks and pipelines, that interconnect refineries, terminals and pipelines. Stream flow rates, properties, operational variables, inventory and facility assignments are determined. Decomposition methods are identified to allow extension of time periods and scenarios.

Integrated refinery scheduling and shipment planning is proposed in Persson *et al.* (2005), to transfer products by ship to storage depots while considering inventories, process schedules and depot demands. Their method uses column generation, valid inequalities, and constraint branching while proposing rolling time horizon, safety stocks and continuous time formulation for future research. Li *et al.* (2005) develop a non-linear model for production planning of fuel products at multi-refinery sites. This is later integrated by Kim *et al.* (2008) into their MILP for the supply network and market planning. Refinery production rates, and transportation routes from distribution centers to markets are determined while maximizing profits, these being the difference between revenue and depot and transportation fixed and variable costs. Al-Qahtani and Elkamel (2008) propose a strategic MILP for planning of multi-refinery networks with process integration and capacity expansion, to minimize total costs, with deterministic availability, crude oil prices, feedstock, chemicals, production costs and market demand. The State Equipment Network (SEN) representation is used to model the tactical intermediate material flows between processing units under different operating modes. Khor and Elkamel (2010) develop a MILP using a state–task network (STN) representation to determine the refinery process topology to produce fixed product quantities from available crude oil while maximizing profits and minimizing environmental impacts.

The downstream PSC was first researched in Sear (1993) who presented a qualitative and partly mathematical formulation for its strategic planning. Planning needs for logistics infrastructure and transportation, focusing fixed and variable costs for road vehicle delivery were discussed. Ronen (1995) identifies significant cost saving potential for transportation and inventory. Their set packing and set partitioning models allocate deliveries to carriers. Escudero *et al.* (1999) develop a two-stage model for supply, transformation and distribution scheduling of multi-operator multi-product PSCs considering product demand, spot supply cost and spot selling prices. Two objectives are minimized: i) over and under supply, production and transformation, using penalties and slack variables for excess and deficit quantities; ii) difference between product supply, transportation, depot stock, transformation and spot supply costs and the spot selling revenue. Capacity usage is maximized for continuous flow capacity of pipelines, capacity blocks of ships, wagons and trucks, and the transportation time.

Relvas *et al.* (2006) present a MILP for pipeline transportation scheduling and inventory management of petroleum products' depots. Continuous-time volume representation obtain product sequences, pumping schedules and batch volumes, that satisfy client demands, quality controls and operating restrictions. Later, MirHassani (2008) considers a capacitated PSC network and proposes a linear programming model for operational planning of the transportation network between the existing refineries and depots to meet customer demand, minimizing total inventory and transportation costs. The static modelling approach also computes inventory shortages and excesses in the presence of transportation route capacity bounds.

Later, Cornillier *et al.* (2008) develop a heuristic for petrol station replenishment to determine product delivery quantities per vehicle and route to each petrol station, to maximize the total profit including revenue minus routing costs. Recently, Guajardo *et al.* (2013) develop a tactical planning model for the downstream PSC where vendors' contributions are maximized with decentralized sales and distribution planning, whereas company costs are minimized with centrally managing production, transportation and inventory planning at refineries, hubs and depots. While total contribution is higher, the individual vendors' contribution can be considerably lower, hence suggest contribution sharing rules to achieve better outcome for company and vendors.

Two reviews have focused on PSC strategic, tactical and operational planning models. Fernandes *et al.* (2011) review the SCM literature with insights for the PSC and An *et al.* (2011) review the PSC models to adopt them for the biofuel industry. Considerable research related to the PSC has focused on refinery planning, while recent literature show an increasing interest in the distribution network. However, such research mainly concerns single company networks. Business needs though, have changed and with it new multi-company PSC configurations and logistic operator networks are arising, bringing in focus new paradigms of competition and cost efficiency involving strategic and tactical planning. This clearly identifies network design and tactical planning for multi-company networks as a new unexplored topic, which in our opinion could be modelled as a single physical network however using multi-entity planning, with important cost benefits and profit improvement for the entities involved.

The strategic MILP model proposed in this chapter is, to the best of the authors knowledge, the first model to consider a general multi-company, multi-echelon and multi-product mathematical formulation for optimal design of downstream PSC networks. At the strategic level, the model determines optimal depot locations, resource capacities, transportation modes and routes while at the tactical level, network affectations of product volumes are determined. Furthermore, the model considers companies' financial participation in refineries, transport and storage depots, while deciding the network planning. Besides the downstream network, refining, importation and exportation are also considered as they play an important role in PSC dynamics. By considering uni-entity and individualistic multi-entity networks, the MILP maximizes the total profits of the entire PSC while assuming jointly-owned installations. Resource utilization and profit-sharing in the multi-entity networks is determined for each entity by their ownership percentages in these structures. We now present the problem description, the MILP formulation, a real-world illustration based on the Portuguese PSC network, presenting associated MILP solutions and a sensitivity analysis, finally concluding with proposals for future research.

2.3 Problem Description

The downstream PSC can be defined as a network of petroleum companies, which act over a system comprised by refineries, distribution depots, transportation modes and routes that cumulatively satisfy the retail customer demands for multiple petroleum products in various regions under study. The downstream PSC network comprises the refining, logistics and commercialization of the petroleum products as presented in Figure 2.1, whereas the upstream PSC includes the activities that allow crude oil to be explored and transformed.

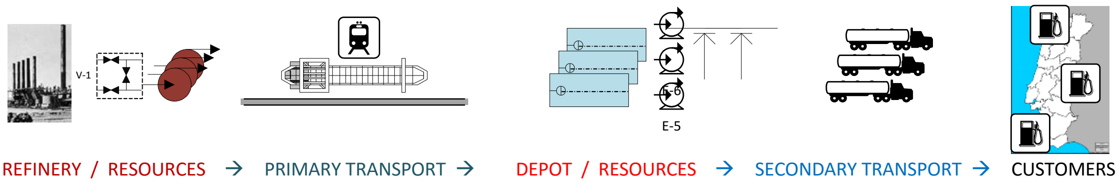


Figure 2.1. Downstream Petroleum Supply Chain

The Petroleum companies are the main stakeholders of the PSC network, as they produce and/or trade petroleum crude and derivatives, transform crude into petroleum derivatives and commercialize them to the retail segment clients. They rely on physical infrastructures, which are wholesale markets, refineries, bulk or primary transport, storage depots, secondary transport and retail stations. These infrastructures may be owned by a single company or multi-company consortiums. Transportation routes are strategic to the PSC and are mainly maritime between wholesale markets and refineries, whereas the primary distribution involves pipeline, maritime or railway transportation between refineries and storage depots, and lastly the secondary distribution is undertaken by pipeline, railway or road transportation between the depots and the retail stations.

The design of a downstream PSC network involves the installation and operation of storage depots and primary and secondary transportation infrastructures that comprise the network. The product transportation volumes are relevant here as they determine the necessary resources and their capacity in the network at refineries and depot installations. These resources may be considered as grouped items such as pump stations, interfaces, vessels and loading bays or individualized as manifolds, arms, pumps, valves, etc. Besides the installations, the resources also imply considerable fixed and operating costs and hence the importance of their being considered separately.

The strategic planning problem associated with the downstream PSC network, involves determining the logistics network topology for the distribution of petroleum products and can be defined as follows:

> **Given** a set of petroleum companies who produce or import a set of oil products at/from a known set of refineries, and commercialize them to a set of customer regions through available and possible sets of depots and transportation connections;
>
> **Determine** the optimal multi-company, multi-echelon, multi-product PSC network consisting of installation or closure of storage depot locations, connecting primary and secondary transportation modes and routes, the storage capacities, and the allocation of volumes for crude oil, refinery production, import and export, depot transfer volumes and customer fulfillment;
>
> **Subject to** customer demands, production, storage depot, transportation and capacity restrictions;
>
> **In order to** maximize the total network profit, comprising the oil companies' revenues minus the transportation and storage costs along the network.

The PSC downstream network may be broken down into activity stages comprising the refining stage, primary distribution, depot storage, secondary distribution and the retail stage. The first stage of the downstream PSC is the refinery, which buys crude oil on the international market and yields the petroleum products. Here the known ex-refinery free on board (FOB) prices for each product and the crude oil and refining costs are used to determine the refining margins. The refineries may utilize their maximum crude processing capacity, thereby trading the product to the petroleum companies for national consumption at the FOB price or may export the product at the wholesale market price, in which case the refinery must support the cost, insurance and freight (CIF) charges to a central wholesale market hub. One such example is the European hub at the Amsterdam-Rotterdam-Antwerp (ARA) refinery cluster in the Netherlands. The product once exited from the refineries is under the ownership of a petroleum company and will stay so until the retail stage. Local and foreign refineries may be considered, whereby foreign refineries are allowed to import product, nevertheless they may not export products. Local refineries, on the other hand, are free to supply locally as well as to export.

The petroleum companies purchase petroleum products from the refineries at the FOB wholesale market price and transport the product up to a petroleum storage depot of convenience in the country. This first transport activity is known as the primary distribution, which involves the refinery to depot and inter-depot transportation, and is generally undertaken using pipeline, maritime or railway transportation mode, since large volumes are involved. The inter-depot transportation for specific product transformation related activities may also be carried by road using tank trucks, as in the case of liquefied petroleum gas bottle-filling, common in southern Europe and other parts of the world. The petroleum companies may also import the products from the wholesale market hub where again the FOB prices are considered and the companies will support the CIF charges up to the depots.

The petroleum storage depots comprise an important stage of the PSC where these depots are allowed to receive the product from local, regional or international refineries on behalf of the companies and will hold the product at their storage facilities until the product is expedited by the petroleum companies to their retail clients. Depots are considered as shared resources by the companies and the stored product is under the ownership of each company. Here, the product is stored in vessels, which are configured using typical vessel capacities that may vary according to the product. The depots are also characterized by the number of vessel rotations per product, which is the number of times a product vessel is refilled in a year. The depots need to be rightly dimensioned in order to maximize the number of rotations within a given inventory management policy, thus implying reduced relative fixed costs, thereby resulting in higher efficiency.

The secondary distribution stage involves the petroleum products transportation from the storage depot to the petrol station or customer zone. This transportation is normally undertaken by road using tank trucks, where the travelling distance and travelling time are determinant to the route profitability. Occasionally, this distribution may also be undertaken using pipeline or railroad transportation, when large volumes are transported to a specific destination, as in the case of Jet Fuel to airports.

Finally, the retail stage comprises of petrol stations, factories, airports, and hospitals, which supply the final consumer. The retail segment receives the product by tank trucks, stores the product in local tanks and sells the products to the end-user, recovering a profit margin from this retail commercial activity. These retail stations may be company owned company operated (COCO), company owned dealer operated (CODO) or dealer owned dealer operated (DODO). Hence based on their vertical integration, the petroleum companies also receive a profit share from the retail activity in addition to a defined product sales margin.

As known, demand curve fluctuations are norm in PSCs, hence the network design must be flexible and adapt over time in order to stay profitable. Yearly operating costs amounting to 20% of the installation cost, can be greatly reduced by re-dimensioning networks. Being a strategic model, the optimal network designed, considers the customer demand for a long time horizon aggregated for large time periods. However, demand previsions may change, leaving some installations with excess capacity and others short. The model’s retrofit design capabilities permit the determination of location and capacity expansion or closure by performing subsequent runs with the changed data. Demand uncertainty management could be modelled with periodic inventory planning, which although not considered in the current model, is identified for further research. Another important aspect is the duration for transportation and product reposal. Specifically, if product pumped into a pipeline requires 36 hours to arrive and 8 hours to repose at the destination depot, the product would be released after 44 hours. Although these can have significant impact on the PSC network capacities, they are more relevant in terms of operational planning and thus lower decision levels and shorter time horizons. By using a capacity fill factor as a design parameter, we allow for an additional capacity buffer to cover the time delay between the product supply and availability at the demand location. Integrating operational planning with time constraints, although not addressed in this chapter, is considered for future research.

The PSC as described above results in the product price formation that is synthesized in Figure 2.2, the PSC value chain. The current study includes all costs, prices and margins at the various activities of the PSC, with exception to the crude price formation, the petroleum excise tax and the value added tax. Also, in this study, the petrol station costs are not broken down into retail station costs and dealer margin but considered aggregately as a service. Cross-country sales and taxation policies are identified as an opportunity for future study.

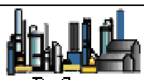
Activity Stages  Crude Oil	Exploration costs +Production costs +Maritime transport	=Crude Oil Spot Price
 Refinery	+Refinery costs +Refining margin	=Refinery Spot Price
 Pipeline	+Primary transport costs +Primary transport	=Primary Spot Price
 Storage Depot	+Storage costs +Storage margin	=Rack Price
 Tanker Trunk	+Secondary transport costs	=Tank Wagon Price
 Petrol Station	+Retail station costs +Dealer/Operator margin	=Retail Price

Figure 2.2. PSC value chain

The above division of the PSC together with the petroleum products price formation is the basis for the development of the structure of costs, tariffs and profits as well as the strategic and tactical planning model. In this study, all downstream activities including purchase of crude oil, refining, sale of product to petroleum companies, delivery of product, and the retail segment commercialization are modelled following a set of assumptions.

2.4 Problem Representation Assumptions

In this section we provide the assumptions taken in consideration when developing the mathematical model. These translate how the real problem is tackled and thus represented in the proposed model formulation. The key idea behind the assumptions presented bellow is to effectively represent a general real PSC, maintaining flexibility for the design and planning of different networks.

- The PSC network considers multiple companies, echelons, products and transportation modes;
- Uni and multi-entity PSC networks are modelled for existing, grassroots and retrofit scenarios;
- Refinery locations, capacities and yields are given and may not be changed;
- Transportation may be undertaken using pipeline, maritime, railway or by road;
- New depots and transportation resources including pipelines, rail stations and maritime docks may be installed at refinery and/or demand nodes or between them;
- Existing storage depots and transportation resources and capacities may be increased or reduced by considering the capacity for operation or closure;
- A fixed flow rate and rotation is considered per depot, transportation resource and product;
- Storage vessels located at depots are configured with typical capacities and rotations per product;
- Company participation percentages in existing refineries, storage depots and transportation facilities are known and will remain unchanged under the scope of the time horizon modelled;
- Company participation percentages in new infrastructures will follow market quotas;
- Depots and transportation resources are shared by owner companies;
- Company participation percentages define the installed and operating resource capacities per entity and product;
- Deterministic customer demand is known and may be grouped by district or municipality capitals and may be converted on the required time basis, such as on yearly or monthly basis;
- Refineries import crude oil, and supply derived products to depots, or may export derived products;
- Refineries as well as depots may import derived products;
- Any exportation/importation action is considered as a trade to/from a region/refinery located outside the boundaries of the region modelled;
- Several refineries and transportation modes may supply a storage depot which then supplies the product to various customer zones;
- Fixed, variable, installation and operating costs are known per resource type while crude oil and product prices are considered per echelon and period;
- Bottle-filling of liquefied petroleum gas and excise and value added taxes are beyond scope.

2.5 Mathematical Formulation

The problem as defined in section 2 considers the strategic and tactical planning for the PSC aimed at defining the PSC multi-entity and multi-echelon network design and transfer volumes for an existing, a grassroots and a retrofit network. Individual revenues and costs for competing entities in a shared network must be computed. To achieve that, a MILP model is developed that maximizes the multi-echelon entity profits, which correspond to the aggregation of the profit margins along the supply chain. At each stage of the PSC value chain presented in figure 2.2, are calculated the stage costs, tariffs, prices and profits, which are aggregated to give the total PSC profits. Total costs include individually the crude supply and refining, importation, exportation, storage, primary and secondary transportation and the unsatisfied demand costs. Fixed tariffs per unit volume of product are assumed for the refining and retail activity stages. The MILP uses these costs, tariffs and final product prices at each activity stage to design an optimized PSC network that consequently reduces the individual activity costs, resulting in lower amounts of tariffs, and hence increased entity profits.

The remainder of this section presents the MILP formulation in detail explaining the notation used, the network constraints, the objective function and lastly the pre-processing.

2.5.1 Notation

2.5.1.1 Sets / Indices

$a \in A$	set of petroleum supply chain activity stages A={CRUDE, REFINING, IMPORT, EXPORT, PRIMARY, STORAGE, SECONDARY, RETAIL }
$c \in C$	set of product characteristics, C={ DENSITY, TYPICAL_M3}
$d \in D$	set of route characteristics, D={ DISTANCE, DURATION }
$e, e1 \in E$	set of petroleum entities/companies E={E1, E2, ..., EN },
$g \in G$	set of GPS coordinates, G={ LATITUDE, LONGITUDE }
$n, o \in N$	set of all nodes,
$p \in P$	set of products P={ P1, P2,..PN },
$r, r1 \in R$	set of resources including services and equipment, R={IMPORT, EXPORT, RETAIL, PIPELINE, MARITIME, RAIL, ROAD, REFINERY, DEPOT, VESSEL, PUMP, INTERFACE, DOCK, STATION, ARM}
$t \in T$	set of time periods, T={1,2,..N}
$v \in V$	Set of costs, V={TARIFF,FIXED,FIXVAR,OPERATING,OPERVAR,UNITARY}
$y \in Y$	set of amortization periods, Y={ 1,2,…,N }

2.5.1.2 Subsets / Indices

$i \in I \subseteq N$	set of refinery nodes,

$IL \subseteq I$	set of local refinery nodes which are allowed to export,
$j \in J \subseteq N$	set of depot nodes,
$k \in K \subseteq N$	set of customer nodes,
$m, m1 \in M \subseteq R$	set of transport modes, M={ PIPELINE, MARITIME, RAIL, ROAD }

2.5.1.3 Subsets unions / Indices

$h \in H = I \bigcup J$	set of refinery and depot nodes,
$l \in L = J \bigcup K$	set of depot and customer nodes,

2.5.1.4 Parameters

BN	big number for number of resources,
BQ	big number for product volume,
HPP	hours per period,
MNA	maximum number of amortization periods,
NCR	number of secondary routes in concentration set,
ND	maximum number of depots,
NFF	fill factor in percentage,
NR	GPS coordinate degree to radian conversion coefficient for distance
TPP	time aggregation in period, TPP=1 for monthly data, TPP=12 for yearly data,
α^{a}	operator margin percentage per activity *a*,
$\Gamma_{n,g}$	GPS (global pointing system) coordinates *g* for each location node *n*,
$\delta_{k,p,e,t}$	customer demand in m^3 at customer node *k*, product *p*, entity *e* and period *t*,
$\varepsilon_{i,p}$	fraction yield of product *p* per unit measure of crude oil at refinery *i*,
$\varphi^{a}_{p,t}$	price of product *p* after activity *a* (Crude, Ex-refinery, retail) at time *t*,
$\eta_{n,o,r,e}$	capital participation percentage by entity *e* in resource *r* between nodes *n,o;* consider *r* = *m* in case of transport mode and *n*=*o* in case of depots,
$\kappa d_{n,y}$	number of completed amortization periods *y* for each existing depot at
$\kappa m_{m,n,r,y,p}$	existing resource units per transport mode *m* and product *p*, per resource *r* at refinery or depot *n,* per completed number of amortization periods *y*,
$\kappa p_{n,r,y,p}$	existing resource units per product *p*, per resource *r* at refinery or depot *n,* per completed number of amortization periods *y*,
$\kappa r_{m,n,r,y}$	existing resource units per transport mode *m*, per resource *r* at refinery or depot *n,* per completed number of amortization periods *y*,

$\kappa t_{n,o,m,y}$	number of completed amortization periods *y* for existing transport infrastructures between *n,o* for mode *m*; road routes are considered as
$\mu_{n,o,m}$	travelling distance between any two nodes *(n,o)* including refineries, depots or customer zones, using transport mode *m*. The condition $(h,l,m):\exists\mu_{h,l,m}$ is used to indicate tuples *(h,l,m)* with available routes referred by travelling
$\theta m_{m,r,p}$	typical capacity of resource *r* for transport mode *m* and product *p*,
$\theta p_{r,p}$	typical capacity of resource *r* associated with product *p*,
$\theta r_{m,r}$	typical capacity of resource *r* associated with transport mode *m*,
ρ_i	crude processing capacity at refinery *i* in m^3 per day,
$\upsilon^a_{m,p}$	maximum permitted distance for activity *a* by transportation mode *m* for the petroleum product *p*; example: fuel-oil transportation by pipeline,
ω^c_p	characteristic *c* per product *p*, e.g. density,
ωm_p	number of vessel stock rotations per product *p*,
ξR^v_r	cost component *v* for resource *r* independent of product, example pipeline,
$\xi P^v_{r,p}$	cost component *v* for resource *r* dependent of product *p*, example vessel,
ζ_e	market quota per entity *e*,

2.5.1.5 Binary Variables

The binary variables determine the strategic decisions where the prefix *I* indicates decisions for installation of depot locations and the transportation routes, the prefix *O* indicates decisions for their operation, and finally *A* associates products to routes. It is assumed that these variables once selected (installed or operating) will stay selected in all periods.

$$AP_{h,j,m,p} = \begin{cases} 1, & \text{if primary route from refinery or depot } h \text{ to depot j with transport mode m is } \textit{available for product p} \\ 0, & \textit{otherwise} \end{cases}$$

$$AS_{j,k,m,p} = \begin{cases} 1, & \text{if secondary route from depot j to customer k with transport mode m is } \textit{available for product p} \\ 0, & \textit{otherwise} \end{cases}$$

$$ID_j = \begin{cases} 1, & \textit{if depot j is to be installed} \\ 0, & \textit{otherwise} \end{cases}$$

$$IT_{n,o,m} = \begin{cases} 1, & \text{if route using transport mode } m \text{ is to be installed between } \textit{node n} \text{ and } \textit{node o} \\ 0, & \textit{otherwise} \end{cases}$$

$$OD_j = \begin{cases} 1, & \textit{if depot j is to be operated} \\ 0, & \textit{otherwise} \end{cases}$$

$$OT_{n,o,m} = \begin{cases} 1, & \text{if route using transport mode } m \text{ is to be } \textit{operated} \text{ between } \textit{node n and node o} \\ 0, & \textit{otherwise} \end{cases}$$

2.5.1.6 Integer Variables

The integer variables determine the depot capacities. The *I* variables determine the installed resources, which include those already existing and defined in the *K* parameters above. To save costs, existing resources may be closed, hence resources strictly required to operate are determined by the *O* variables. A positive difference between corresponding *I* variables and *K* parameters provides the number of new resources to install, whereas a positive difference between *I* and *O* resource variables provides the number of such resources to close. Capacity, once installed is assumed to be constant along all periods of the time horizon.

$IRA_{m,n,r}$ number of resources *r* to install per transport mode *m* at each depot node *n*,

$IRM_{m,n,r,p}$ number of resources *r* to install per transport mode *m* per depot *n* for product

$IRP_{n,r,p}$ number of resources *r* to install per depot *n* for product *p*,

$ORA_{m,n,r}$ number of resources *r* to operate per transport mode *m* at each depot node *n*,

$ORM_{m,n,r,p}$ number of resources *r* to operate per transport mode *m* per depot *n* for product

$ORP_{n,r,p}$ number of resources *r* to operate per depot *n* for product *p*,

2.5.1.7 Positive Variables

These are tactical time-dependent decision variables which determine the product volumes *(Q)* at various activity stages, nodes and operating routes of the PSC.

$QCP_{i,t}$ volume of crude processed by refinery *i,* in period *t*,

$QE_{i,p,t}$ volume exported by refinery *i* of product *p* in period *t*,

$QI_{j,p,e,t}$ volume imported by depot *j* of product *p* and entity *e* in period *t*,

$QP_{h,j,m,p,e,t}$ primary distribution volume transported from refinery/depot *h* to depot *j* using transport mode *m* of product *p* by entity *e* in period *t*,

$QR_{i,p,t}$ planned production volume per refinery *i* of product *p* in period *t*,

$QS_{j,k,m,p,e,t}$ secondary distribution volume transported from depot *j* to customer zone *k* using transport mode *m* of product *p* by entity *e* in period *t*,

$QU_{k,p,e,t}$ unsatisfied demand volume per customer *k*, for product *p,* entity *e* and period *t*,

2.5.1.8 Continuous Variables

The following continuous variables determine the sub-values of the objective function.

$\Pr ofit$ objective function = total PSC profits,

$CDFD_{h,t}$ depot fixed costs for depot *h* in period *t*,

$CDO_{h,t}$ depot operating costs for depot *h* in period *t*,

$CDU_{h,p,e,t}$ depot unitary costs at depot *h* for product *p*, for entity *e* in period *t*,

$CE_{i,p,e,t}$ exportation costs per refinery *i*, product *p*, entity *e* in period *t*,

$CI_{j,p,e,t}$ importation costs per depot *j*, product *p*, entity *e* in period *t*,

$CRFD_{h,t}$ depot resources fixed costs for depot *h* in period *t*,

$CRO_{h,t}$ depot resources operating costs for depot *h* in period *t*,

$CTFD_{h,l,m,t}$ transportation fixed costs between nodes *h,l* for transport mode *m* in period *t*,

$CTO_{h,l,m,t}$ transportation operating costs between nodes *h,l* for transport mode *m* in

$CTU_{h,l,m,p,e,t}$ transportation unitary costs from nodes *h,l* for transport mode *m* product *p* in

$ML_{k,p,e,t}$ retail margin per customer location *k*, product *p*, entity *e* in period *t*,

$MR_{i,e,t}$ refinery margin per refinery *i*, entity *e* in period *t,*

$TDE_{h,e,t}$ storage depot tariffs at depot *h* for entity *e* in period *t*,

$TTE_{h,l,m,e,t}$ transportation tariff between nodes *h,l* for transport mode *m* for entity *e* in

2.5.2 Constraints

The model considers various constraints that govern the installation, operation and capacity requirements of depots and transportation infrastructures. Moreover, volume balance equations secure the production and transportation volumes at each PSC stage, thereby influencing the strategic decisions for the PSC.

2.5.2.1 Volume balance equations

Refinery mass balance: The various volume balance equations pull the final customer demand to higher stages, thereby determining the crude oil necessary for refinery production. The crude processed at each refinery and period $QCP_{i,t}$ is restricted by eq. (2.1) to its crude processing capacity ρ_i. Besides, each refinery has a known fractional yield, which determines the petroleum product volumes obtained from a unit measure of crude oil. Consequently, eq. (2.2) restricts the refined volumes $QR_{i,p,t}$ derived per refinery, product and time to the processed crude $QCP_{i,t}$ multiplied by fractional yield $\varepsilon_{i,p}$.

$$QCP_{i,t} \le \rho_i, \qquad \forall i \in I, t \in T \tag{2.1}$$

$$QR_{i,p,t} \le QCP_{i,t} \times \varepsilon_{i,p}, \qquad \forall i \in I, p \in P, t \in T \tag{2.2}$$

Secondly, for each refinery, product and period, the sum of the refined volume $QR_{i,p,t}$ and the sum of the import volumes for all entities $\sum_{e \in E} QI_{i,p,e,t}$ is enforced by the import/export balance equations eq. (2.3) to equal the sum of the total traded volumes $\sum_{(j,m):\exists\mu_{i,j,m},e\in E} QP_{i,j,m,p,e,t}$ and the total exported product $QE_{i,p,t}$. The condition expressed in the summation of the first term of the right hand side of equation (2.3), $(h,l,m):\exists\mu_{h,l,m}$, implies summations only over tuples *(h,l,m)* present in available routes $\mu_{h,l,m}$.

$$QR_{i,p,t} + \sum_{e\in E} QI_{i,p,e,t} = \sum_{(j,m):\exists\mu_{i,j,m},e\in E} QP_{i,j,m,p,e,t} + QE_{i\in IL,p,t}, \qquad \forall i\in I, p\in P, t\in T \tag{2.3}$$

Product exports: Exportation of surplus production $QE_{i,p,t}$ is permitted by constraints (eq. 2.3) above. However, it is sometimes required to study the network from the importation perspective or to analyze the network for a single country or region. This is achieved by prohibiting exportation to refineries other than the local ones $i \not\subset IL$ as enforced in eq. (2.4).

$$QE_{i,p,t} = 0, \qquad \forall i \not\subset IL \tag{2.4}$$

Individualistic operation: Entities may only distribute products through self-owned or partially owned depots and may not exceed the volume determined by their participation. While the participation in existing depots is known, the percentage participation per entity for the new depots is assumed to be equal to the market quota. Eq (2.5). enforces this for existing depots, where the total secondary volume supplied by depot k per entity, transport mode, product and period, may not exceed the product of the number of operating vessels, the vessel flow capacity per hour and the percentage participation of the entity in that depot. Eq. (2.6) enforces the entity capacity limits for newly installed depots.

$$\sum_{(k,m):\exists\mu_{j,k,m}} \frac{QS_{j,k,m,p,e,t}}{\omega m_{m,p}} \le ORP_{j,r,p}\times\theta p_{r,p}\times TPP\times\eta_{j,j,r1,e}, \tag{2.5}$$
$$\forall j\in J\wedge\eta_{j,j,r1,e}, m\in M, r\in R, r1=DEPOT, p\in P\wedge(r,p)\subset\theta p_{r,p}, e\in E, t\in T$$

$$\sum_{(k,m):\exists\mu_{j,k,m}} \frac{QS_{j,k,m,p,e,t}}{\omega m_{m,p}} \le ORP_{j,r,p}\times\theta p_{r,p}\times TPP\times\zeta_{e}, \tag{2.6}$$
$$\forall j\in J\wedge NOT\,\eta_{j,j,r1,e}, m\in M, r\in R, r1=DEPOT, p\in P\wedge(r,p)\subset\theta p_{r,p}, e\in E, t\in T$$

Storage Depot balance: In either case, the depot must fulfill the secondary distribution demand coming from the customer balance equations. The PSC may be modelled as a direct transportation network as presented in Figure 2.3 or a routed network as shown in Figure 2.4.

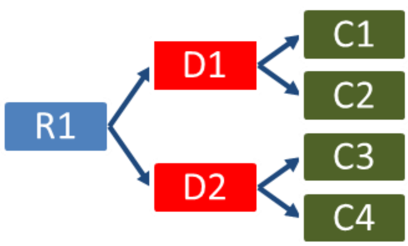


Figure 2.3. Direct Refinery-Depot

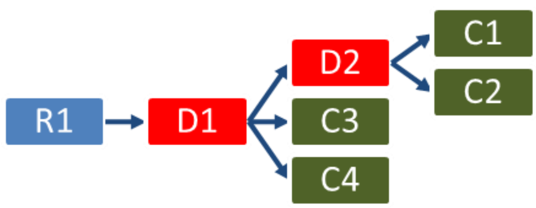


Figure 2.4. Inter-depot transportation

Constraints (2.7) enforce this for the direct network. At each period, the sum of primary volume received by each depot $QP_{h,j,m,p,e,t}$ and the imported volume $QI_{j,p,e,t}$ terminating at the depot must be equal to the sum of the secondary distribution volume $QS_{j,k,m,p,e,t}$ originating from that depot. In this case, the depot receives product from the refinery and supplies directly to the customer.

$$\sum_{(h\subset I,m):\exists\mu_{h,j,m}} QP_{h,j,m,p,e,t} + \sum_{e\in E} QI_{j,p,e,t} = \sum_{(k,m):\exists\mu_{j,k,m}} QS_{j,k,m,p,e,t}, \qquad \forall j\in J, p\in P, e\in E, t\in T \quad (2.7)$$

The model can be easily altered to handle routed networks. This could be particularly relevant for big multi-country networks. In this case, eq. (2.7) will be rewritten as (2.8) and (2.9). Eq. (2.8) is the same as eq. (2.7), except that they are only valid for refinery located depots $j\subset I$. Eq. (2.9) is only valid for non-refinery depots $j\not\subset I$, in which case, the right hand side must add primary distribution volumes $QP_{j,h,m,p,e,t}$ exiting the depot.

$$\sum_{(h\subset I,m):\exists\mu_{h,j,m}} QP_{h,j,m,p,e,t} + \sum_{e\in E} QI_{j,p,e,t} = \sum_{(k,m):\exists\mu_{j,k,m}} QS_{j,k,m,p,e,t}, \qquad \forall j\in J\wedge j\subset I, p\in P, e\in E, t\in T \quad (2.8)$$

$$\sum_{(h\subset I,m):\exists\mu_{h,j,m}} QP_{h,j,m,p,e,t} + \sum_{e\in E} QI_{j,p,e,t} = \sum_{(h\not\subset I,m):\exists\mu_{j,h,m}} QP_{j,h,m,p,e,t} + \sum_{(k,m):\exists\mu_{j,k,m}} QS_{j,k,m,p,e,t}, \quad \forall j\in J\wedge j\not\subset I, p\in P, e\in E, t\in T \quad (2.9)$$

Customer Demand balance: At the retail activity stage, the satisfied demand is enforced by equations (2.10) to be equal to the sum of the demand $\delta_{k,p,e,t}$ minus the unsatisfied demand $QU_{k,p,e,t}$ for each customer zone, product, entity and period, which is the sum of the secondary distribution volumes $QS_{j,k,m,p,e,t}$ coming from all depots and transport modes.

$$\sum_{(j,m):\exists\mu_{j,k,m}} QS_{j,k,m,p,e,t} = \delta_{k,p,e,t} - QU_{k,p,e,t}, \qquad \forall k\in K, p\in P, e\in E, t\in T \quad (2.10)$$

2.5.2.2 Transportation network

Transportation route retrofit installation: Transportation restrictions play an important role in the selection of transport modes and routes. The transportation network comprises of the primary and secondary distribution where the former transports the petroleum product from refinery to storage depots and the later transports the product from the depot to the petrol station or customer zones. All strategic decisions regarding transportation installation and operation are made considering a single network whereas the tactical decisions of transported volumes are modelled distinctly per entity. Retrofit design is considered, hence existing transport routes and capacities are installed as enforced by constraint (2.11).

$$\sum_{y\in Y} \kappa t_{h,l,m,y} \le IT_{h,l,m} \times BQ, \qquad \forall h\in H, l\in L, m\in M \wedge (h,l,m):\exists\mu_{h,l,m} \quad (2.11)$$

However, an existing route may be closed (not operated). Hence, constraint (2.12) enforces that only installed routes may be operated.

$$OT_{h,l,m} \leq IT_{h,l,m}, \qquad \forall h \in H, l \in L, m \in M \wedge (h,l,m) : \exists \mu_{h,l,m} \tag{2.12}$$

Distribution route operation: The primary and secondary distribution volumes activate primary and secondary product allocations on transportation routes as in eqs. (2.13) and (2.14). Constraint (2.15) enforces that product allocations are made only to operating routes. For each product, a route between two nodes will use at most one transport mode, which will be selected either for primary or for secondary distribution. Eqs. (2.16) implement these cuts, thereby greatly reducing the model size and time. It is assumed that the route once selected will be operating in all periods.

$$\sum_{e \in E} \sum_{t \in T} QP_{h,j,m,p,e,t} \leq AP_{h,j,m,p} \times BQ, \qquad \forall h \in H, j \in J, m \in M \wedge (h,j,m) : \exists \mu_{h,j,m}, p \in P \tag{2.13}$$

$$\sum_{e \in E} \sum_{t \in T} QS_{j,k,m,p,e,t} \leq AS_{j,k,m,p} \times BQ, \qquad \forall j \in J, k \in K, m \in M \wedge (j,k,m) : \exists \mu_{j,k,m}, p \in P \tag{2.14}$$

$$AP_{h,l,m,p} + AS_{h,l,m,p} \leq OT_{h,l,m}, \qquad \forall h \in H, l \in L, m \in M \wedge (h,l,m) : \exists \mu_{h,l,m}, p \in P \tag{2.15}$$

$$\sum_{m \in M} \left(AP_{h,l,m,p} + AS_{h,l,m,p} \right) \leq 1, \qquad \forall h \in H, l \in L \wedge l \not\subset I, p \in P \tag{2.16}$$

The transportation network equations are created only for available routes $(h,l,m) : \exists \mu_{h,l,m}$.

2.5.2.3 Storage Depot Capacities

Installation, operation and sizing of resources: The main focus of strategic decisions at the storage depots, is the installation, operation and capacity modelling of resources. Resources are typified as dynamic or continuous flow equipment having a typical flow capacity in cubic meters per hour as in the case of pumps and arms; and static or non-continuous flow equipment with typical volume capacity as in the case of storage vessels. The later are characterized by number of capacity rotations which in the case of vessels is the average number of times it is refilled per period. The model is provided with the typical capacity of resources related to transport mode (e.g. pipeline) in $\theta r_{m,r}$, those related to transport mode and product (e.g. arm) in $\theta m_{m,r,p}$, and those related to product (e.g. vessel) in $\theta p_{r,p}$. Constraints (2.17), (2.18) and (2.19) therefore determine the resource units needed to operate per depot, transport mode and/or product based on the transport volumes. Here, the product volumes transferred from the depot per period $\sum_{j:\exists \mu_{h,j,m}} \sum_{p \in P} \sum_{e \in E} QP_{h,j,m,p,e,t} + \sum_{k:\exists \mu_{h,k,m}} \sum_{p \in P} \sum_{e \in E} QS_{h,k,m,p,e,t}$ must be less than the installation's operating resource capacity which is the product of the resource units $ORA_{m,h,r}$, the resource capacity $\theta r_{m,r}$ and the hours per period HPP. In the case of product dependent static resources $ORP_{h,r,p}$, this capacity is augmented by dividing the transfer volumes by the number of resource rotations $\omega m_{m,p}$.

$$\sum_{j:\exists\mu_{h,j,m}}\sum_{p\in P}\sum_{e\in E}QP_{h,j,m,p,e,t}+\sum_{k:\exists\mu_{h,k,m}}\sum_{p\in P}\sum_{e\in E}QS_{h,k,m,p,e,t}\leq ORA_{m,h,r}\times\theta r_{m,r}\times HPP,\ \forall m\in M,h\in H,r\in R,t\in T,(m,r):\exists\theta r_{m,r} \quad (2.17)$$

$$\sum_{j:\exists\mu_{h,j,m}}\sum_{e\in E}QP_{h,j,m,p,e,t}+\sum_{k:\exists\mu_{h,k,m}}\sum_{e\in E}QS_{h,k,m,p,e,t}\leq ORM_{m,h,r,p}\times\theta m_{m,r,p}\times HPP,\forall m\in M,h\in H,r\in R,p\in P,t\in T,(m,r,p):\exists\theta m_{m,r,p} \quad (2.18)$$

$$\sum_{(j,m):\exists\mu_{h,j,m}}\sum_{e\in E}\frac{QP_{h,j,m,p,e,t}}{\omega m_{m,p}}+\sum_{(k,m):\exists\mu_{h,k,m}}\sum_{e\in E}\frac{QS_{h,k,m,p,e,t}}{\omega m_{m,p}}\leq ORP_{h,r,p}\times\theta p_{r,p}\times TPP,\forall h\in H,r\in R,p\in P,t\in T,(m,r,p):\exists\theta p_{r,p} \quad (2.19)$$

As expected, resources may only operate if installed as enforced by constraints 20, 21 and 22.

$$ORA_{m.h.r}\leq IRA_{m,h,r}, \qquad \forall m\in M,h\in H,r\in R,(m,r):\exists\theta r_{m,r} \quad (2.20)$$

$$ORM_{m,h,r,p}\leq IRM_{m,h,r,p}, \qquad \forall m\in M,h\in H,r\in R,p\in P,(m,r,p):\exists\theta m_{m,r,p} \quad (2.21)$$

$$ORP_{h,r,p}\leq IRP_{h,r,p}, \qquad \forall h\in H,r\in R,p\in P,(r,p):\exists\theta p_{r,p} \quad (2.22)$$

Equations (2.17), (2.18) and (2.19) above determine the resource units to operate and consequently their installation through (2.20), (2.21) and (2.22). Retrofit design guarantees the installation of the existing resources through (2.23), (2.24) and (2.25). A positive difference between installed $IRA_{m,h,r}$ and existing $\sum_{y\in Y}\kappa r_{m,h,r,y}$ resources gives the number of new resources to open and a positive difference between the installed $IRA_{m,h,r}$ and operating $ORA_{m.h.r}$ resources gives the number of such resources to close.

$$\sum_{y\in Y}\kappa r_{m,h,r,y}\leq IRA_{m,h,r}, \qquad \forall m\in M,h\in H,r\in R,t\in T,(m,r):\exists\theta r_{m,r} \quad (2.23)$$

$$\sum_{y\in Y}\kappa m_{m,h,r,y,p}\leq IRM_{m,h,r,p}, \qquad \forall m\in M,h\in H,r\in R,p\in P,t\in T,(m,r,p):\exists\theta m_{m,r,p} \quad (2.24)$$

$$\sum_{y\in Y}\kappa p_{h,r,y,p}\leq IRP_{h,r,p}, \qquad \forall h\in H,r\in R,p\in P,t\in T(m,r,p):\exists\theta p_{r,p} \quad (2.25)$$

Eq. (2.17) through (2.25) are defined per depot for transport mode, resource and product combinations defined in the resource capacities $\theta r_{m,r}$, $\theta m_{m,r,p}$ and $\theta p_{r,p}$. The operating resources determined above require that the source depot be operated. This is enforced by constraints (2.26) up to (2.28).

$$ORA_{m.h.r}\leq OD_{h}\times NB, \qquad \forall m\in M,h\in H,r\in R \quad (2.26)$$

$$ORM_{m,h,r,p}\leq OD_{h}\times NB, \qquad \forall m\in M,h\in H,r\in R,p\in P \quad (2.27)$$

$$ORP_{h,r,p}\leq OD_{h}\times NB, \qquad \forall h\in H,r\in R,p\in P \quad (2.28)$$

Furthermore, constraints (2.29),(2.30) and (2.31) enforce that resources are installed only in installed depots.

$$IRA_{m.h.r} \leq ID_h \times NB, \qquad \forall m \in M, h \in H, r \in R \tag{2.29}$$

$$IRM_{m,h,r,p} \leq ID_h \times NB, \qquad \forall m \in M, h \in H, r \in R, p \in P \tag{2.30}$$

$$IRP_{h,r,p} \leq ID_h \times NB, \qquad \forall h \in H, r \in R, p \in P \tag{2.31}$$

Storage Depot installation and operation: Secondary distribution volumes obligate that the originating depot be operating as enforced by constraint (2.32).

$$\sum_{(h,k,m):\exists \mu_{h,k,m}} \sum_{p \in P, e \in E, t \in T} QS_{h,k,m,p,e,t} \leq OD_h \times BN, \qquad \forall h \in H \tag{2.32}$$

Moreover, existing and operating depots must be installed as enforced in constraints (2.33) and (2.34).

$$\sum_{y \in Y} \kappa d_{h,y} \leq ID_h, \qquad \forall h \in H, y \in Y, (h,y) : \exists \kappa d_{h,y} \tag{2.33}$$

$$OD_j \leq ID_j, \qquad \forall j \in J \tag{2.34}$$

Total depots: Finally, constraint (2.35) imposes an upper bound on maximum installed depots.

$$\sum_{j \in J} ID_j \leq ND \tag{2.35}$$

The constraints presented here provide the feasible solutions wherein the optimal solution is determined by the objective function presented in the following section.

2.5.3 Objective Function

An important feature of this MILP model is that it considers various entities along the PSC, thereby adding to its complexity but also utility in that it can design and analyze the PSC network under distinct business settings. Such a network design has three interesting challenges. The first is to analyze various business partnership strategies between the entities and operators. The second is to define an imputation model for the product, operator and service costs among the various entities. Finally, the third is to develop a model that maximizes the total network profits while providing some considerations for optimizing individual entity profits.

The first problem is considered using an individualistic or non-collaborative strategy. Here, the petroleum entities are in strict competition with each other thus may only use the capacity determined by their percentage financial participation in self-owned or partially owned structures. Current and retrofit networks consider existing infrastructures. The fixed, operating and unitary costs are calculated per infrastructure and resource and imputed to the product owner entities based on the transported volumes. The fixed costs are amortizations per equipment which are imputed only in non-amortized periods. The operator tariffs are then calculated by adding an operator markup α^{a} defined per activity, to the abovementioned costs.

The cost structure allows the definition of an activity as an entity-owned infrastructure or as a service. In the first case, the fixed, operating and unitary costs must be defined while in the later it would suffice to define only the unitary tariff. Typically, depots and pipelines are entity-owned while maritime, railway and road transportation are considered as services. Nevertheless, installation and operating costs with resources required at the originating depots or refineries need to be considered.

Another innovation of the MILP is the computation of petroleum company earnings along the supply chain that is determinant to the resulting design of the PSC network. Although the model can design a single network maximizing individual entity profits, a key advantage is the possibility to design and optimize a multi-entity PSC network, including various entities and logistic operators. Here the main difficulty is how to distribute the fixed installation and operating costs that are calculated per installation and must be allocated per entity.

The fixed and operating cost allocation per entity could follow either of three criteria: using market quotas; infrastructure capital participation quotas, or a flat fee. The flat fee criteria for fixed cost allocation could negatively influence the utilization of a participated infrastructure that generates good returns. Here, an entity with significant participation in the capital and alternative logistic options could tend to lower its utilization to reduce the imputed fixed costs, while simultaneously taking advantage of the infrastructure generated profits due to its participation in the capital. In contrast, a harmonious design could be expected if the market quota is used as criteria for fixed cost allocation as the model would thus generate solutions where expedited volumes would follow market quota distributions and not be inversely biased by the capital participation. Finally, the capital participation criteria could result in increased entity tariffs if lower volumes were transferred through a depot, and inversely reduced tariffs if were transported higher volumes than their market quotas or capital participation.

The current model distributes the fixed, fixed variable, operating and operating variable costs, based on the market quota and affects the unitary costs directly to the entities as per the expedited volumes. The model assumes market quota defined globally per entity, however it can be easily modified to consider the market quota per customer zone and per product. The market quota may be calculated based on planned volumes per entity or on the basis of statistical market data. The entity participation in current depot and transportation infrastructures is assumed to be known.

The model maximizes the PSC global profits being the difference between the total revenues and the total costs computed per stage and per entity. Entity profits are individually accounted as the constraints force the entity distribution planning to be performed in competition. Furthermore, the customer demand volumes are given per entity and per product, hence all the volumes, revenues, costs and consequent margins for refining, retail, importation and exportation are calculated individually per echelon node per entity, and whenever possible per product. Allocation of profits and costs to entities is important to obtain various insights for the calculation of individual profits, costs and tariffs.

Eq. (2.36) provides the objective function that maximizes the total PSC profits, which is the aggregation of six terms: the refinery margins $MR_{i,e,t}$ minus the exportation costs $CE_{i,p,e,t}$, the importation costs $CI_{j,p,e,t}$; plus the retail segment margin $ML_{k,p,e,t}$ minus the transportation tariffs $TTE_{h,l,m,e,t}$ and the storage depot tariffs $TDE_{j,t}$. The objective function terms are hereafter described in full detail.

$$\text{Profit} = Max\sum_{e\in E}\left(\begin{array}{l}\sum_{i\in I}\sum_{t\in T}MR_{i,e,t}-\sum_{i\in I}\sum_{p\in p}\sum_{t\in T}CE_{i,p,e,t}-\sum_{j\in J}\sum_{p\in p}\sum_{t\in T}CI_{j,p,e,t}\\+\sum_{k\in K}\sum_{p\in P}\sum_{t\in T}ML_{k,p,e,t}-\sum_{(h,l,m):\exists\mu_{h,l,m}}\sum_{t\in T}TTE_{h,l,m,e,t}-\sum_{j\in J}\sum_{t\in T}TDE_{j,e,t}\end{array}\right) \quad (2.36)$$

2.5.3.1 Refinery margin (MR)

Each refinery produces derivatives that are either traded to the petroleum entities for local consumption or exported when production is surplus. Hence, the refinery margin given by eq. (2.37) is the difference between refining revenue: product of the refined volume and the difference between the ex-refinery price $\varphi^{REFINING}_{p,t}$ and the refining costs $\xi P^{v=t}_{r=REFINERY,p}$; and the crude oil supply costs given by $QCP_{i,t}\times\varphi^{CRUDE}_{CRUDE,t}$. This margin is distributed per entity based on their participation percentage $\eta^{REFINERY}_{i,i,e}$ in the refinery capital.

$$MR_{i,e,t}=\left(\sum_{p\in P}QR_{i,p,t}\left(\varphi^{REFINING}_{p,t}-\xi P^{v=t}_{r=REFINERY,p}\right)-QCP_{i,t}\times\varphi^{CRUDE}_{CRUDE,t}\right)\times\eta^{REFINERY}_{i,i,e},\forall i\in I,e\in E,t\in T \quad (2.37)$$

2.5.3.2 Exportation costs (CE)

As inventories are not considered, assuming a lean supply chain, excess production not traded to entities for national consumption, is considered to be exported. This implies an exportation cost, which is the product of the export volume and the unitary export tariff and the entity participation as given in eq.(2.38). The export tariff is the cost of insurance and freight for transporting the product to the nearest wholesale market hub.

$$CE_{i,p,e,t}=QE_{i,p,t}\times\xi P^{v=t}_{r=EXPORT,p}\times\eta^{REFINERY}_{i,i,e}, \qquad \forall i\in I,p\in P,e\in E,t\in T \quad (2.38)$$

2.5.3.3 Importation costs (CI)

Refineries and depots may import product that can be economically sourced from refineries external to the network. Also, product volumes that are not economically viable to produce due to the refinery yield of product mix or capacity restrictions, may be imported. Importation costs are computed as per Eq. (2.39) being the product of the import volume and the cost of product insurance and freight (CIF). The later is the sum of the ex-refinery or wholesale cost of product and the unitary importation tariff for transporting the product from the nearest wholesale market hub.

$$CI_{j,p,e,t} = QI_{j,p,e,t} \times \left(\varphi_{p,t}^{REFINING} + \xi P_{r=IMPORT,p}^{v=t} \right), \qquad \forall j \in J, p \in P, e \in E, t \in T \quad (2.39)$$

2.5.3.4 Retail Margin (ML)

At each customer zone and period, the product supplied by an entity is the customer demand less the unmet demand; the unitary retail value is the difference between the retail price and retail tariff; and the supply cost is the supplied product evaluated at the product's ex-refinery price. The retail margin per customer location, product, entity and period is given by eq. (2.40) which is the difference between the product of the supplied product and the unitary retail value and the product supply cost.

$$ML_{k,p,e,t} = \left(\delta_{k,p,e,t} - QU_{k,p,e,t}\right)\left(\varphi_{p,t}^{RETAIL} - \xi P_{r=RETAIL,p}^{v=t}\right) - \sum_{(j,m):\exists \mu_{j,k,m}} QS_{j,k,m,p,e,t} \times \varphi_{p,t}^{REFINING}, \qquad \forall k \in K, p \in P, e \in E, t \in T \quad (2.40)$$

Unsatisfied demand cost: The cost of unsatisfied demand is already considered in eq. (2.40) as the unmet demand implies lost revenue.

2.5.3.5 Transportation tariff (TTE)

The transportation tariff which is payable by the product proprietor is computed in eq. (2.41) per transport route, mode, entity and period. This is the sum of the unamortized fixed costs $CTFD_{h,l,m,t}$, operating costs $CTO_{h,l,m,t}$ and unitary costs $CTU_{h,l,m,p,e,t}$ which is then indexed by the operator margin $\alpha^{TRANSPORT}$ percentage. In the case of secondary transportation, which is realized by road, the installation and operating costs are not incurred, however the formulation permits all transportation modes. Tariff components are calculated only for available transport routes and modes $(h,l,m):\exists \mu_{h,l,m}$.

$$TTE_{h,l,m,e,t} = \left(1 + \alpha^{TRANSPORT}\right) \times \left(\zeta_e \times \left(CTFD_{h,l,m,t} + CTO_{h,l,m,t}\right) + \sum_{p \in P} CTU_{h,l,m,p,e,t} \right), \forall h \in H, l \in L, m \in M \wedge (h,l,m):\exists \mu_{h,l,m}, e \in E, t \in T \quad (2.41)$$

The installation of a transport mode between two nodes incurs a fixed installation cost, its operation incurs a fixed operating cost and the volumes transported will incur a linear variable cost. Eq. (2.42) determines the fixed installation cost where the binary expression $IT_{h,l,m} - \sum_{y \in Y} \kappa t_{\mathrm{h,l,m,y}}$ gives the difference between installed and existing routes and is true only for new and existing non-amortized routes. The second term $\sum_{y \in Y} \kappa t_{\mathrm{h,l,m,y}}$ with the condition $t \in T \wedge \mathrm{t} \leq \mathrm{MNA} \wedge \mathrm{y} + \mathrm{t} > \mathrm{MNA}$ is true only in the amortized periods. Moreover, the route installation cost is given by $\xi R_m^{v=f} + \mu_{h,l,m} \times \xi R_m^{v=fv}$ which is the sum of the fixed installation cost and the fixed variable installation cost per kilometer multiplied by the route distance.

$$CTFD_{h,l,m,,t} = \left(IT_{h,l,m} - \sum_{y \in Y} \kappa t_{h,l,m,y} \right) \left(\xi R_m^{v=f} + \mu_{h,l,m} \times \xi R_m^{v=fv} \right), \quad \forall (h,l,m) : \exists \mu_{h,l,m}, t \in T \wedge t \leq MNA \wedge y + t > MNA \quad (2.42)$$

Secondly, an operating route incurs a fixed operating cost as in eq. (2.43). This is calculated per available transport route, mode and period, as the sum of a fixed operating cost $\xi R_m^{v=o}$ and the product of the route distance and its operating cost per kilometer $\mu_{h,l,m} \times \xi R_m^{v=ov}$, for operating routes $OT_{h,l,m}$.

$$CTO_{h,l,m,t} = OT_{h,l,m} \times \left(\xi R_m^{v=o} + \mu_{h,l,m} \times \xi R_m^{v=ov} \right) \qquad \forall (h,l,m) : \exists \mu_{h,l,m}, t \in T \quad (2.43)$$

Finally, the volume dependent unitary transportation cost given by eq. (2.44), is the product of the transported volume (primary and secondary), the route distance and the unitary cost per product and transport mode.

$$CTU_{h,l,m,p,e,t} = \left(QP_{h,l,m,p,e,t} + QS_{h,l,m,p,e,t} \right) \times \mu_{h,l,m} \times \xi P_{m,p}^{v=u}, \qquad \forall (h,l,m) : \exists \mu_{h,l,m}, p \in P, e \in E, t \in T \quad (2.44)$$

2.5.3.6 Storage Depot tariffs (TDE)

These are modularized so resource capacities can be added as needed by transported volumes. Transporting volumes through a depot incurs fixed installation and operating costs related to the depot; fixed installation and operating costs concerning the depot resources and variable costs related to the transferred product. The decisions: install a new depot only implies the fixed installation cost; operate the depot adds a fixed operating cost; transport product volumes through a depot adds the resource capacity fixed and operating costs and a unitary cost linearly dependent to the transported volume. Inversely, an installed depot does not obligate its operation, neither installation of capacity resources nor transfer of volumes. Such decisions result from the objective function. The product owner entity pays the depot tariffs as per eq. (2.45). This is calculated per depot and period, by indexing with the storage percentage margin, the product of the entity market quota and the sum of the depot fixed and operating costs, resource fixed and operating costs plus the depot unitary costs.

$$TDE_{h,e,t} = \left(1 + \alpha^{STORAGE} \right) \times \left(\zeta_e \times \left(CDFD_{h,t} + CDO_{h,t} + CRFD_{h,t} + CRO_{h,t} \right) + \sum_{p \in P} CDU_{h,p,e,t} \right), \quad \forall h \in H, e \in E, t \in T \quad (2.45)$$

Likewise eq. (2.42), the depot fixed installation cost is calculated in eq. (2.46) only for periods where the installation costs have not been amortized. Parameters $\xi R_{DEPOT}^{v=f}$ defines the depot fixed installation cost and the binary expression $ID_h - \sum_{y \in Y} \kappa d_{h,y}$ gives the difference between installed and existing number of depots and is true only for new and existing depots in the non-amortized periods.

$$CDFD_{h,t} = \left(ID_h - \sum_{y \in Y,(y+t)>MNA} \kappa d_{h,y} \right) \times \xi R_{DEPOT}^{v=f}, \qquad \forall h \in H, t \in T \wedge \mathrm{t} \leq \mathrm{MNA} \tag{2.46}$$

An operating depot incurs a fixed operating cost per depot node and period, as per eq. (2.47), which computes the fixed depot operating cost only for operating depots.

$$CDO_{h,t} = OD_h \times \xi R_{DEPOT}^{v=o}, \qquad \forall h \in H, t \in T \tag{2.47}$$

The costs for installing and operating storage resources are considerable. Resource installation incurs additional fixed and operating costs for the required number of typical capacity resources. The fixed cost calculated per installed resource is imputed to the product proprietor entity by inclusion in the tariff. These are only incurred if the resource has not been amortized in that period. This cost is calculated per entity as the product of the number of installed resources per depot and a fixed resource installation cost, defined per resource or per product. Eq.(2.48) computes the fixed resource costs payable by the product proprietor.

$$\begin{aligned} CRFD_{h,t} = &\sum_{m,r} \left(IRA_{m,h,r} - \sum_{y \in Y \wedge (y+t)>MNA} \kappa r_{m,h,r,y} \right) \xi R_r^{v=f} \\ &+ \sum_{r,p} \left(\sum_m \left(IRM_{m,h,r,p} - \sum_{y \in Y \wedge (y+t)>MNA} \kappa m_{m,h,r,y,p} \right) + \left(IRP_{h,r,p} - \sum_{y \in Y \wedge (y+t)>MNA} \kappa p_{h,r,y,p} \right) \right) \left(\xi P_{r,p}^{v=f} + \xi R_r^{v=f} \right), \forall h \in H, t \in T \wedge t \leq MNA, \xi P_{r,p}^{v=f} \vee \xi R_r^{v=f} \end{aligned} \tag{2.48}$$

An installed resource does not imply that it must be operated. However, if operated eq. (2.49) calculates a fixed operating cost which is the product of the resource's operating cost and the number of operating resources per depot and resource type, transport mode and/or product. The resource cost is given in $\xi R_r^{v=o}$ and when product dependent, in $\xi P_{r,p}^{v=o}$.

$$CRO_{h,t} = \sum_{m \in M, r \in R} ORA_{m,h,r} \xi R_r^{v=o} + \sum_{r \in R, p \in P} \left(\sum_{m \in M} ORM_{m,h,r,p} + ORP_{h,r,p} \right) \left(\xi P_{r,p}^{v=o} + \xi R_r^{v=o} \right), \quad \forall h \in H, t \in T, \xi P_{r,p}^{v=o} \vee \xi R_r^{v=o} \tag{2.49}$$

The total volume dependent unitary cost is specified in eq. (2.50). It is the product of the depot's transported volume and the depot unitary cost $\xi P_{DEPOT,p}^{v=u}$.

$$CDU_{h,p,e,t} = \sum_{l \in L} \sum_{m \in M} QS_{h,l,m,p,e,t} \times \xi P_{DEPOT,p}^{v=u}, \qquad \forall h \in H, p \in P, e \in E, t \in T \tag{2.50}$$

2.5.4 Pre-processing

2.5.4.1 Secondary route candidates

As the problem is large-scale, a p-median concentration set is constructed as per Rosing and ReVelle (1997), to reduce the problem size, by defining a restricted set of routes which are good candidates in the final solution. A concentration set of possible secondary transport routes is defined for each customer location. A list of depots is created in ascending order of their road distance to the customer location. The first lowest distance NCR routes are enrolled into the secondary route concentration set. The heuristic is produced below in eqs. (2.51).

$$
\begin{array}{l}
\forall (k \in K)\{ \\
\quad \forall (j \in J \ \wedge m = 'ROAD' \wedge orderby\ \mu_{j,k,m})\{ \\
\quad\quad If\ (LCR <= NCR)\{ \\
\quad\quad\quad \mu_{j,k,m} = \mu_{j,k,m};\ !*add\ secondary\ route \\
\quad\quad\quad LCR = LCR + 1; \\
\quad\quad \}else\{ \\
\quad\quad\quad \mu_{j,k,m} = 0; \qquad !*drop\ secondary\ route \\
\quad\quad \} \\
\quad \} \\
\}
\end{array} \tag{2.51}
$$

Route suppression: Route distances between any two nodes per transportation mode are obtained forefront. Similarly, distance estimates for new pipelines are computed based on GPS coordinates. Zero distance routes $(h,j,m):\neg\exists\mu_{l,j,m}$, are non-valid routes and hence constraint (2.52) suppresses their installation.

$$OT_{h,j,m} = IT_{h,j,m} = 0, \qquad \forall h \in H, j \in J, m \in M, (h,j,m):\neg\exists \mu_{l,j,m} \tag{2.52}$$

2.5.4.2 Transportation stage, mode and product restrictions

Although a valid route may be installed, not all transport modes are available to all products. Parameter $\upsilon^{a}_{m,p}$ defines the upper bound for the transport distance per transport stage, mode and product. For example, road transportation is not permitted for primary distribution, as also pipeline transportation for fuel-oil without heating is limited to small distances. Similarly, only road transportation may be used for secondary distribution. However, Jet Fuel may be transported by pipeline or railway to airports. Both situations can be modelled with permitted distances per product for primary and secondary distribution as in constraints (2.53) and (2.54).

$$QP_{h,j,m,p,e,t} = 0 \qquad \forall h \in H, j \in J, m \in M, p \in P, e \in E, t \in T, \wedge\left(\mu_{h,j,m} > \upsilon^{PRIMARY}_{m,p}\right) \tag{2.53}$$

$$QS_{j,k,m,p,e,t} = 0 \qquad \forall j \in J, k \in K, m \in M, p \in P, e \in E, t \in T, \wedge\left(\mu_{j,k,m} > \upsilon^{SECONDARY}_{m,p}\right) \tag{2.54}$$

Pre-processing considerably reduces the model size and execution time for large-scale instances. The MILP model is now illustrated using the Portuguese PSC to study its response to real-world situations, identifying insights and opportunities for further research.

2.6 A Case-Study example

A real case scenario, based on the Portuguese downstream PSC network is now used to test the strategic MILP presented in section 4. The network under study comprises refineries, distribution centres and customer zones in Portugal as shown in Figure 2.5. Two national refineries currently supply the petroleum products' consumption in Portugal. One is located on the north coast at Matosinhos and the other at the south coast at Sines. Portugal may also import product from regional refineries in neighboring Spain or from the Amsterdam-Rotterdam-Antwerp (ARA) refinery cluster. The PSC market in Portugal is currently operated by four major petroleum companies which include national and multinational companies. These also supply the smaller companies and hypermarket chains, hence their demand is included in the major company market quotas. The companies purchase the oil products from the local refineries or import them from external sources. The product is then transported first using the primary distribution to eleven currently operating storage depots where it is stored and then transported to the retail segment using the secondary distribution channels. The retail segment is organized as customer zones, corresponding to the eighteen districts in Portugal. The GPS coordinates of district capitals are used for the localization, land route and distance computations.

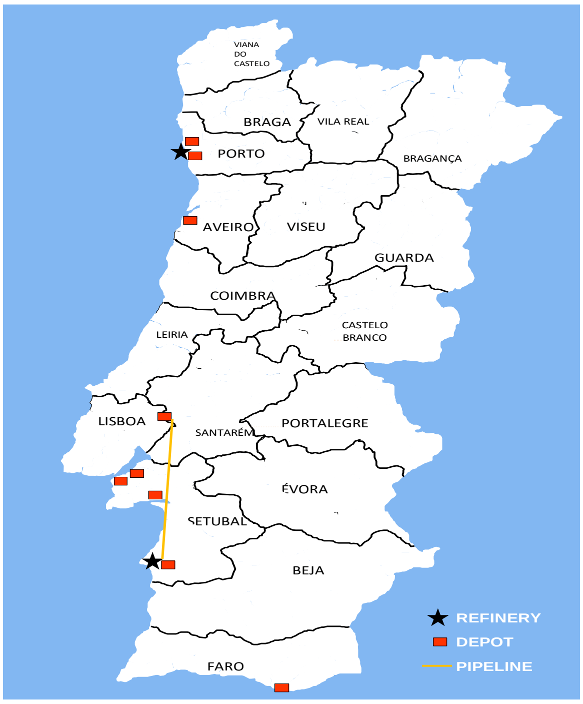


Figure 2.5. The Portuguese PSC

The PSC network considers the distribution of 8 oil products in Portugal (GO, A1, U8, U5, C4, C3, FU, B5), including liquefied petroleum gases (LPG – C3 and C4). However, particularities relating to the LPG bottle-filling and the archipelagos of Madeira and Azores have not been included in this study. Transportation is undertaken by pipeline, maritime or railway for the primary distribution and by road for the secondary distribution. An exception is JET A1 which may be transported by pipeline or railway. Crude oil is purchased to feed refinery production plans, which are restricted by their production capacity. Excess production is exported and deficit products are imported to fulfill the market requirements.

In order to perform this study, a statistical database is constructed from the quantitative data published by the economics ministry of Portugal through the Direcção Geral de Energia e Geologia (DGEG, 2015), Instituto Nacional de Estatística (INE, 2014) and Autoridade de Concorrência (AdC 2014,2015); and the performance indicators published by the European Commission's Directorate-General for Energy (Eurostat, 2015) and Purvin & Gertz Inc. (2014). The Google Maps application program interfaces (Google Inc., 2014) provide the road, rail and maritime travelling distances, whereas the pipeline distances are calculated using the GPS coordinates and the big circle formula (Weisstein, 2014).

The real world parameters are provided in Appendix A and cover a twelve year period from 1999 to 2010, being the most recent available data, collected from Portuguese and European statistics This has been specifically collected from Portuguese and European statistics and when not directly available, inferred and standardized with respect to their monetary and measurement units. The documentation and correlation of the PSC data by itself is an important contribution which has involved substantial effort and to the best of our knowledge has not been realized before. It is therefore presented here and will be refined in future articles so as to make available such a database for use in future research developments for the PSC. These include:

i) the product characteristics such as density used for mass to volume conversions and vessel typical dimensions;

ii) the demand per product, customer district and period in cubic meters;

iii) the current market quotas per entity used to calculate demand volumes per entity;

iv) the crude processing capacities per day per installed refinery in cubic meters;

v) each refinery's typical yield of product mix per unit of crude oil;

vi) each entity ownership percentages per refinery, storage depots and transportation infrastructure;

vii) paid amortization periods per existing depot infrastructure;

viii) paid amortization periods per existing transportation infrastructure in order to address the retrofit network problem;

ix) installed transport mode dependent resource units; x) installed transport mode and product dependent resource units; xi) installed product dependent resource units; - defined per location, resource and per number of paid amortizations;

xii) typical capacities of transport mode dependent resources; xiii) typical capacities of transport mode and product dependent resources; xiv) typical capacities of product dependent resources; - given in cubic meters as vessel capacity or the flow rate per hour;

xv) the vessel rotations per product;

xvi) product dependent resource costs typified as fixed, fixvar (fixed variable), operating, opervar (operating variable) and unitary;

xvii) product independent resource costs;

xviii) price per product at the exit of each activity stage;

xix) maximum distances per distribution stage, transportation mode and product;

xx) rail distances; xxi) pipeline distances xxii) maritime distances xxiii) road travelling distances; - between refinery, depot and customer locations;

xxiv) the GPS latitude and longitude for each customer district, depot and refinery location.

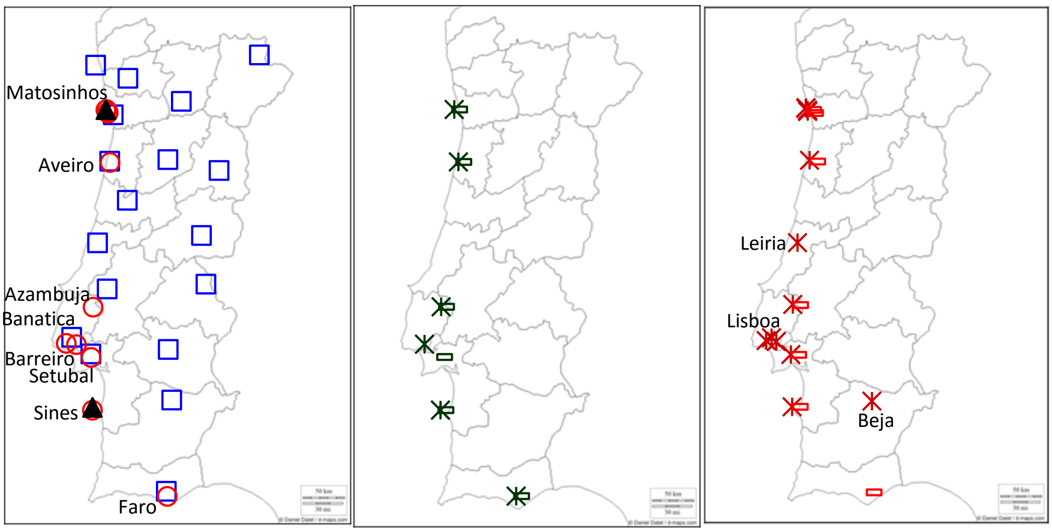
The MILP results based on this data are now presented.

2.7 Computational Results

The MILP model is implemented using GAMS 23.7.3 64-bit and solved with MILP commercial solver CPLEX 12.3 using 4 parallel threads on a dual-core AMD Opteron 2.4 Ghz CPU with 8GB RAM. The stopping criterion is 0.1% optimality gap or 8 hours of CPU time. The MILP models six different possible networks for the Portuguese PSC maximizing profits for each setting.

2.7.1 Strategic decisions for the PSC

Figure 2.6 presents the Portuguese PSC current (a), and MILP generated grassroots (b) and retrofit (c) designs. The MILP first models each of these designs as uni-entity networks where all resources and product demand are assumed to be owned by a single entity; and later as multi-entity individualistic networks where the resources are owned by various entities who are in competition. The current network (a) uses existing eleven depots, while networks (b) and (c) depart from twenty-six possible locations (the ones used in the current depots plus the district capitals). The current network permits the operation of only existing depots, capacities and routes, however transfer volumes may be freely assigned. The grassroots starts from scratch and designs the optimal network of depots and transportation routes for the given conditions. The retrofit permits opening, expansion and closing of existing depots, capacities and routes.



Refineries Current depots Customers; Grassroots Uni-entity Multi-entity depots; Retrofit Uni-entity Multi-entity depots

Figure 2.6. Depot locations for Current(a), Grassroots(b) and Retrofit(c) networks

The current Portuguese PSC can be observed in Figure 2.6(a), where some particularities are worth a mention. Some depots are owned by multiple entities, whereas some locations harbor various depots owned by one or more entities. For instance, 4 depots are currently located at a single location, Matosinhos, Here the major entity E1 owns two depots, one for liquid products and another for LPG gases, E2 and E3 share the third depot, and yet E4 owns the fourth depot. Also around the Lisbon area, three depots (Banatica, Barreiro and Setubal) are owned or contracted by E2, E3 and E1 respectively. Another reality is that not all depots maintain storage facilities for all products. Liquid fuels are present at most depots however LPG gases and fuel-oil vessels are infrequent. Also, the build dates vary for existing depots and resources, hence the incurred amortizations will differ. For the current network modelling, product volumes may only be affected to existing refineries, depots, transport routes and customer zones.

Figure 2.6(b) shows the grassroots design results, where it may be observed that only six of the eleven existing depot locations are selected in both uni-entity and multi-entity networks. Setubal is switched to Barreiro in the later. The selection of existing depots re-validates these locations. All depots are seen to be located in high demand districts.

Figure 2.6(c) shows the retrofit design where only seven existing depots are selected in the uni-entity network. However, in the multi-entity network, thirteen depot locations are selected, including ten current locations, and three new ones at Lisboa, Beja and Leiria. Differences in selected transport modes, routes, resources, transfer volumes and profits will be seen later.

The above results identify that the current network is not efficient as both grassroots and uni-entity retrofit use less depots, whereas the multi-entity retrofit opens more depots. Some existing depots are not selected due to their lower profitability to the PSC as a whole. This is because the model does not permit changes to entity participation percentages in existing depots. Hence new depots are opened where participation percentages assume market quotas. The later is also the rule for the depot participations in grassroots design. Changes to entity participation percentages, is therefore identified as a research opportunity.

It may be observed that operating depots and connecting routes are more numerous in multi-entity networks when compared to uni-entity networks. This implies higher operating costs as is confirmed later in Figure 10. Another observation is that rail transport is only selected in the current network, clearly identifying its inefficiency therefore closure in the grassroots and retrofit redesign. Under the multi-entity retrofit design, the new depots Lisboa, Leiria and Beja are connected by pipeline to Sines. This is due to the percentage mismatch and lower efficiency of transporting the product from Sines to other existing entity depots by maritime and thereafter by road to the customers. It may also be noted that the current networks mainly undertake maritime transportation for primary distribution and road transportation for secondary distribution. However, in the Grassroots and Retrofit networks, pipelines are opted for primary transportation, and in the case of JET also for secondary distribution. Hence, the pipeline transport is seen to be better choice when considering a long term project.

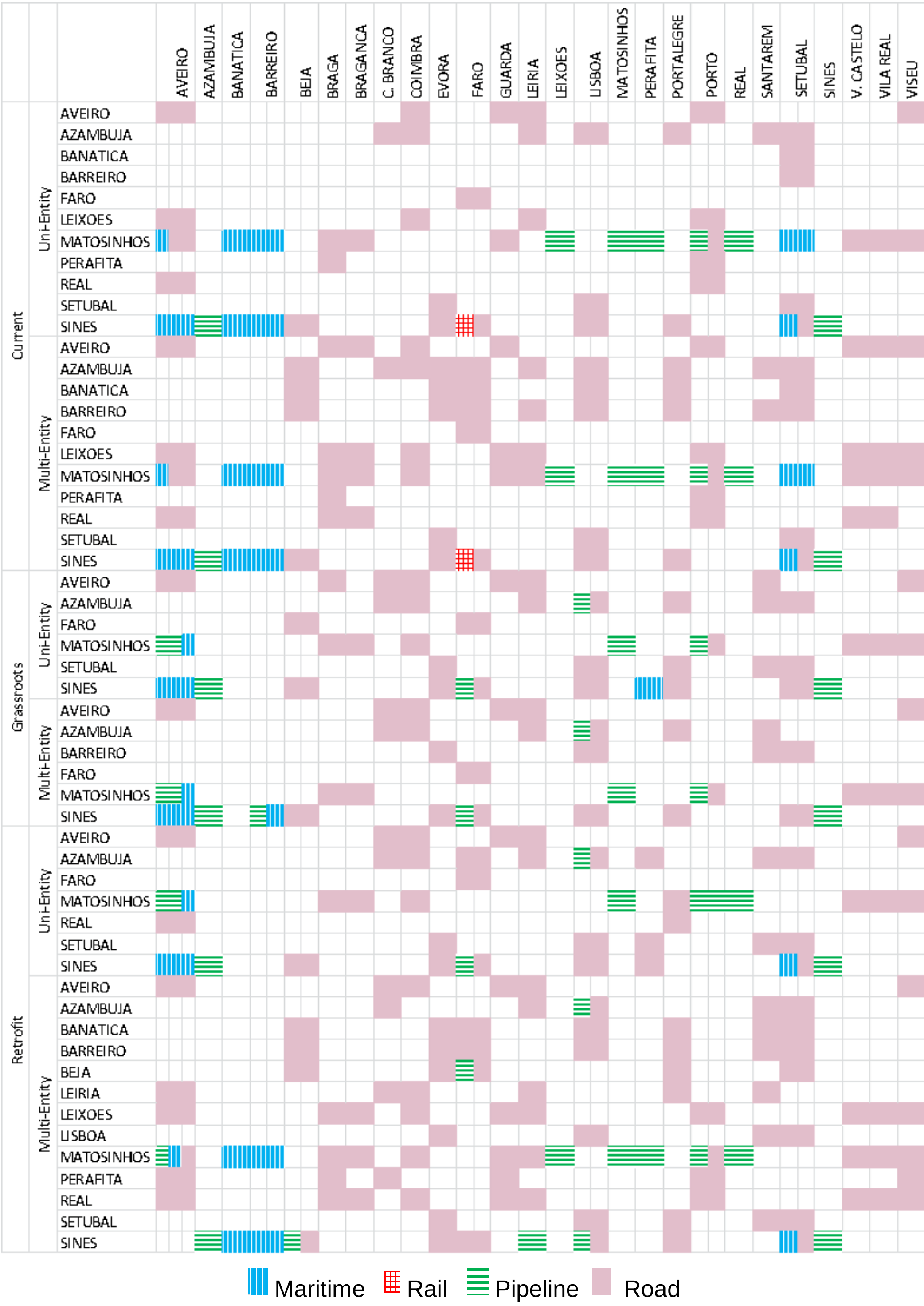


Figure 2.7. Transport links per network

Figure 2.8 represents the vessel units (number along the color bars) operated per depot and product for all six networks. The multi-entity retrofit network is seen to significantly resemble the current network capacities, thereby confirming the multi-entity and individualistic nature of the current Portuguese PSC. In contrast to the current network, the grassroots design shows significant reduction in the number of depots, consequently increasing the number of vessels per product. The retrofit design revalidates current depots, and vessels per depot are greatly reduced in both (uni and multi-entity) indicating excess capacity in the current network. The number of depots increases in the retrofit from uni-entity to multi-entity network. In the former, depots Perafita and Real are combined into Real. Both Grassroots and the uni-entity retrofit show increased influence of Aveiro, Matosinhos and a relative augment of capacities at Azambuja. In the Lisbon area, Setubal increases its influence and Barreiro considerably reduces in the uni-entity networks and the inverse for the multi-entity networks. The main reason here is that Barreiro is affected to a single entity while other multi-entity facilities exist in that area. Finally, it may be observed that only depots with maritime access such as Banatica, Barreiro, Setubal, Sines and Matosinhos are selected for fuel-oil (FU) storage while the remaining depots show a regular distribution of other products. Finally, the grassroots and retrofit designs show a reduction of depot capacities for most products except C3 and FU.

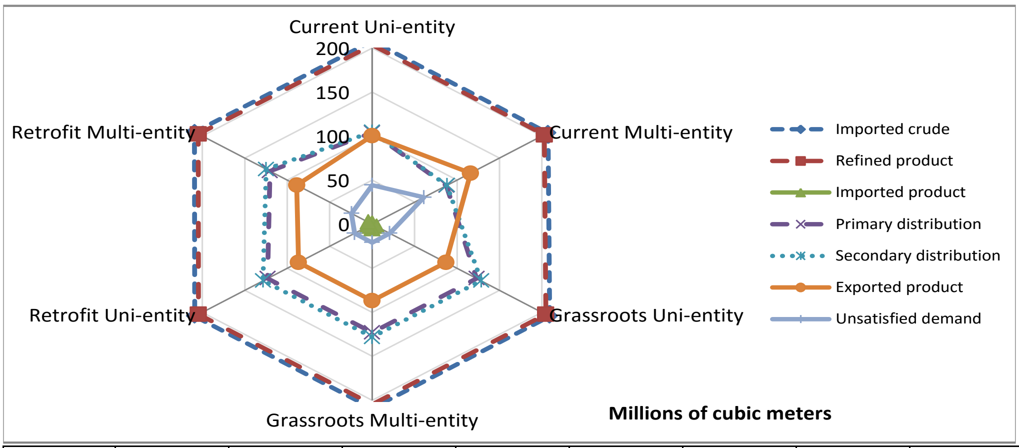
Design	Network	Depot	C3	C4	U5	U8	GO	FU	A1	B5
Current	Uni-entity	AVEIRO			1	1	1			1
		FARO	1	1						
		SETUBAL			1	1	2	1		1
		MATOSINHOS	4	3	5	2	5	7	4	3
		AZAMBUJA	8	6	4	3	8		4	3
		SINES	5	4	6	4	6	6	5	4
		PERAFITA	5	2						
		BANATICA	2	2			2			1
		BARREIRO			1	1	1	1		1
		REAL	3	1	2		2			
		LEIXOES			2		2	1		2
	Multi-entity	AVEIRO			1	1	1			1
		FARO	1	1						
		SETUBAL			1	1	2	1		1
		MATOSINHOS	4	3	5	2	5	7	4	3
		AZAMBUJA	8	6	4	3	8		4	3
		SINES	5	4	6	4	6	6	5	4
		PERAFITA	5	2						
		BANATICA	2	2			2			1
		BARREIRO			1	1	1	1		1
		REAL	3	1	2		2			
		LEIXOES			2		2	1		2
Grassroots	Uni-entity	AVEIRO	4	9	3	1	4	2		3
		FARO	2	2	1	1	1			1
		SETUBAL			2	1	2	14		2
		MATOSINHOS	8	13	6	3	7	6	1	5
		AZAMBUJA	7	9	5	2	7		2	2
		SINES	7	7	3	1	4	7	3	2
	Multi-entity	AVEIRO	4	9	3	1	4	2		3
		FARO	1	2	1	1	1			1
		MATOSINHOS	8	13	7	3	7	6	1	6
		AZAMBUJA	4	4	4	2	7		2	2
		SINES	7	7	3	1	4	8	3	2
		BARREIRO	4	5	3	1	2	15		2
Retrofit	Uni-entity	AVEIRO	3	8	3	1	4	2		3
		FARO	1	2	1	1	1			1
		SETUBAL			2	1	2	14		1
		MATOSINHOS	6	11	5	2	5	6	1	5
		AZAMBUJA	8	9	6	2	7		2	3
		SINES	7	7	3	1	4	7	3	2
		REAL	3	2	1	1	2			
	Multi-entity	AVEIRO			1	1	1	1		1
		BEJA	1		2	1	2		1	1
		LEIRIA	1	2	2	1	4			2
		LISBOA		4	5	2	6		6	2
		SETUBAL			1	1	1	7		1
		MATOSINHOS	5	9	4	1	4	4	1	3
		AZAMBUJA	8	6	2	1	2		3	1
		SINES	6	6	3	1	4	6	2	2
		PERAFITA	1	1	1	1	1	1		1
		BANATICA	2	1	1		1	2		1
		BARREIRO	1		1	1	1	3		1
		REAL	4	6	3	1	3	2	1	2
		LEIXOES	1	2	1	1	1	1	1	1

Figure 2.8. Depot storage capacities (number of vessels) per product

Changes in depot locations alter the selection of transportation routes. Figure 2.7 shows the transport mode allocations from refineries to depots and depots to customer districts for each network. Sometimes various modes are selected due to transport mode restrictions on some products. This is especially true for primary transportation between the refineries and Setubal or Aveiro where maritime is selected for fuel-oil and pipeline for other products. The same occurs with secondary transportation for the depot/district pairs Matosinhos-Porto, Azambuja-Lisboa and Beja-Faro where JET is transported by pipeline and other products by road.

2.7.2 Tactical decisions for the PSC

The processed crude, refined, imported, primary, secondary, exported product and unsatisfied demand volumes are compared in Figure 2.9. The imported crude and the refined product volumes are seen to use maximum refinery capacities as inventories are not considered, identifying that the difference between the product and crude prices cover the refining and transportation costs. Imported product together with the primary (refinery) volumes, feed the secondary volumes (i.e. satisfied demand). Comparatively, the secondary volumes increase from multi-entity to uni-entity and significantly from current to grassroots and retrofit design with unsatisfied demand following an inverse trend. The imported product although low, is seen to increase from current to grassroots to retrofit design, whereas export and unsatisfied demand decrease. The later is higher when compared the multi-entity to the uni-entity network. Hence, the current design primary and secondary distribution volumes are lower when compared with grassroots and retrofit designs. This is true in some cases where the gross retail margin does not cover the depot and transport costs, based on the defined costs.



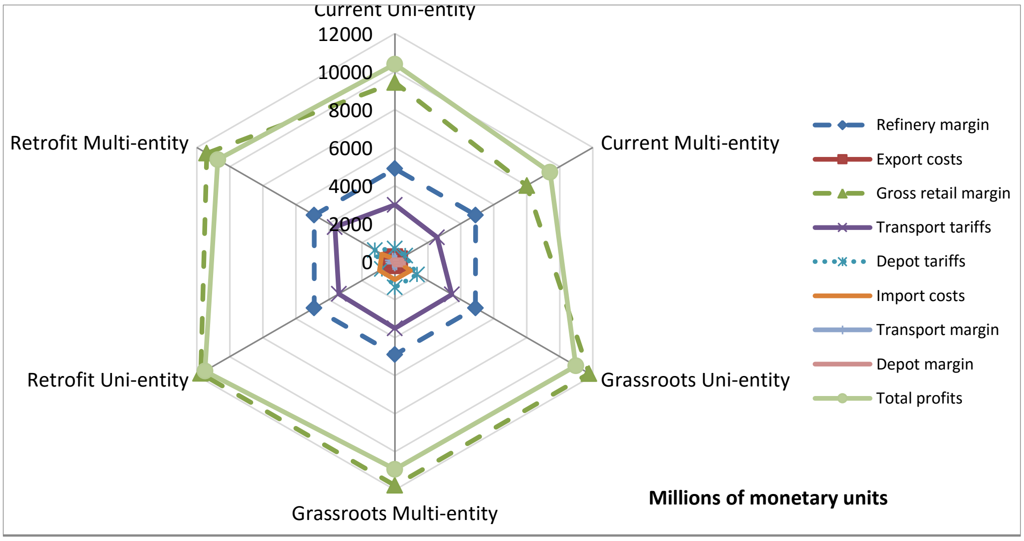
Design	Network	Imported crude	Refined product	Imported product	Primary distribution	Secondary distribution	Exported product	Unsatisfied demand
Current	Uni-entity	207.871	203.505	0.795	103.887	104.682	100.414	44.413
	Multi-entity	207.103	202.754	0.670	87.428	88.098	115.997	60.997
Grassroots	Uni-entity	208.994	204.605	5.319	123.129	128.448	86.795	20.647
	Multi-entity	208.937	204.550	5.317	122.977	128.294	86.890	20.801
Retrofit	Uni-entity	208.937	204.550	5.283	123.144	128.427	86.690	20.668
	Multi-entity	209.159	204.766	4.415	120.350	124.765	88.831	24.330

Figure 2.9. Volumes per network in millions of cubic meters

2.7.3 Value contributions and Performance Statistics

In all six networks, significant product volumes are exported. Note that the demand and price parameters cover a twelve year period from 1999 to 2010 (latest available), where product demand in the initial years was considerably low. As current refinery capacities are used, which in reality were lower in the early years, higher export volumes are generated. It would have been interesting to compare the model results with real distribution volumes, however such information is not available. Besides, the network has considerably changed in the course of the twelve year period.

Figure 2.10 presents the objective function (total profits) and its components, namely the refinery margin, exportation costs, gross retail margin, transportation costs, depot tariffs, importation costs, and the dealer transport and depot margins. The gross retail margin is the difference between the retail price, the ex-refinery price and the retail station cost. This excludes the transportation and storage costs which are shown separately. Non-tangible costs such as the entity publicity and trading costs are not considered. As inventory holding efficiencies are not considered, the refinery margins are fairly constant.



Design	Network	Refinery margin	Export costs	Gross retail margin	Transport tariffs	Depot tariffs	Import tariffs	Transport margin	Depot margin	Total profits
Current	Uni-entity	4890.438	256.298	9421.475	2986.356	681.054	148.011	54.049	88.833	10383.076
	Multi-entity	4892.430	296.073	7996.456	2558.129	636.897	120.782	47.514	83.073	9407.593
Grassroots	Uni-entity	4887.521	221.537	11771.277	3455.762	1344.624	946.206	101.709	175.386	10967.765
	Multi-entity	4887.668	221.780	11763.831	3503.950	1353.385	943.984	117.240	176.528	10922.168
Retrofit	Uni-entity	4887.668	221.269	11765.773	3392.559	788.191	938.971	93.900	102.808	11509.158
	Multi-entity	4887.094	226.735	11405.973	3632.413	1207.172	791.896	134.893	157.457	10727.201

Figure 2.10. Value contribution per network in millions of monetary units

The current networks have a little higher total refinery margin and relatively higher total exportation costs, when compared to the grassroots and retrofit, which have significantly higher gross retail margins, depot and transport tariffs. The last two are higher in grassroots and retrofit when compared to the current design as the satisfied volumes are also higher, however the retrofit uni-entity show relatively low tariffs as existing depots are operated. Highest total profits are observed for retrofit design in uni-entity networks, and for grassroots design in multi-entity networks. Uni-entity margins and costs are higher than in multi-entity networks as the satisfied demand is also higher.

The model statistics are presented in table 2.1. The number of equations, continuous variables and CPU time are considerably higher in multi-entity when compared to uni-entity networks. CPU time increases from current, to retrofit to grassroots design. The MILP solution or overall profits are higher in the uni-entity networks than the multi-entity counterparts. Differences between retrofit and grassroots profits range within 5%, however are 6-16% better than the current design. Although the retrofit would be expected to perform better, they are observed to be tied up by existing entity participation percentages which may not be changed. The grassroots designs are more efficient as their infrastructure participations resemble entity market quotas. As the right costs are crucial for the performance of the PSC, a sensitivity analysis to cost variations is now presented.

Table 2.1. Performance results

Design	Network	Equations	Variables	Discrete	Objective	GAP (%)	CPU (s)
Current	Uni-Entity	33766	87891	12103	10.383×10^9	0.00	9
	Multi-Entity	45454	313953	12103	9.408×10^9	0.00	19
Grassroots	Uni-Entity	49976	90289	14501	10.968×10^9	0.09	11915
	Multi-Entity	61664	316351	14501	10.922×10^9	1.36	28821
Retrofit	Uni-Entity	47261	90260	14472	11.509×10^9	0.10	1865
	Multi-Entity	58949	316322	14472	10.727×10^9	0.57	28823

2.8 Sensitivity Analysis

The PSC is characterized by large volumes and small margins. Crude and product prices are interdependent and highly volatile, influenced by the international markets and petroleum reserves. PSC configurations and profits are highly sensitive to the infrastructure and resource installation and operating costs which do not vary. The above study was based on regional cost estimates, which have reduced in recent times. Hence, it would be interesting to study the effect of cost reduction on the network.

2.8.1 Impact on Volumes

A sensitivity analysis is performed for three scenarios where depot, transport and resource costs are reduced by 0%, 10% and 20%. The comparison of resulting networks against earlier designs is presented for volumes in figure 2.11 and objective function components in figure 2.12. While refined, primary and secondary volumes increase when cost is reduced by 0% up to 20%, the unsatisfied demand decreases. The imported and exported volumes do not vary much with cost reduction. An additional depot Leiria supplied by pipeline from Sines, is opened for Grassroots design with 20% cost reduction in both uni-entity and multi-entity networks. Depot Viseu supplied by pipeline from Matosinhos, substitutes Aveiro and Perafita, in multi-entity retrofit with 10% and 20% reduced costs. Depot Leiria substitutes Real in uni-entity retrofit with 20% cost reduction.

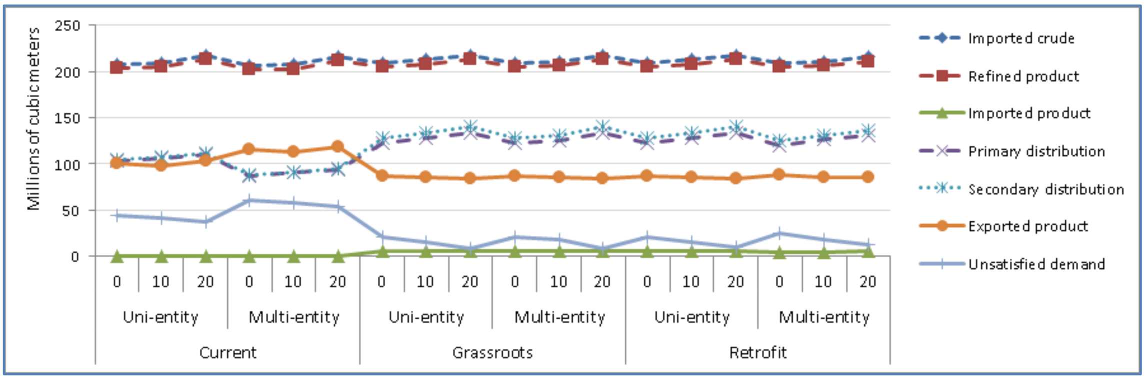


Figure 2.11. Impact on volumes with 0-10-20% reduction of actual costs

Figure 2.12 shows the evolution of MILP volumes for the current network during twelve years. A steady increase in primary and secondary distribution volumes is observed with simultaneous decrease in the unsatisfied demand. The refined and exportation volumes remain constant and near maximum production for all years, except 2000, where they fall drastically for 0% and 10% cost reduction, although for 20% cost reduction, they are comparable to other years. This only occurs in 2000, as the difference between the refined product prices and the crude oil prices does not compensate the refinery cost plus transportation cost. Hence, the model decides not to export product, thereby reducing refined volumes.

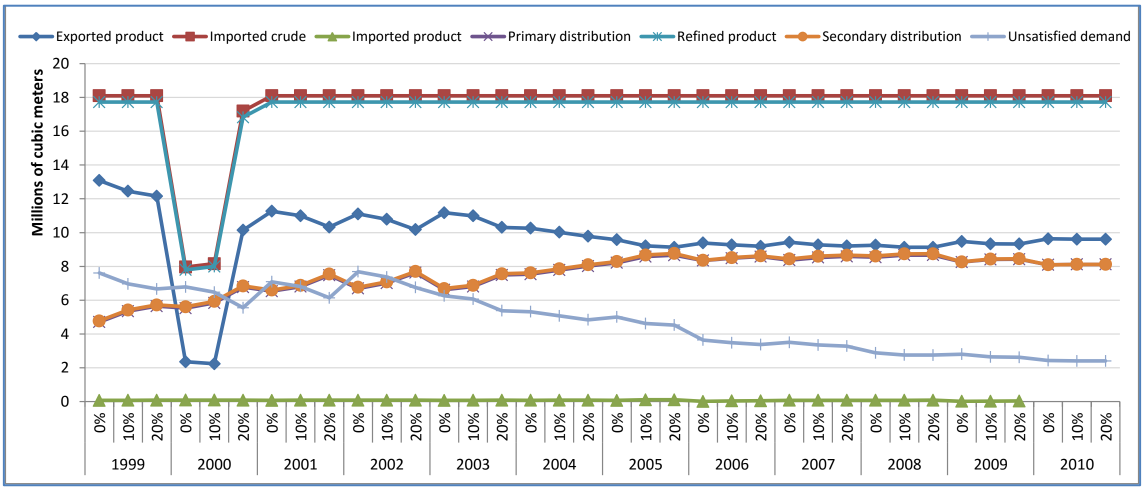


Figure 2.12. Yearly volumes with 0-10-20% cost reduction

2.8.2 Impact on Value Contributions

Figure 2.13 shows the objective value variation, where a 10% cost reduction increases the total profits by 8% to 9.5% for all networks. From benchmark costs to 20% cost reduction, the total profits increase between 16.8% and 19.5%. The profit variation is lowest for the current networks, and highest for the uni-entity grassroots and the multi-entity retrofit networks. In all cases, the reduction of resource costs reflects in increased refinery margins and significantly better gross retail margins, as a result of reduced installation and operation costs. The total storage, importation and exportation costs are comparatively stable under the 0%, 10% and 20% scenarios, meaning that the cost reduced supports the increased volumes.

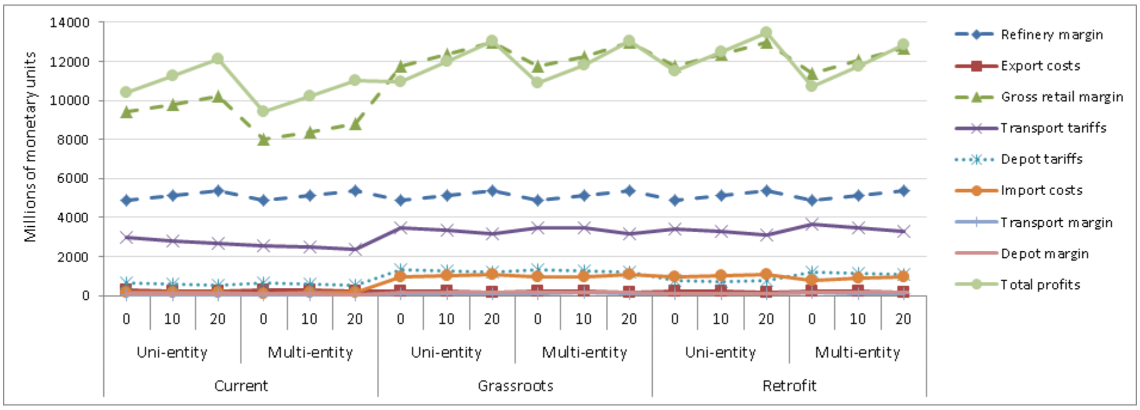


Figure 2.13. Value creation with 0-10-20% cost reduction

While the model maximizes total PSC profits, it would be interesting to study the impact of cost reduction on individual entity profits and contributions in the multi-entity network. Figure 2.14 presents the profits and costs per entity and network with cost reduction. In all networks, the refinery margins are affected only to the main entity. However, and importation costs for other entities are also observed in current and grassroots networks. Individual entity profits and costs are seen to follow an increase or decrease trend in consonance with the total profits. As the model maximizes total profits, this may not always be true.

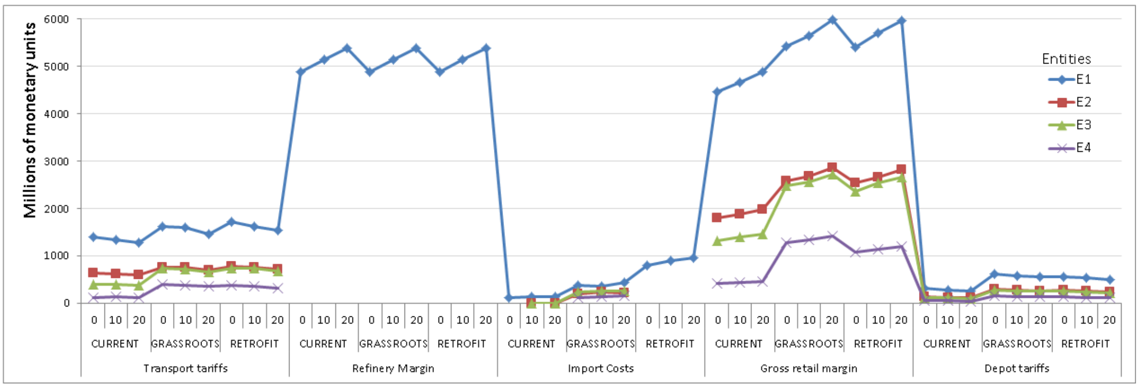


Figure 2.14. Value creation by entity with 0-10-20% cost reduction

Finally, figure 2.15 below shows the model results for the current network for a 12 year horizon considering benchmark costs. Refinery margins are seen to decrease sharply following the fall in product prices in 2000, then rise gradually along with product prices until 2005, and decrease again until 2010. The gross retail margin initially increases along with the refinery margin beyond 2005 until 2008 favored by the increase in demand, after which it reduces as the demand falls. The total transport tariffs thereby increase steadily following expedited volumes. However, the total depot tariffs are constant as most installations are already amortized. The depot and transport dealer margins have little impact on overall profits as they are indexed to the tariffs which are relatively low for the current network as the investments have been amortized. The entity incorporated depot margins are higher than transport margins as entities may only incorporate pipeline margins.

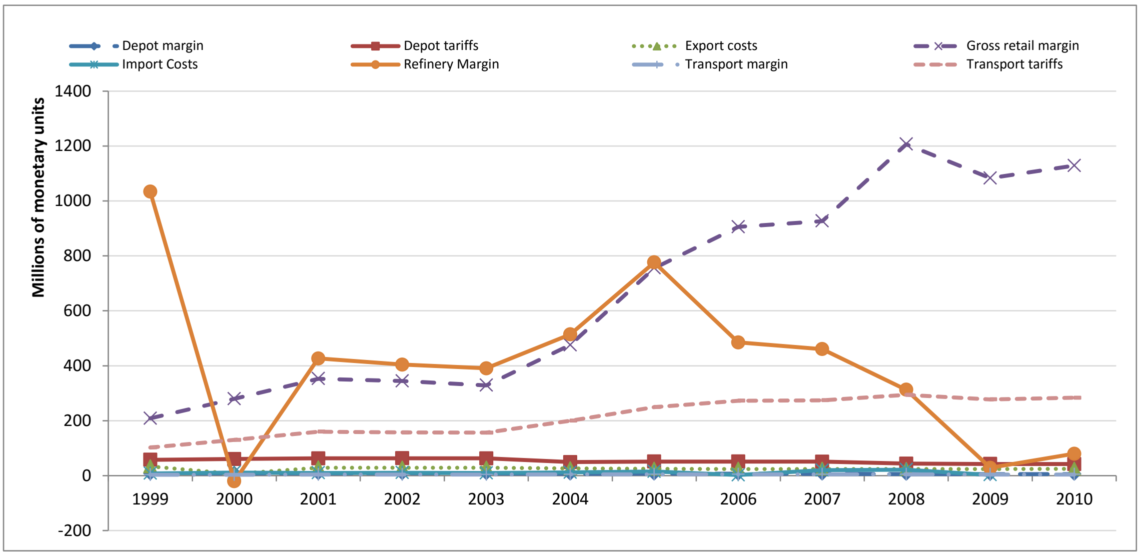


Figure 2.15. PSC stage yearly value contributions with actual costs

Overall, the sensitivity analysis results imply that the right entity participation percentages and the fixed, operating and unitary costs play a significant role in network topologies and profits. Another implication is that inventory buffers could be used to hedge against fluctuating prices.

2.9 Conclusions and future research

The current chapter presents a new MILP model for strategic planning of downstream PSC networks without considering inventories. The model determines depot locations, transport modes, routes and resource capacities and network affectations besides crude oil and petroleum products' importation, exportation, production, storage and transportation volumes. The MILP solves the design of uni-entity and multi-entity networks, permitting existing, grassroots and retrofit redesign. The model test runs for the Portuguese PSC create better network designs with higher total PSC profits when compared the existing network to grassroots and retrofit design for uni-entity and multi-entity networks under individualistic operation.

Uni-entity networks are common in big countries where PSC networks are built and managed by a single entity. However, in smaller countries as in Portugal, the market is small and petroleum entities are in competition. Hence, multi-entity networks with individualistic strategies have been built in Portugal. Here, the logistic costs are observed to be higher than in uni-entity networks. The results suggest that a network owned and run by a single entity improves overall profits implying lower tariffs. The MILP sensitivity analysis to cost reduction, installs additional depots with a positive effect on product volumes and total profits with reduction of unmet demand.

Several insights have been identified along this study: First, collaborative design and planning could significantly improve the PSC performance. Secondly, the crude oil and petroleum derivative prices vary significantly with time. Hence, various stage inventories could improve the PSC profits, thereby optimizing the procurement and exportation costs. Thirdly, the PSC entities have participation in transportation and depot operator capital. It would be interesting to study the effect of entity incorporation of operator profits, on the design of the PSC networks. Finally, most existing PSC facilities were built for multi-entity individualistic operation. Some inefficiencies and rigidity of the current network has been identified, due to misaligned entity participations in depot and transportation infrastructures. It would be interesting to study the network efficiency by changing entity participations in existing depots.

The above considerations could significantly improve the costs, tariffs and profits for the PSC members and hence are identified for further research. Future model improvements will focus collaborative network design, tactical planning concerning time-dependent inventories, incorporation of participated logistic and transport operator profits, entity financial participation, cross-country sales and taxation policies, and treatment for uncertainty.

CHAPTER 3. Collaborative Design and Tactical Planning of Downstream Petroleum Supply Chains

3.1 Abstract

In this chapter, a dynamic MILP is presented for the PSC collaborative design and tactical planning with multi-stage inventories, extending the work presented in the previous chapter. The model determines the optimal design with multi-entity supply, installations and resource capacity sharing with prices incorporating economies of scale while fulfilling customer demands for a multi-entity, multi-echelon, multi-transportation and multi-product downstream PSC. Collaborative planning permits multi-echelon profit maximization where the entities incorporate profits from partly owned refineries, storage depots and transportation and retail infrastructures. The dynamic MILP is tested for the real-case Portuguese PSC network, considering production at local refineries and supply from a regional hub, obtaining current, grassroots and retrofit designs for the uni and multi-entity networks.

3.2 Introduction

The Petroleum Supply Chain (PSC) as the basis of modern economy, influences global and national economies. The PSC comprises of upstream - exploration and production of crude oil - and downstream -refining and delivery of petroleum products to consumers. Although the upstream has been fairly researched, the downstream remains much unexplored as competition was low and detailed information unavailable, hence PSC networks seldom changed. However, high crude costs, dynamic markets and stiff competition have awakened an interest for flexible PSC networks. Recent investigation Al-Qahtani and Elkamel (2008), Shah *et al.* (2011), and Guajardo *et al.* (2013) identify the study of downstream PSC design and planning as an opportunity for improving costs and competitiveness. Its importance increases with the surge of new operators, fractioned markets, new product specifications and plant change requirements due to environmental regulations. Therefore, collaborative multi-entity network design with tactical planning, incorporating existing networks, long-term supply, demand, prices, costs and geographical information has become a key issue. Although literature has focused various stages of the PSC, few articles specifically address the network design problem.

In the downstream PSC, a qualitative and partly mathematical strategic planning formulation was first presented in Sear (1993). Using road vehicle delivery fixed and variable costs, planning needs for logistics infrastructure and transportation were discussed. Later on, Escudero *et al.* (1999) presented a two-stage model encompassing the supply, transformation and distribution scheduling of multi-operator multi-product PSCs activities. Product demand, spot supply cost and selling prices were considered. As main objectives the work aimed at minimizing the under and over amounts of the different considered activities (supply, production and transformation). Ross (2000) studied the incorporation of performance criteria to the PSC. A maximization of profit was considered for the resource allocation in the downstream PSC distribution planning. Order performance criteria for distribution centers and delivery vehicles are used to illustrate its benefits. Dempster *et al.* (2000) propose deterministic and stochastic programming approaches for the depot and refinery tactical optimization problem. They use STOCHGEN, a general purpose stochastic problem generator for generating a scenario tree with product demand and supply cost uncertainties. Later, Cornillier *et al.* (2008) presented a heuristic approach applied to the replenishment problem of petrol stations which determines the product delivery quantities per vehicle and route to each petrol station. As main objective the heuristics aims at maximizing the total profit. Recently, Guajardo *et al.* (2013) studied the tactical planning model at the downstream PSC. Vendors' contributions are maximized with distribution planning and decentralized sales while costs are minimized. Different costs are considered namely, production, transportation and inventory planning, hubs and depots.

In the transportation problem within the downstream PSC, Cafaro and Cerdá (2004) developed a MILP for transportation scheduling of a multiproduct pipeline from an oil refinery to several supply points, adopting a continuous time and volume mathematical representation. The result is a problem description where the model size and processing time are much lower when compared with discrete representations, thereby permitting longer time horizons and increased slug pumping flexibility. Integrated pipeline transportation scheduling coupled with inventory management of petroleum products' depots is modelled in Relvas *et al.* (2006). The mathematical modelling representation aims at obtaining product sequences, pumping schedules and batch volumes, that satisfy customers' demands, quality control constraints and operating restrictions. The same authors, Relvas *et al.* (2007) addressed the rescheduling problem where variability of real ordering plans or production changes were studied and their influences in the pumping schedule and inventory management were explored. Boschetto *et al.* (2010) studied through an optimization model a real-world pipeline network, by addressing the associated network scheduling activities. A hierarchical approach is developed which explores the integration between an MILP model and a set of heuristic modules. More recently, Cafaro *et al.* (2011) improved the continuous-time MILP formulation for detailed schedule of single-source to multiple distribution terminals pipelines to minimize the total flow restart volumes and the number of single-delivery pumping runs, by incorporating heuristic rules for selecting the receiving terminal, thereby obtaining substantial savings in energy consumption and pump maintenance costs. Recently, Relvas *et al.* (2013) studied the planning of oil derivatives distribution through a multiproduct pipeline, using an alternative and holistic modelling concept so as to obtain faster solutions within reduced time frames. The proposed approach allows the possibility of obtaining medium planning term solutions through the solution of a single model in efficient form avoiding decomposition approaches.

When considering the effect of uncertainty and contingency planning, different works arise. MirHassani (2008) proposed a linear programming model for operational planning of a capacitated transportation network. The system considers existing refineries and depots to meet customer demand, minimizing total inventory and transportation costs. The modelling approach also computes inventory shortages and excesses in the presence of transportation route capacity bounds. Mota *et al.* (2013) propose a MILP model to integrate tactical decisions of determining influence areas per distribution center and operational decisions of daily distribution planning. Contingencies of the Portuguese PSC are studied, providing distribution decisions for abnormal scenarios of product shortage or reduced fleet availability.

Concerning the petroleum products supply chain investment planning problem, Oliveira *et al.* (2013) proposed a two-stage mixed-integer linear stochastic program for planning under demand uncertainty. They use a Lagrangian decomposition based approach to generate scenarios considering network design and discrete capacity expansion under demand uncertainty. Later, Oliveira *et al.* (2014) developed acceleration techniques using stochastic Benders decomposition to strengthen the generated cuts, in order to improve the quality and performance of the solutions.

Recently, four reviews have discussed the strategic, tactical and operational planning models for the PSC. While An *et al.* (2011) study PSC models for application to the biofuel industry, Leiras *et al.* (2011) categorize oil chain planning models as per their segment (upstream, midstream, or downstream), planning level (strategic, tactical, or operational), and problem type (linear, non-linear, mixed integer linear, or mixed integer non-linear) and model type (deterministic or stochastic). Shah *et al.* (2011) focus on the methodologies used to address scheduling, planning, and supply chain management of oil refinery operations. Fernandes *et al.* (2011) study the SCM literature for insights to the PSC. The paper analyzes different publications in the area and identifies the future challenges to be addressed: improve models for refinery operations; integrated models for supply chain management. The potential on the use enterprise-wide optimization is clearly recognized and the need of developing comprehensive optimization models and computational tools to address the complex problems present in PSC was identified. On these reviews it can be concluded that refinery planning has been the main focus, nevertheless recent literature has targeted the distribution network. Strategic-tactical collaborative planning for multi-company PSCs and logistics operator networks was identified as an opportunity. Following this challenge, Fernandes *et al.* (2013) developed a novel MILP formulation for the strategic planning of multi-company, multi-echelon and multi-product downstream PSC networks. Strategic decisions including depot locations, resource capacities, transportation modes and routes, and tactical decisions of network affectations of product volumes including crude importation, refining, and product importation and exportation were determined. PSC total profits were maximized for uni-entity and individualistic multi-entity networks with jointly-owned installations. Resource utilization and profit-sharing was achieved through companies' financial participation in refineries, transport structures and storage depots.

In comparison to the individualistic strategic MILP presented in the chapter 2, the current chapter extends the base model to a dynamic model for collaborative design, resource sharing, economy of scale tariffs, operator-profit incorporation and tactical inventory planning of the downstream PSC. Resource modelling is improved and the depot and transportation tariffs now incorporate piecewise cost linearization, based on total investment costs or only non-amortized costs. Considering collaborative multi-entity networks, the consolidated PSC profits are maximized, incorporating shared profits from financially participated members who operate the individual infrastructures. The new MILP is tested for the Portuguese PSC, wherein the networks are compared with respect to the strategic decisions of depot locations, resource capacities, and transportation route selection as well as tactical decisions of dynamic inventories and product transfer volumes. The resulting costs, tariffs and profits as well as the resource capacities per depot and route are analyzed.

The following sections present the problem description, the dynamic MILP formulation, a real-world example based on the Portuguese PSC network, used to test the proposed MILP, wherein the results are presented, concluding and presenting proposals for future research.

3.3 The Downstream PSC Problem in Study

The downstream PSC is defined as a complex network managed by petroleum companies, encompassing refineries, distribution depots, transportation modes and routes that act in coordination to satisfy the retail customer demands for multiple petroleum products in various regions under study. Such a network comprises the refining, logistics and commercialization of the petroleum products as presented in Figure 3.1.

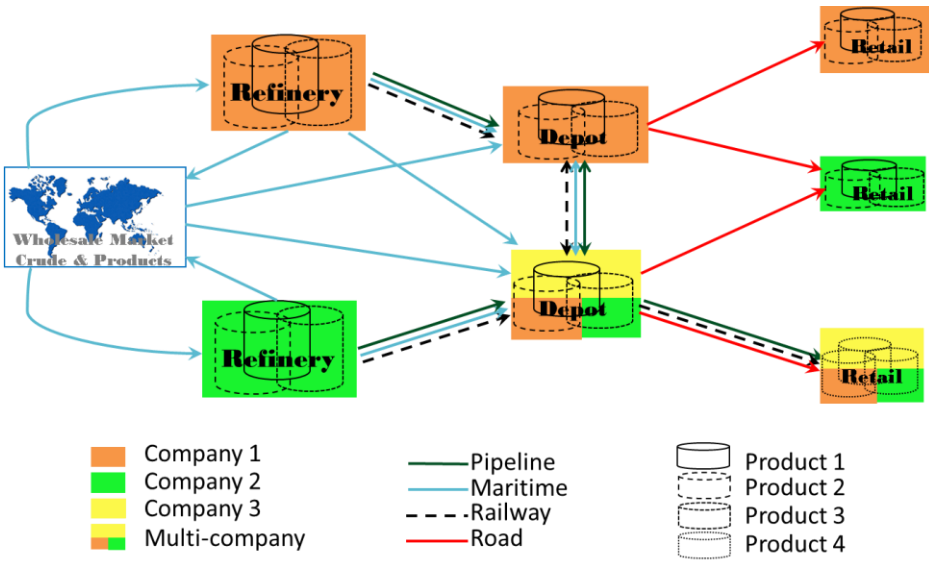


Figure 3.1. The downstream Petroleum Supply Chain

At the heart of the PSC network are the Petroleum companies, who produce and/or trade petroleum crude and derivatives, transform these into petroleum products and commercialize them to the retail clients. To realize this, they rely on physical infrastructures which are wholesale markets, refineries, bulk or primary transport, storage depots, secondary transport and retail stations. These infrastructures may be owned by a single company or multi-company consortiums and may be designed to manage various products of a single company, products of various companies or a single product for various companies. Although the installations may be owned by various companies and sometimes competitors, product is handled as batches belonging to a single petroleum company until it is sold to the retail client. Transportation routes are strategic to the PSC and are normally maritime to link with the wholesale markets, could be pipeline, maritime or railway between refineries and storage depots as well as inter-depot transportation, and finally pipeline, railway or road between the depots and the retail stations. Recently, logistics operators have emerged to provide one or more of these activities as external services, however, it is common that their capital is totally or partially owned by the petroleum companies. Finally the retail stations may be company or dealer owned and/or operated and normally, but not compulsorily, receive products from an exclusive company. To achieve a truly collaborative network where all entities benefit, it is crucial to have the right locations, sizes, prices, quantities and participations of the different infrastructures. Hence the mission of the petroleum companies is to design, construct and manage collaborative networks that maximize the profits and reduce risks for the entire PSC.

The design of a downstream PSC network concerns the installation and operation of storage depots and primary and secondary transportation infrastructures. The determination of the necessary product transportation volumes affects the selection of the required resources and dimensioning their optimal capacity in the network at refineries and depot installations. These resources may be considered as grouped items such as pump stations, interfaces, vessels and loading bays or individualized as manifolds, arms, pumps, valves, etc. Like the installations, the resources also imply considerable fixed and operating costs and hence the importance of being considered separately.

The downstream PSC strategic and tactical planning problem therefore consists of determining the logistics network topology for the distribution of petroleum products, considering a set of petroleum companies who produce or import a set of oil products at/from a known set of refineries, and commercialize them to a set of customer regions through available and possible sets of depots and transportation connections. The objective is to determine the optimal network involving the closure, installation and operation of storage depot locations (under a retrofit solution), individual costs and tariffs, the storage capacities definition, the establishment of transportation modes and routes (primary and secondary), and the allocation of volumes for crude oil, refinery production, import, export and depot transfers, various stage inventories, and customer fulfillment; accounting for production, storage depot, transportation and capacity restrictions and satisfying customer demands while maximizing the total network profit comprising of the oil companies' revenues in addition to their shared revenues in participated infrastructures minus the crude, refining, transportation, storage and retail costs along the network.

Fundamental to these decisions is the product price formation that is synthesized in Figure 3.2, the PSC value chain. The current study includes all costs, prices and margins at the various activities of the PSC, with exception to the crude price formation, the petroleum excise tax and the value added tax. Also, in this study, the petrol station costs are not broken down into retail station costs and dealer margin but considered aggregately as a service. Cross-country sales and taxation policies are identified as an opportunity for future study.

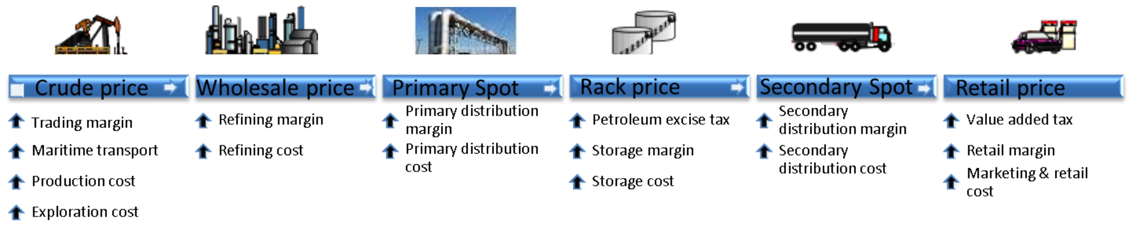


Figure 3.2. PSC value chain

The cost structure and tariffs as well as profits, are based on the value chain of the PSC depicted in Figure 3.2 and considering how the price of petroleum products is formed. Along with our previous work (Fernandes *et al.* 2013), all downstream activities are considered. Although the strategic decisions for installation and operation of equipment, resources and network affectations involve investments, the inventory decisions also have considerable impact on the total costs of the PSC. These tactical decisions relating to dynamic multi-stage inventory planning, play an important role in increasing efficiencies and can improve the strategic decisions. Inventory buffers also mitigate against price and supply uncertainty, and improve the procurement savings by purchasing crude oil at lower prices and trading the derivatives at better prices. The MILP developed, models many of the PSC activities whose assumptions are presented in the following section.

3.4 Model Development Assumptions

When addressing the above described problem a set of assumptions had to be considered. These specify how the real problem is tackled and thereafter represented in the proposed model formulation. Multi-entity PSC networks are modelled for current, grassroots and retrofit designs in order to compare with previous uni-entity network. The key idea behind the assumptions presented below is to effectively represent a general real PSC while maintaining flexibility for the design and planning of networks under different contexts.

- The PSC network considers multiple companies, echelons, products and transportation modes;
- Refinery locations, capacities and yields are given and may not be changed;
- Product yields per refinery are fixed as a single type of crude oil is considered;
- Different transportation modes may be considered: pipeline, maritime, railway or by road;
- New depots and transportation resources including pipelines, rail stations and maritime docks may be installed at refinery and/or demand nodes or between them;
- Existing storage depots and transportation resources and capacities may be increased or reduced by determining the capacity for operation or closure;

- Storage vessels located at depots are configured with typical capacities and rotations per product;
- Fixed flow rates and inventory rotation are considered per depot, transportation resource and product; Deterministic customer demand per product, grouped by district or municipality, is known on the required time basis, namely yearly or monthly given the focus of the problem;
- Crude oil is imported by refineries which supply derived products to depots, or may export them;
- Refineries and depots may import derived products;
- Any exportation/importation flow is considered as a trade to/from a region/refinery located outside the boundaries of the region modelled;
- A storage depot may receive product from several refineries and by various transportation modes and supply it to various customer zones;
- Depots and transportation resources are shared by owner companies;
- Fixed, variable, installation and operating costs are known per resource type as well as crude oil and product prices per echelon and period;
- Each infrastructure resource capacities and consequently costs are determined per location based on total multi-entity transfer volumes;
- Entity service tariffs are determined per product and storage depot location or transportation route based on existing participation, resources and economies of scale;
- Installation and fixed operating costs per infrastructure are incorporated into entity tariffs using a piecewise linearization function, based on coefficients related to transfer volume thresholds;
- Infrastructure profits are computed per depot location or transportation route as the difference between the multi-entity revenues and costs. Company participation percentages determine the profits incorporated by the owner entities;
- Participating and non-participating entities may transfer volumes through infrastructures, not limited by their participation, but must pay the corresponding fixed installation and operating costs;
- Bottle-filling of liquefied petroleum gas is considered as bulk. Excise and value added taxes are considered beyond scope;
- Time is assumed to be represented in a discrete time scale, where the evolution of product volumes and inventories along the network occurs at the fixed time intervals boundaries;
- Company participation percentages in existing refineries, storage depots and transportation facilities are known and will remain unchanged under the scope of the time horizon modelled;
- Company participation percentages in new infrastructures will follow market quotas.

3.5 Model Mathematical Formulation

The strategic and tactical planning problem defined in the previous section, was first modelled for the PSC using a MILP in Fernandes *et al.* (2013) to obtain multi-entity, multi-echelon and multi-entity PSC designs with transfer volumes for existing, grassroots and retrofit networks. The current formulation extends the model in Fernandes *et al.* (2013) by adding collaborative design, operator-profit incorporation, standardized resources, multi-stage and dynamic inventories, and piecewise price linearization. The current model improves also transportation generalization and uses transport mode families and product families associated with resources. The families may contain one or many transport modes or products. A resource independent of transport modes will be defined associated with a transport mode family that includes all transport modes. An example is resource VESSEL associated to transport family ANY that includes all transport modes. The same is the case for the product families. This standardization simplifies and reduces the number of parameters, variables, equations and the objective function constructs.

Secondly, in a multi-entity network, unit tariffs should be inversely proportional to the transfer volumes per entity and infrastructure. Diverging tariffs have been modelled in Baumgartner *et al.* (2012) using a piecewise linearization approach. The current multi-entity and collaborative MILP permits entities flexible utilization of each other's infrastructures. Consequently, the tariff includes imputation of piecewise linearized installation and operating costs, in addition to the volume-dependent unitary cost. This motivates ownership participations in infrastructures as well as maximization of capacity utilization.

Figure 3.3 illustrates our piecewise cost linearization function. We consider a depot construction that has a nominal capacity of 100,000 m^3 with a fixed cost of 200,000 monetary units (m.u.) and a variable cost of 1.6 m.u/m^3. The capacity is divided into 10 volume thresholds that increase linearly from 10 to 100%. Only one threshold may be selected per entity implying a fixed cost that varies from initial 20% to maximum 100% of the resource installation cost. Hence for 10% utilization (10,000 m^3), a 20% fixed cost is imputed, resulting in the total fixed cost of 40,000 m.u., or a unitary fixed cost of 4 m.u which when added to the variable cost of 1.6 derives the tariff of 5.6 m.u./m^3. If an entity uses 100% capacity, its tariff would be 3.6 m.u./m^3.

resource fraction	volume m3	% fixed costs	variable unit cost	instalation fixed cost	fixed cost	variable cost	tariff
1	10000	0.2	1.6	200000	40000	16000	5.6
2	20000	0.36	1.6	200000	72000	32000	5.2
3	30000	0.49	1.6	200000	98000	48000	4.9
4	40000	0.6	1.6	200000	120000	64000	4.6
5	50000	0.69	1.6	200000	138000	80000	4.4
6	60000	0.77	1.6	200000	154000	96000	4.2
7	70000	0.84	1.6	200000	168000	112000	4.0
8	80000	0.9	1.6	200000	180000	128000	3.9
9	90000	0.95	1.6	200000	190000	144000	3.7
10	100000	1	1.6	200000	200000	160000	3.6

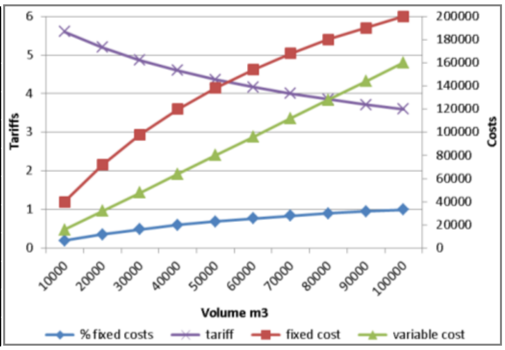


Figure 3.3. Piecewise cost linearization function

The problem here is therefore to assign an entity to its respective threshold at each depot, resource and transport route (in the case of pipelines, docks or rail stations which imply installation costs). This is achieved by creating binary variables, ZD and ZT, that define the threshold for installation and operation of depots and transport routes respectively. An index *u* defines the number of linearly increasing volume thresholds per infrastructure. Besides, each resource is considered to be partitioned into *u* partitions each with capacity fraction of *1/u*. Integer variables ZR determine the number of resource capacity partitions required per entity. In either case, a capacity threshold or number of resource partitions is assigned to each product entity based on the transported volumes. Consequently, entity tariffs will include a corresponding percentage of installation and operating fixed costs. The above variables determine the capacity used, thereby implying a coefficient for the resource cost to be imputed. Also, installation and operating costs are imputed to the product entities, whereas the profits are collected by the owner entities as per their participation in the infrastructure, thereby motivating their utilization. Product entities without participations are imputed installation costs as per the transferred volumes. Secondly there are two methods for imputing the installation costs into product entity tariffs: differed project costs or non-amortized costs. The differed project cost is the fixed amortizations during the amortization period, whereas the latter includes only the non-amortized costs. Hence, the product entities will pay the installed and operating (Z) percentage costs by way of tariff as determined above, whereas the profits are incorporated by the infrastructure owner entities based on their ownership percentages in the infrastructures.

We now present the dynamic MILP formulation in detail explaining the notation used, the model decisions, the network constraints, the objective function and lastly the pre-processing.

3.5.1 Notation

3.5.1.1 Sets / Indices

$a \in A$	set of petroleum supply chain activity stages
$c \in C$	set of product characteristics, C={ DENSITY, TYPICAL_M3}
$e, e1 \in E$	set of petroleum entities/companies E={E1, E2, ..., EN }
$fc \in FC$	set of trip atributes, FC={CAPACITY, WORK, DAYS, BEFORE, AFTER, WAYS}
$g \in G$	set of GPS coordinates, G={ LATITUDE, LONGITUDE }
$n, o \in N$	set of all nodes
$p \in P$	set of products P={ P1, P2,..PN }
$r, r1 \in R$	set of resources including services and equipment
$t \in T$	set of time periods, T={1,2,..N}
$u \in U$	set of volume thresholds U={1,2,..N} for piecewise price linearization

$v \in V$	set of costs, V={ TARIFF, FIXED, FIXVAR, OPERATING, OPERVAR, UNITARY }
$y \in Y$	set of amortization periods, Y={ 1,2,…,N }

3.5.1.2 Subsets / Indices

$i \in I \subseteq N$	set of refinery nodes
$IL \subseteq I$	set of local refinery nodes which are allowed to export
$j \in J \subseteq N$	set of depot nodes
$k \in K \subseteq N$	set of customer nodes
$m, m1 \in M \subseteq R$	set of transport modes, M={ PIPELINE, MARITIME, RAIL, ROAD }
$MC \subseteq M$	set of continuous flow transport modes MC={ PIPELINE }
$MP \subseteq M$	set of participated transport modes MP={ PIPELINE }

3.5.1.3 Subset unions / Indices

$fm \in FM = M \bigcup \{ANY\}$	set of transport mode families,
$fp \in FP = P \bigcup \{ANY\}$	set of products families FP={ P1, P2,..PN, ANY }
$h \in H = I \bigcup J$	set of refinery and depot nodes
$l \in L = J \bigcup K$	set of depot and customer nodes

3.5.1.4 Parameters

BC_i	beginning inventory of crude at refinery *i,*
$BD_{j,p,e}$	beginning inventory at depot *j*, for product *p,* entity *e,*
$BL_{k,p,e}$	beginning inventory at customer *k*, for product *p,* entity *e*,
$BR_{i,p,e}$	beginning inventory at refinery *i*, for product *p,* entity *e*,
BN	big number for number of resources
HPP	hours per time period
MD_m	distance multiplicator per transportation mode to account for route curves,
MNA	maximum number of amortization periods
MV_m	medium velocity per transportation mode *m*
$FM_{fm,m}$	association of transportation mode family *fm* with transportation mode *m*
$FP_{fp,p}$	association of product family *fp* with product *p*
$FR_{m,fc}$	trip parameters per transportation mode *m* and trip characteristic *fc*
NCD	number of depots in concentration set
NCP	number of capacity partitions per resource available to entity
NFF	fill factor in percentage
NP	total number of periods
NR	degree to radian coefficient for distance calculations

OCC	operating cost coefficient
RCD	radius for concentration set for demand at depot
TPP	time aggregation in period, TPP=1 / 12 for monthly / yearly data
α^{a}	operator margin percentage per activity *a*
β^{a}_{Min}	minimum safety stock percentage per activity a
β^{a}_{Max}	maximum safety stock percentage per activity a
γ	working average cost of capital (WACC) coefficient for inventory costs
$\Gamma_{n,g}$	GPS (global pointing system) coordinates *g* for each location node *n*
$\delta_{k,p,e,t}$	customer demand in m^3 at customer node *k*, product *p*, entity *e* and period *t*
$\varepsilon_{i,p}$	fraction yield of product *p* per unit measure of crude oil at refinery *i*
$\varphi^{a}_{p,t}$	price of product *p* after activity *a* (crude, ex-refinery, retail) at time *t*
$\eta_{n,o,r,e}$	capital participation percentage by entity *e* in resource *r* between nodes *n,o;* consider *r* = *m* in case of transport mode and *n*=*o* in case of depots
$\kappa d_{n,y}$	number of completed amortization periods *y* for each existing depot at
$\kappa r_{n,fm,r,y,fp}$	current resource units per transport mode family *fm* and product family *fp*, per resource *r* at refinery or depot *n,* per completed number of amortization
$\kappa t_{n,o,m,y}$	number of completed amortization periods *y* for current transport infrastructures between *n,o* for mode *m*; road routes are considered as
$\mu_{n,o,m}$	transport distance between any two nodes *(n,o)* including refineries, depots or customer zones, using transport mode *m*
$\mu'_{n,o,m}$	transport duration between any two nodes *(n,o)* including refineries, depots or customer zones, using transport mode *m*
$\theta r_{fm,r,fp}$	typical capacity of resource *r* for transport mode family *fm* and product family
ρ_{i}	crude processing capacity at refinery *i* in m^3 per day
τc_{u}	piecewise linearization function coefficients per step *u*
$\tau t_{h,u}$	piecewise linearization function thresholds per depot *h* and step *u*
$\upsilon^{a}_{m,p}$	maximum permitted distance for activity *a* by transportation mode *m* for the petroleum product *p*; example: fuel-oil transportation by pipeline
ω^{c}_{p}	characteristic *c* per product *p*, e.g. density
$\omega m_{m,c,p}$	characteristic *c* per transport mode *m* per product *p*
ξR^{v}_{r}	cost component *v* for resource *r* independent of product, example pipeline
$\xi P^{v}_{r,p}$	cost component *v* for resource *r* dependent of product *p*, example vessel
ζ_{e}	market quota per entity *e*

3.5.2 Model Decisions

The model supports strategic decisions which are constant throughout the model time horizon, and tactical decisions that are time-dependent. The decision variables are briefly synthesized and then presented.

i) the strategic binary decisions determine the depots and transportation routes to install (*I* prefixed) and to operate (*O* prefixed), besides determining the tariff and fixed cost linearization coefficients (Z prefixed) per depot or transportation route, entity and product;

ii) the strategic integer decisions determine the expansion and reduction of capacities, determining the number resources to install (*I* variables) and to operate (*O* variables) per depot. Note that the *K* parameters define existing resources, hence a positive difference between corresponding I variables and K parameters determines how many new resources are installed, and a positive difference between *I* and *O* variables determines the number of resources to close. Besides, the tariff and fixed cost linearization coefficients (Z prefixed) are determined as the number of resource partitions per depot resource, entity and product;

iii) the non-negative tactical decisions determine the transfer volumes (Q prefixed) and the inventory volumes (S prefixed) per activity stage, node and transportation route;

iv) the continuous objective sub-function variables determine the activity stage profits and margins per node, transportation route and time period;

v) the non-negative objective sub-function variables determine the tariffs and costs per activity stage, node, transportation route and time period.

3.5.2.1 Binary Strategic Decision Variables

$$ID_j = \begin{cases} 1, & \text{if depot } j \text{ is to be installed} \\ 0, & \text{otherwise} \end{cases}$$

$$IT_{h,l,m} = \begin{cases} 1, & \text{if route using transport mode } m \text{ is to be installed between node } h \text{ and node } l \\ 0, & \text{otherwise} \end{cases}$$

$$OD_j = \begin{cases} 1, & \text{if depot } j \text{ is to be operated} \\ 0, & \text{otherwise} \end{cases}$$

$$OT_{h,l,m} = \begin{cases} 1, & \text{if route using transport mode } m \text{ is to be operated between node } h \text{ and node } l \\ 0, & \text{otherwise} \end{cases}$$

$$ZD_{j,e,u} = \begin{cases} 1, & \text{if } \pi_{j,u-1} < QP_{j,l,m,p,e,t} + QS_{j,l,m,p,e,t} < \pi_{j,u} \\ 0, & \text{otherwise} \end{cases}$$

$$ZT_{h,l,m,e,u} = \begin{cases} 1, & \text{if } \pi_{h,u-1} < QP_{h,l,m,p,e,t} + QS_{h,l,m,p,e,t} < \pi_{h,u} \\ 0, & \text{otherwise} \end{cases}$$

3.5.2.2 Integer Strategic Decision Variables

$IR_{j,fm,r,fp}$ number of resources *r* to install per depot *j* per transport mode family *fm* for product family *fp*,

$OR_{j,fm,r,fp}$ number of resources *r* to operate per depot *j* per transport mode family *fm* for product family *fp*,

$ZR_{j,fm,r,fp,e}$ number of resource partitions to impute installation and operating costs per entity *e*, resource *r,* depot *j,* transport mode family *fm* and product family f*p*,

3.5.2.3 Non-negative Tactical Decision Variables

$QCI_{i,t}$ Crude volume imported by refinery *i,* in period *t*,

$QCP_{i,t}$ crude volume processed by refinery *i,* in period *t*,

$QE_{i,p,t}$ exported volume by refinery *i* of product *p* in period *t*,

$QI_{j,p,e,t}$ imported volume by depot *j* of product *p* and entity *e* in period *t*,

$QP_{h,j,m,p,e,t}$ primary volumes of product *p* transported by mode *m* from refinery/depot *h* to depot *j* by entity *e* in period *t*,

$QR_{i,p,t}$ production volume per refinery *i* of product *p* in period *t*,

$QS_{j,k,m,p,e,t}$ secondary volumes of product *p* transported by mode *m* from depot *j* to customer *k* by entity *e* in period *t*,

$QU_{k,p,e,t}$ unsatisfied demand per customer *k*, for product *p,* entity *e* at end of period *t,*

$SC_{i,t}$ crude inventory at refinery *i* at end of period *t,*

$SD_{j,p,e,t}$ inventory at depot *j*, for product *p,* entity *e* at end of period *t,*

$SL_{k,p,e,t}$ inventory at customer *k*, for product *p,* entity *e* at end of period *t*,

$SR_{i,p,e,t}$ inventory at refinery *i*, for product *p,* entity *e* at end of period *t*,

3.5.2.4 Continuous Objective Sub-function Variables

$Profit$ objective function or total PSC profits,

$MDE_{j,e,t}$ margin per depot *j*, entity *e* in period *t*,

$ML_{k,p,e,t}$ margin per retail customer *k*, product *p*, entity *e* in period *t*,

$MR_{i,e,t}$ margin per refinery *i*, entity *e* in period *t,*

$MTE_{h,l,m,e,t}$ transportation margin between nodes *h,l* for mode *m* and entity *e* in period *t*,

3.5.2.5 Non-negative Objective Sub-function Variables

$CDD_{h,t}$ depot costs at depot *h* in period *t*,

$CDFD_{h,t}$ depot fixed costs for depot *h* in period *t*,

$CDFE_{h,e,t}$ depot fixed costs to entity *e* for depot *h* in period *t*,

$CDOD_{h,t}$ depot operating costs for depot *h* in period *t*,

$CDOE_{h,e,t}$ depot operating costs to entity *e* for depot *h* in period *t*,

$CDU_{h,p,e,t}$ unitary costs at depot *h* for product *p*, for entity *e* in period *t*,

$CE_{i,p,e,t}$ exportation costs per refinery *i*, product *p*, entity *e* in period *t*,

$CI_{j,p,e,t}$ importation costs per depot *j*, product *p*, entity *e* in period *t*,

$CNC_{i,p,e,t}$ inventory costs for crude at refinery *i,* product *p*, and entity *e* in period *t*,

$CND_{j,p,e,t}$ inventory costs at depot *j,* product *p*, and entity *e* in period *t*,

$CNL_{k,p,e,t}$ inventory costs at customer *k,* product *p*, and entity *e* in period *t*,

$CNR_{i,p,e,t}$ inventory costs at refinery *i,* product *p*, and entity *e* in period *t*,

$CRFD_{h,fp,t}$ resources fixed costs for depot *h,* resource *fp* in period *t*,

$CRFE_{h,fp,e,t}$ resources fixed costs to entity *e* for depot *h*, resource *fp* in period *t*,

$CROD_{h,fp,t}$ resources operating costs for depot *h,* resource *fp* in period *t*,

$CROE_{h,fp,e,t}$ resources operating costs to entity *e* for depot *h* resource *fp* in period *t*,

$CTD_{h,l,m,t}$ transportation costs between nodes *h,l* for mode *m* in period *t*,

$CTFD_{h,l,m,t}$ transportation fixed costs between nodes *h,l* for mode *m* in period *t*,

$CTFE_{h,l,m,e,t}$ transportation fixed costs to entity *e* between nodes *h,l* for mode *m* in period *t*,

$CTOD_{h,l,m,t}$ transportation operating costs between nodes *h,l* for mode *m* in period *t*,

$CTOE_{h,l,m,e,t}$ transportation operating costs to entity *e* between nodes *h,l* for mode *m* in

$CTU_{h,l,m,p,e,t}$ transportation unitary costs between nodes *h,l* for mode *m* and product *p* in

$TDE_{h,e,t}$ depot tariffs at depot *h* for entity *e* in period *t*,

$TTE_{h,l,m,e,t}$ transportation tariff between nodes *h,l* for mode *m* for entity *e* in period *t*,

3.5.3 Constraints

Strategic and tactical decisions are enforced at each stage through installation, operation, capacity and volume balance equations. Customer demand transfers to higher stages of the PSC through volume balance equations, thereby determining the crude oil and product supply needs at various nodes and periods. The current model improves our earlier model with new constraints for multi-stage inventories, transportation and storage depot entity price linearization thresholds, minimum and maximum inventories, routed networks and collaborative design and operation. With the exception of equations (3.3), (3.4), (3.13), (3.25), (3.71), (3.72) and (3.73) which were originally proposed in Fernandes *et al.* (2013), all other equations are newly created or modified.

3.5.3.1 Volume balance equations

Crude volume balance: For the crude oil procurement, the initial time period eq. (3.1) enforces for each refinery that the sum of the initial crude inventory BC_i and the imported crude $QCI_{i,t}$ provides the sum of the processed crude $QCP_{i,t}$ and the crude inventory $SC_{i,t}$ transiting to the next period. In eq. (3.2), for the remaining time intervals, the previous period crude inventory $SC_{i,t-1}$ is added to the LHS.

$$BC_i + QCI_{i,t} = QCP_{i,t} + SC_{i,t}, \qquad \forall i \in I, t \in T \wedge t \leq 1 \tag{3.1}$$

$$SC_{i,t-1} + QCI_{i,t} = QCP_{i,t} + SC_{i,t}, \qquad \forall i \in I, t \in T \wedge t > 1 \tag{3.2}$$

Refinery mass balance: Eq. (3.3) enforces at each refinery and period, that the processed crude $QCP_{i,t}$ may not exceed the refinery capacity ρ_i. Besides, eq. (3.4) restricts the refined volumes $QR_{i,p,t}$ per refinery, product and period to the product of the processed crude $QCP_{i,t}$ and the refinery fractional yield $\varepsilon_{i,p}$.

$$QCP_{i,t} \leq \rho_i, \qquad \forall i \in I, t \in T \tag{3.3}$$

$$QR_{i,p,t} = QCP_{i,t} \times \varepsilon_{i,p}, \qquad \forall i \in I, p \in P, t \in T \tag{3.4}$$

Besides, for each refinery, product and period, the import/export balance equations eq. (3.5) at initial time period and eq. (3.6) at remaining periods, enforce that the sum of the previous period inventories $SR_{i,p,t-1}$ (or beginning inventory $BR_{i,p}$ in case of the initial period), the refined volumes $QR_{i,p,t}$ and entity imported volumes $\sum_{e \in E} QI_{i,p,e,t}$ equals the sum of the traded volumes $\sum_{(j,m)|\exists \mu_{i,j,m}, e \in E} QP_{i,j,m,p,e,t}$, the exported product $QE_{i,p,t}$ and the inventory $SR_{i,p,t}$ transiting to the following period. The traded volumes only exist where a route is available, hence the condition $(j,m)|\exists \mu_{i,j,m}$. Exportation of surplus production $QE_{i,p,t}$ is restricted to local refineries $i \in IL$, in order to study the network from an importation, single country or single region perspective.

$$BR_{i,p} + QR_{i,p,t} + \sum_{e \in E} QI_{i,p,e,t} = \sum_{(j,m):\exists \mu_{i,j,m}, e \in E} QP_{i,j,m,p,e,t} + QE_{i \in IL,p,t} + SR_{i,p,t}, \forall i \in I, p \in P, t \in T \wedge t \leq 1 \tag{3.5}$$

$$SR_{i,p,t-1} + QR_{i,p,t} + \sum_{e \in E} QI_{i,p,e,t} = \sum_{(j,m)|\exists \mu_{i,j,m}, e \in E} QP_{i,j,m,p,e,t} + QE_{i \in IL,p,t} + SR_{i,p,t}, \forall i \in I, p \in P, t \in T \wedge t > 1 \tag{3.6}$$

Storage Depot balance: The MILP allows modelling of the PSC as a direct network (refinery-depot-customer) as well as a routed network that includes inter-depot transfers. If a direct network is chosen, constraints (3.7) enforce the initial period balances, where the sum of the beginning inventory $BD_{j,p,e}$, the primary volumes $QP_{h,j,m,p,e,t}$ and imported volumes $QI_{j,p,e,t}$ at each depot equate to the sum of the secondary volumes $QS_{j,k,m,p,e,t}$ expedited from that depot and the inventory $SD_{j,p,e,t}$ transiting to the following period. Eq. (3.8) enforces the remaining period balances by adding the previous period inventory $SD_{j,p,e,t-1}$.

$$BD_{j,p,e} + \sum_{(h \in H \cap I, m) | \exists \mu_{h,j,m}} QP_{h,j,m,p,e,t} + QI_{j,p,e,t} = \sum_{(k,m)|\exists \mu_{j,k,m}} QS_{j,k,m,p,e,t} + SD_{j,p,e,t}, \forall j \in J, p \in P, e \in E, t \in T, t \leq 1 \quad (3.7)$$

$$SD_{j,p,e,t-1} + \sum_{(h \in H \cap I, m) | \exists \mu_{h,j,m}} QP_{h,j,m,p,e,t} + QI_{j,p,e,t} = \sum_{(k,m)|\exists \mu_{j,k,m}} QS_{j,k,m,p,e,t} + SD_{j,p,e,t}, \forall j \in J, p \in P, e \in E, t \in T, t > 1 \quad (3.8)$$

While the above equations are still valid for refinery depots $j \in H \cap I$ in the routed network formulation where inter-depot transportation is permitted, eqs. (3.31) and (3.32) will be additionally defined for the non-refinery depots $j \in H \setminus I$.

Customer Demand balance: For each customer, product, entity and period, the initial period customer demand balance equations (3.9) enforce that the sum of the beginning inventory $BL_{k,p,e}$, the secondary volumes $QS_{j,k,m,p,e,t}$ received from all depots and modes equate to the sum of the satisfied customer demand $\delta_{k,p,e,t} - QU_{k,p,e,t}$ and the inventory $SL_{k,p,e,t}$ transiting to the next period . In the following period balance equations (3.10), the retail inventory transiting from the previous period $SL_{k,p,e,t-1}$ is added to the LHS.

$$BL_{k,p,e} + \sum_{(j,m)|\exists \mu_{j,k,m}} QS_{j,k,m,p,e,t} = \delta_{k,p,e,t} - QU_{k,p,e,t} + SL_{k,p,e,t}, \forall k \in K, p \in P, e \in E, t \in T \wedge t \leq 1 \quad (3.9)$$

$$SL_{k,p,e,t-1} + \sum_{(j,m)|\exists \mu_{j,k,m}} QS_{j,k,m,p,e,t} = \delta_{k,p,e,t} - QU_{k,p,e,t} + SL_{k,p,e,t}, \forall k \in K, p \in P, e \in E, t \in T \wedge t > 1 \quad (3.10)$$

3.5.3.2 Transportation network

Distribution route operation: The binary variables $ZT_{h,l,m,e,u}$ determine the entity thresholds per transportation infrastructure. On each route, at most one $ZT_{h,l,m,e,u}$ variable is selected per entity based on the threshold parameter $\pi_{h,u}$. For each entity transport route and mode having fixed or operating costs, eq. (3.11) enforces that the average product volumes on this route are lower than the threshold volume $\pi_{h,u}$ of the selected threshold $ZT_{h,l,m,e,u}$ and higher than the previous threshold volume $\pi_{h,u-1}$.

$$\sum_{u \in U} \pi t_{h,u-1} ZT_{h,l,m,e,u} \leq \sum_{t \in T} \sum_{p \in P} \left(QP_{h,l,m,p,e,t} + QS_{h,l,m,p,e,t} \right) / NP \leq \sum_{u \in U} \pi t_{h,u} ZT_{h,l,m,e,u}, \quad \forall (h,l,m) \mid \exists \mu_{h,l,m} \wedge \exists \xi r_{v \in \{\text{FIXED,FIXVAR,OPERATING,OPERVAR}\},m}, e \in E \tag{3.11}$$

Constraint (3.12) enforces at most one operating capacity threshold per entity and operating route.

$$\sum_{u \in U} ZT_{h,l,m,e,u} \leq OT_{h,l,m}, \quad \forall (h,l,m) \mid \exists \mu_{h,l,m} \wedge \exists \xi r_{v \in \{\text{FIXED,FIXVAR,OPERATING,OPERVAR}\},m}, e \in E \tag{3.12}$$

Moreover, for all available routes $(h,l,m) \mid \exists \mu_{h,l,m}$, constraints (3.13) enforce that only installed routes may operate while eqs.(3.14) enforce that operating routes and modes have corresponding resources operating at the originating depot.

$$OT_{h,l,m} \leq IT_{h,l,m}, \quad \forall (h,l,m) \mid \exists \mu_{h,l,m} \tag{3.13}$$

$$OT_{h,l,m} \leq \sum_{fm=m,r,fp} OR_{h,fm,r,fp}, \quad \forall (h,l,m) \mid \exists \mu_{h,l,m} \tag{3.14}$$

Likewise, for each available transportation route and mode, constraints (3.15) enforce that installed routes are only present when the originating depot node has corresponding installed resources.

$$IT_{h,l,m} \leq \sum_{fm=m,r,fp} IR_{h,fm,r,fp}, \quad \forall (h,l,m) \mid \exists \mu_{h,l,m} \tag{3.15}$$

Route retrofit installation: To permit retrofit design, at least current capacity is installed, as enforced by constraint (3.16). However, this capacity may be selected for closure (not operated) as explained above.

$$\kappa t_{h,l,m,y} \leq IT_{h,l,m}, \quad \forall (h,l,m) \mid \exists \mu_{h,l,m}, y \in Y \tag{3.16}$$

3.5.3.3 Storage Depot network

Resource capacity, installation and operation: Strategic decisions at storage depots involve the capacity installation and operation of two types of resources. Dynamic or continuous flow equipment capacities are specified as flow in cubic meters per hour as in the case of pumps and arms. Static or non-continuous flow equipment are specified as volume capacities and number of rotations or refills per period, as in the case of storage vessels. Resource capacities $\theta r_{fm,r,fp}$ are defined per transport mode family and/or per product family. As presented in figure 3.3, each resource capacity is dividing by parameter NCP to obtain NCP resource partitions of equal capacity. Entity operating resource partitions $ZR_{h,fm,r,fp,e}$ per depot, transport mode family and/or product family are determined by constraints (3.17) based on the transported volumes. The sum of the depot transfer volumes (divided by the number of resource rotations $\omega m_{m,p}$ for static resources) and the carry forward inventory volumes divided by the number of periods, determines the resource partitions strictly required per product and entity.

This is the product of the resource partitions $ZR_{h,fm,r,fp,e}$, the resource capacity $\theta r_{fm,r,fp}$ and HPP (hours per period) for dynamic resources or TPP (times per period) for static resources divided by parameter NCP. The resource partitions operating per entity $ZR_{h,fm,r,fp,e}$ imply their installation and operation costs, thereby their inclusion into entity tariffs.

$$\begin{aligned}&(ZR_{h,fm,r,fp,e}-1)\times\theta r_{fm,r,fp}\times((1-k)\times HPP+k\times TPP)/NCP \leq \\ &\sum_{t\in T}\sum_{p\in FP_{fp,p},m\in FM_{fm,m}}\left(\left(\left(\sum_{j\in\mu_{h,j,m}}QP_{h,j,m,p,e,t}+\sum_{k\in\mu_{h,k,m}}QS_{h,k,m,p,e,t}\right)\left((1-k)+\frac{k}{\omega m_{m,p}}\right)+SD_{h,p,e,t}\,k\right)\right)\Big/NP \leq \\ &ZR_{h,fm,r,fp,e}\,\theta r_{fm,r,fp}((1-k)HPP+k\times TPP)/NCP,\ \forall h\in H, e\in E, k=1, (fm,r,fp)\,|\,\exists\,\theta r_{fm,r,fp}\wedge fm=ANY\end{aligned} \tag{3.17}$$

Although the resource partitions required to operate per entity are determined in equations (3.17), the number of physical resources to install and operate per depot are determined in $IR_{h,fm,r,fp}$ and $OR_{h,fm,r,fp}$ respectively. A positive difference between installed $IR_{h,fm,r,fp}$ and existing $\sum_{y\in Y}\kappa r_{h,fm,r,y,fp}$ resources gives the number of new resource to open and a positive difference between the installed $IR_{h,fm,r,fp}$ and operating $OR_{h,fm,r,fp}$ resources gives the number of resources to close. As retrofit design is considered, eq. (3.18) enforces that at least the current resource capacities are installed.

$$\sum_{y\in Y}\kappa r_{h,fm,r,y,fp}\leq IR_{h,fm,r,fp},\qquad \forall h\in H, fm\in FM, r\in R, fp\in FP, (fm,r,fp)\,|\,\exists\,\theta r_{fm,r,fp} \tag{3.18}$$

At each depot, eq. (3.19) enforces the sum of entity operating resource partitions to be contained in physically operating resources.

$$\sum_{e\in E}ZR_{h,fm,r,fp,e}\leq OR_{h,fm,r,fp}\times NCP,\qquad \forall h\in H, fm\in FM, r\in R, fp\in FP, (fm,r,fp)\,|\,\exists\,\theta r_{fm,r,fp} \tag{3.19}$$

Likewise, constraints (3.20) enforce that resources operate only in operating depots. Constraints (3.21) enforce that resources are installed only in installed depots. Constraints (3.22) enforce that resources are installed in order to operate. Eqs. (3.19-3.22) are defined per depot and only for the transport mode families, resources and product families with associated resource capacities $\theta r_{fm,r,fp}$.

$$OR_{h,fm,r,fp}\leq OD_h\times NB,\qquad \forall h\in H, fm\in FM, r\in R, fp\in FP, (fm,r,fp)\,|\,\exists\,\theta r_{fm,r,fp} \tag{3.20}$$

$$IR_{h,fm,r,fp}\leq ID_h\times NB,\qquad \forall h\in H, fm\in FM, r\in R, fp\in FP, (fm,r,fp)\,|\,\exists\,\theta r_{fm,r,fp} \tag{3.21}$$

$$OR_{h,fm,r,fp}\leq IR_{h,fm,r,fp},\qquad \forall h\in H, fm\in FM, r\in R, fp\in FP, (fm,r,fp)\,|\,\exists\,\theta r_{fm,r,fp} \tag{3.22}$$

Depot installation and operation: Constraints (3.23) determine the volume thresholds per entity $ZD_{h,e,u}$, such that these accommodate the entity's resource partitions at the depot. The volume thresholds are used to impute the depot installation and operating costs to entities.

$$\sum_{u \in U} \pi t_{h,u-1} ZD_{h,e,u} \leq \sum_{fm,r,fp} ZR_{h,fm,r,fp,e}\, \theta r_{fm,r,fp} \times TPP/NCP \leq \sum_{u \in U} \pi t_{h,u} ZD_{h,e,u}, \forall h \in H, e \in E, r = 'VESSEL' \quad (3.23)$$

Eq. (3.24) enforces that at most one depot volume threshold is selected per entity per operating depot. Likewise, eq. (3.25) enforces that only installed depots may operate.

$$\sum_{u \in U} ZD_{h,e,u} \leq OD_h, \qquad \forall h \in H, e \in E \quad (3.24)$$

$$OD_h \leq ID_h, \qquad \forall h \in H \quad (3.25)$$

The variables ZD, ZT and ZR implement the imputation of piecewise linearized installation and operating costs to product entity tariffs. These variables may be set for entities even in non-participated depots, thereby providing collaborative design.

3.5.3.4 Minimum and Maximum inventory constraints

Crude working stock: Parameters β^{α}_{Min} and β^{α}_{Max}, defined per activity, enforce safety stocks per PSC stage. These may be adjusted to enhance entity profits and improve PSC efficiency. Equations (3.26) and (3.27) enforce the crude oil inventory $SC_{i,t}$ at each period, within a percentage of the refinery's crude processing capacity ρ_i.

$$SC_{i,t} \geq \rho_i \times \beta^{CRUDE}_{Min}, \qquad \forall i \in I, t \in T \quad (3.26)$$

$$SC_{i,t} \leq \rho_i \times \beta^{CRUDE}_{Max}, \qquad \forall i \in I, t \in T \quad (3.27)$$

Refinery safety stock: Equation (3.28) enforces a minimum for the processed crude based on the refinery's production capacity ρ_i. Constraints (3.29) enforce for each refinery, product and period, a maximum inventory based on the refinery's product yield $\varepsilon_{i,p}$.

$$QCP_{i,t} \geq \rho_i \times \beta^{REFINING}_{Min}, \qquad \forall i \in I, t \in T \quad (3.28)$$

$$SR_{i,p,t} \leq \rho_i \times \varepsilon_{i,p} \times \beta^{REFINING}_{Max}, \qquad \forall i \in I, p \in P, t \in T, t > 1 \wedge t \leq T-1 \quad (3.29)$$

Retail Working Stock: Equations (3.30) enforces that inventory buffers $SL_{k,p,e,t}$ at each retail station, are limited to a percentage of the following period customer demand $\delta_{k,p,e,t+1}$.

$$SL_{k,p,e,t} \leq \delta_{k,p,e,t+1} \times \beta^{RETAIL}_{Max}, \qquad \forall k \in K, p \in P, e \in E, t \in T, t > 1 \wedge t < T \quad (3.30)$$

3.5.3.5 Model extensions

Routed networks: An alternate routed network formulation is proposed for use with large networks. Here, eqs. (3.7,3.8) are still valid for refinery depots, however initial period eqs. (3.31) and following period eqs. (3.32) are defined for the non-refinery depots $j \in H \setminus I$. The later account for the inter-depot transportation volumes $QP_{j,h,m,p,e,t}$.

$$BD_{j,p,e} + \sum_{(h \in H \cap I, m) | \exists \mu_{h,j,m}} QP_{h,j,m,p,e,t} + QI_{j,p,e,t} = \sum_{(h \in H \setminus I, m) | \exists \mu_{j,h,m}} QP_{j,h,m,p,e,t} + \sum_{(k,m) | \exists \mu_{j,k,m}} QS_{j,k,m,p,e,t} + SD_{j,p,e,t}, \quad \forall j \in H \setminus I, p \in P, e \in E, t \in T, t \leq 1 \tag{3.31}$$

$$SD_{j,p,e,t-1} + \sum_{(h \in H \cap I, m) | \exists \mu_{h,j,m}} QP_{h,j,m,p,e,t} + QI_{j,p,e,t} = \sum_{(h \in H \setminus I, m) | \exists \mu_{j,h,m}} QP_{j,h,m,p,e,t} + \sum_{(k,m) | \exists \mu_{j,k,m}} QS_{j,k,m,p,e,t} + SD_{j,p,e,t}, \quad \forall j \in H \setminus I, p \in P, e \in E, t \in T, t > 1 \tag{3.32}$$

Individualistic operation: The PSC may be studied from the individualistic perspective where the entities may only distribute products from participated depots, not exceeding the volumes determined by their participation. Eqs. (3.33-3.35) enforce the depot, transportation and resources operating thresholds per entity to their respective participation percentages.

$$ZD_{h,e,u} \pi_{h,u} \leq \eta_{h,e}, \quad \forall h \in H, e \in E, u \in U \tag{3.33}$$

$$ZR_{h,fm,r,fp,e} \leq kr_{h,fm,r,fp} \eta_{h,e}, \quad \forall h \in H, fm \in FM, r \in R, fp \in FP, e \in E \tag{3.34}$$

$$ZT_{h,l,m,e,u} \pi_{h,u} \leq \eta_{h,l,m,e}, \quad \forall h \in H, l \in L, m \in M, e \in E, u \in U \tag{3.35}$$

3.5.4 Objective Function

The current model objective function includes many features: i) the analysis of various business objectives, namely maximize profits, maximize revenues, and minimize costs. ii) The PSC can be optimized for all individual entities as the cost, tariff and profit components per entity are obtained individually and finally aggregated. Here, a weighted multi-objective function based on entity market quotas could aggregate entity objective values. iii) Entities play two roles in PSCs: product entities are product owners who utilize the network services and hence pay service tariffs to the logistics operators; and capital entities who own logistics operator installations and hence incorporate a share of their profits. iv) Costs, tariffs and profits are determined per depot or route, and product for each product entity. A detailed cost structure is modelled using fixed, fixed variable, operating, operating variable and unitary (volume based) costs. The fixed variable and operating variable costs may be defined per unit of distance or duration or both.

The model is versatile and its structure permits granular cost and profit imputation per activity, depot, route, entity and product; and when not directly imputable, secures their distribution based on the entity/product market quotas. v) Tariffs per product entity, depot or route, and product are calculated based on the determined costs per product entity, indexed by a markup margin α^{a} defined per activity. vi) The tariffs calculated above are paid by the product entities to the logistics operators, and hence constitute their revenue. The logistics operator costs are calculated per depot or transportation route and product. The difference between the revenue and costs provide the logistics operator profits for each depot or transportation route and product. vii) The capital entities incorporate the logistics operator profits based on their capital participation in the depot and transportation infrastructures. viii) The disaggregation of the objective function into individual costs, tariffs and profits per product entity, depot, route, and product, makes them available for further study or direct use. ix) As entities use shared resources, the individual entity costs and consequently tariffs must be properly calibrated so as to reflect economies of scale. Here, a piecewise linear approximation approach is used to affect the fixed and operating costs to the product entities. Product entities are expected to select wholly or partially owned structures. x) Finally, the model permits study of the PSC and definition of the costs and tariffs using either the projected costs or only the non-amortized costs. In the first case, the tariffs and consequently the incorporated profits are higher, whereas in the later both will be lower. The net profit (incorporated profits minus tariffs) for uni-entity networks are the same for both cases, however, in a multi-entity network, the net profit in the projected case will benefit entities with major capital participations. Note that depots and pipelines are entity-owned, therefore will incur fixed, operating and unitary costs while maritime, railway and road transportation are third-party services where only unitary costs are defined.

$$
Profit = Max \sum_{t \in T, e \in E} \left(\begin{array}{l} \sum_{i \in I} MR_{i,e,t} - \sum_{i \in I} \sum_{p \in p} CE_{i,p,e,t} - \sum_{j \in J} \sum_{p \in p} CI_{j,p,e,t} \\ - \sum_{p \in P} \left(\sum_{i \in I} CNC_{i,p,e,t} + \sum_{i \in I} CNR_{i,p,e,t} + \sum_{j \in J} CND_{j,p,e,t} + \sum_{k \in K} CNL_{k,p,e,t} \right) \\ + \sum_{k \in K} \sum_{p \in P} ML_{k,p,e,t} - \sum_{h \in H} TDE_{h,e,t} - \sum_{(h,l,m) | \exists \mu_{h,l,m}} TTE_{h,l,m,e,t} - \sum_{k \in K} \sum_{p \in P} CS_{k,p,e,t} \\ + \sum_{h \in H} MDE_{h,e,t} + \sum_{(h,l,m) | \exists \mu_{h.l.m}} MTE_{h,l,m,e,t} \end{array} \right) \quad (3.36)
$$

The objective function in eq. (3.36) maximizes the total PSC profits, which is the sum of individual entity profits, each aggregating thirteen terms: the refinery margins $MR_{i,e,t}$ less the exportation $CE_{i,p,e,t}$ and importation costs $CI_{j,p,e,t}$; less the crude $CNC_{i,p,e,t}$, refinery $CNR_{i,p,e,t}$, depot $CND_{j,p,e,t}$, and retail $CNL_{k,p,e,t}$ inventory costs; plus the retail segment margin $ML_{k,p,e,t}$ minus the transportation tariffs $TTE_{h,l,m,e,t}$, the storage depot tariffs $TDE_{j,t}$ and the product shortage cost $CS_{k,p,e,t}$; plus the entities' participation in the storage depot operator $MD_{j,e,t}$ and the transportation operator $MT_{h,l,m,e,t}$ profits. The objective function maximizes revenues if only M-prefixed terms are included; or minimizes costs if only the C-prefixed and T-prefixed terms are included. The function can accommodate weight parameter λ_e for the different objective terms which could take equal weights, or entity market quotas, or include only one entity, or exclude any entity to study the impact of new entries, closures and fusions. The objective function terms are hereafter described in full detail.

3.5.4.1 Refinery margin (MR)

The refinery products are either traded to the petroleum entities for local consumption or exported when production is surplus. Eq. (3.37) provides the refinery margin which is the difference between refining revenue: product of the refined volume and the difference between the ex-refinery price $\varphi_{p,t}^{a1}$ and the refinery costs $\xi P_{r1,p}^{v=t}$; and the crude costs $QCI_{i,t} \times \varphi_{p2,t}^{a2}$. This margin is directly incorporated by entities based on their participation $\eta_{i,i,e}^{r1}$ in the refinery capital.

$$MR_{i,e,t} = \left(\sum_{p \in P} QR_{i,p,t} \left(\varphi_{p,t}^{a1} - \xi P_{r1,p}^{v=t} \right) - QCI_{i,t} \times \varphi_{p2,t}^{a2} \right) \eta_{i,i,e}^{r1}, \forall i \in I, e \in E, t \in T, a1 = r1 = REFINERY, a2 = p2 = CRUDE \quad (3.37)$$

3.5.4.2 Exportation costs (CE)

Besides production for trading to entities and replenishing inventories for national consumption, refineries may produce for exportation. Eq. (3.38) gives the exportation cost, which is the product of the export volumes and the export tariff and the entity participation in refinery. The export tariff is the cost of insurance and freight (CIF) for transporting the product to the regional wholesale market hub.

$$CE_{i,p,e,t} = QE_{i,p,t} \, \xi P_{r1,p}^{v=t} \eta_{i,i,e}^{r2}, \qquad \forall i \in I, p \in P, e \in E, t \in T, r1 = EXPORT, r2 = REFINERY \quad (3.38)$$

3.5.4.3 Importation costs (CI)

Refineries and depots may import product from refineries external to the network. Besides, refinery yields of product mix and capacity restrictions may imply product importation. Eq. (3.39) provides the importation costs as the product of the import volume and the cost of product, insurance and freight. The latter is the sum of the ex-refinery or wholesale cost of product and the unitary importation tariff that covers the transportation costs from the nearest wholesale market hub.

$$CI_{h,p,e,t} = QI_{h,p,e,t} \times \left(\varphi_{p,t}^{a1} + \xi P_{r1,p}^{v=t} \right), \qquad \forall h \in H, p \in P, e \in E, t \in T, a1 = REFINING, r1 = IMPORT \quad (3.39)$$

3.5.4.4 Inventory costs (CN)

Every stage (procurement, refinery, depot and retail) and location hold inventories to improve production and distribution efficiencies, besides securing safety stocks. Inventory costs per location, entity and product are calculated in eqs. (3.40-3.43) as the product of the inventory tariff γ, the inventory volume and the product stage price. At all stages, the inventory costs are imputed to the product owners with exception to the refinery, where they are imputed as per the entity participation $\eta_{i,e}^{r1}$.

$$CNC_{i,p,e,t} = \gamma \times SC_{i,t} \times \varphi_{p1,t}^{a1} \times \varepsilon_{i,p} \times \eta_{i,e}^{r1}, \qquad \forall i \in I, p \in P, e \in E, t \in T, a1 = p1 = CRUDE, r1 = REFINERY \quad (3.40)$$

$$CNR_{i,p,e,t} = \gamma \times SR_{i,p,t} \times \varphi_{p,t}^{a1} \times \eta_{i,e}^{r1}, \qquad \forall i \in I, p \in P, e \in E, t \in T, a1 = r1 = REFINERY \quad (3.41)$$

$$CND_{j,p,e,t} = \gamma \times SD_{j,p,e,t} \times \varphi^{a1}_{p,t}, \qquad \forall j \in J, p \in P, e \in E, t \in T, a1 = REFINERY \tag{3.42}$$

$$CNL_{k,p,e,t} = \gamma \times SL_{k,p,e,t} \times \varphi^{a1}_{p,t}, \qquad \forall k \in K, p \in P, e \in E, t \in T, a1 = RETAIL \tag{3.43}$$

3.5.4.5 Retail Margin (ML)

The retail margin per customer location, product, entity and period is obtained in eq. (3.44) as the product of satisfied demand and unitary retail value, minus the product supply cost for each customer zone and period. The satisfied demand is the difference between customer demand and unmet demand; the unitary retail value is the difference between the retail price and retail tariff; and the supply cost is the supplied product valued at the product's ex-refinery price.

$$ML_{k,p,e,t} = \left(\delta_{k,p,e,t} - QU_{k,p,e,t}\right)\left(\varphi^{a1}_{p,t} - \xi P^{v=t}_{r1,p}\right) - \sum_{(j,m)\mid\exists\mu_{j,k,m}} QS_{j,k,m,p,e,t} \times \varphi^{a2}_{p,t},$$
$$\forall k \in K, p \in P, e \in E, t \in T, a1 = r1 = RETAIL, a2 = REFINERY \tag{3.44}$$

3.5.4.6 Unsatisfied demand cost (CS)

Lost revenue for unsatisfied demand is accounted for in eq. (3.44), however, eq. (3.45) provides a penalization if the cost minimization objective is considered.

$$CS_{k,p,e,t} = QU_{k,p,e,t} \times \xi P^{v=t}_{r1,p}, \qquad \forall k \in K, p \in P, e \in E, t \in T, r1 = UNMET \tag{3.45}$$

3.5.4.7 Transportation tariff (TTE)

Eq.(3.46) computes the transportation tariff per product entity, transport route, mode and period as the sum of the fixed costs $CTFE_{h,l,m,e,t}$, operating costs $CTOE_{h,l,m,e,t}$ and unitary costs $CTU_{h,l,m,p,e,t}$. These are estimated per entity and marked up by the operator margin α^{a1}. Eqs. (3.46-3.49) are defined only for available transport routes and modes $(h,l,m) \mid \exists \mu_{h,l,m}$.

$$TTE_{h,l,m,e,t} = \left(1 + \alpha^{a1}\right)\left(CTFE_{h,l,m,e,t} + CTOE_{h,l,m,e,t} + \sum_{p \in P} CTU_{h,l,m,p,e,t}\right), \forall h \in H, l \in L, m \in M \wedge (h,l,m) \mid \exists \mu_{h,l,m}, e \in E, t \in T, a1 = TRANSPORT \tag{3.46}$$

The fixed installation cost per product entity and route is determined in eq.(3.47) for the amortization period $t \in T \wedge t \leq MNA$. The expression $\sum_{u \in U} \left(ZT_{h,l,m,e,u}\, \tau c_u\right)$ gives the entity cost threshold coefficient and $\xi R^{v=f}_m + \mu_{h,l,m} \times \xi R^{v=fv}_m$ is the route installation cost which includes the fixed and the distance variable fixed installation cost.

$$CTFE_{h,l,m,e,t} = \sum_{u \in U} \left(ZT_{h,l,m,e,u}\, \tau c_u\right)\left(\xi R^{v=f}_m + \mu_{h,l,m} \times \xi R^{v=fv}_m\right), \qquad \forall (h,l,m) \mid \exists \mu_{h,l,m}, e \in E, t \in T \wedge t \leq MNA \tag{3.47}$$

The operating cost per product entity and route is determined in eq. (3.48) as the product of the entity cost threshold coefficient $\sum_{u \in U} ZT_{h,l,e,m}\, \tau c_u$ and the route operating cost. The later is the sum of the fixed resource operating cost $\xi R_m^{v=o}$ and the distance-based resource operating cost $\mu_{h,l,m} \times \xi R_m^{v=ov}$.

$$CTOE_{h,l,m,e,t} = \sum_{u \in U} ZT_{h,l,m,e,u} \tau c_u \left(\xi R_m^{v=o} + \mu_{h,l,m} \times \xi R_m^{v=ov}\right), \quad \forall (h,l,m) \mid \exists \mu_{h,l,m}, e \in E, t \in T \quad (3.48)$$

The volume dependent cost is given by eq. (3.49), as the product of the transported volume (primary and secondary), the route distance and the transport unitary cost.

$$CTU_{h,l,m,p,e,t} = \left(QP_{h,l,m,p,e,t} + QS_{h,l,m,p,e,t}\right) \times \mu_{h,l,m} \times \xi P_{m,p}^{v=u}, \quad \forall (h,l,m) \mid \exists \mu_{h,l,m}, p \in P, e \in E, t \in T \quad (3.49)$$

3.5.4.8 Storage Depot tariffs (TDE)

Depots initially have no storage, where resource capacities are added as justified by transported volumes. The storage depot tariffs per product entity, depot and period are computed in eq. (3.50), by marking up with the storage margin, the aggregate of the depot fixed and operating costs, resource fixed and operating costs and the depot unitary costs.

$$TDE_{h,e,t} = \left(1 + \alpha^{a1}\right)\left(CDFE_{h,e,t} + CDOE_{h,e,t} + \sum_{fp \in FP}\left(CRFE_{h,fp,e,t} + CROE_{h,fp,e,t}\right) + \sum_{p \in P} CDU_{h,p,e,t}\right), \forall h \in H, e \in E, t \in T, a1 = STORAGE \quad (3.50)$$

The depot fixed installation cost per product entity and depot is calculated in eq. (3.51) during the amortization periods. The expression $\sum_{u \in U} ZD_{h,e,u} \tau c_u$ is the product entity cost threshold coefficient and parameter $\xi R_{r1}^{v=f}$ is the fixed cost per depot.

$$CDFE_{h,e,t} = \sum_{u \in U} ZD_{h,e,u} \times \tau c_u \times \xi R_{r1}^{v=f}, \quad \forall h \in H, e \in E, t \in T, t \leq MNA, r1 = DEPOT \quad (3.51)$$

Likewise, eq. (3.52) gives a depot operating cost per product entity, depot node and period.

$$CDOE_{h,e,t} = \sum_{u \in U} ZD_{h,e} \times \tau c_u \times \xi R_{r1}^{v=o}, \quad \forall h \in H, e \in E, t \in T, r1 = DEPOT \quad (3.52)$$

Eq. (3.53) calculates the entity resource fixed cost per product entity and depot as the product of the installed entity resource partitions $\sum_{fm,r} ZR_{m,h,r,p,e}$ and a resource partition installation cost $\left(\xi P_{r,p}^{v=f} + \xi R_r^{v=f}\right)/NCP$, defined per resource and product.

$$CRFE_{h,fp,e,t} = \sum_{fm,r} ZR_{m,h,r,p=fp,e} \times \left(\xi P_{r,p=fp}^{v=f} + \xi R_r^{v=f}\right)/NCP, \quad \forall h \in H, fp \in FP, e \in E, t \in T, t \leq MNA, \xi P_{r,p}^{v=f} \vee \xi R_r^{v=f} \quad (3.53)$$

Similarly, the entity resource operating cost per product entity and depot is computed eq. (3.54) as the product of the operating entity resource partitions $\sum_{r \in R} \sum_{fm \in FM} ZR_{h,fm,r,fp,e}$ and a resource partition operating cost $\left(\xi P_{r,p}^{v=o} + \xi R_r^{v=o}\right)/NCP$ defined per resource and product.

$$CROE_{h,fp,e,t} = \sum_{r \in R} \sum_{fm \in FM} \left(ZR_{h,fm,r,fp,e}/NCP\right) \times \left(\xi P_{r,p=fp}^{v=o} + \xi R_r^{v=o}\right), \quad \forall h \in H, fp \in FP, e \in E, t \in T, \xi P_{r,p}^{v=o} \vee \xi R_r^{v=o} \quad (3.54)$$

Eq. (3.55) provides the volume dependent cost as the product of the transported volume and a depot unitary cost $\xi P_{r1,p}^{v=u}$.

$$CDU_{h,p,e,t} = \sum_{l \in L} \sum_{m \in M} \left(QP_{h,l,m,p,e,t} + QS_{h,l,m,p,e,t} \right) \times \xi P_{r1,p}^{v=u}, \qquad \forall h \in H, p \in P, e \in E, t \in T, r1 = DEPOT \quad (3.55)$$

Unamortized costs: Product entity tariffs computed in the eqs.(3.46) and (3.50) reflect the projected costs as determined in (3.47)(3.51)(3.53). The tariffs can be lower if only unamortized costs are used as in eqs. (3.56-3.58). Here, the fixed cost per product entity is calculated by eqs. (3.56,3.58) only in the unamortized periods as the product of the difference between installed and existing thresholds $ZT_{h,l,m,e,u} \tau c_u \left(1 - \eta_{h,l,m,e} \sum_{y \in Y} \kappa t_{\mathrm{h,l,m,y}}\right)$ and the resource cost. Note that the second term is subtracted only in the amortized periods $\mathrm{t \le MNA \wedge y + t > MNA}$.

$$CTFE_{h,l,m,e,t} = \sum_{u \in U} \left(ZT_{h,l,m,e,u} \tau c_u \left(1 - \eta_{h,l,m,e} \sum_{y \in Y} \kappa t_{\mathrm{h,l,m\,y}} \right) \right) \left(\xi R_m^{\mathrm{v=f}} + \mu_{h,l,m} \times \xi R_m^{v=fv} \right), \quad (3.56)$$
$$\forall (h,l,m) \mid \exists \mu_{h,l,m}, e \in E, t \in T \wedge \left(\mathrm{t \le MNA} \vee \left(\neg \eta_{\mathrm{h,l,m\,e}} \wedge \mathrm{y + t > MNA} \right) \right)$$

$$CDFE_{h,e,t} = \sum_{u \in U} \left(ZD_{h,e,u} \tau c_u \left(1 - \eta_{h,h,r1,e} \sum_{(y+t) > MNA} \kappa d_{h,y} \right) \right) \xi R_{r1}^{v=f}, \forall h \in H, e \in E, t \in T \wedge \left(\mathrm{t \le MNA} \vee \neg \eta_{\mathrm{h,h,r1,e}} \right), r1 = DEPOT \quad (3.57)$$

$$CRFE_{h,fp,e,t} = \sum_{fm,r} \left(k \times ZR_{h,fm,r,p,e} \right) \left(\xi P_{r,p}^{v=f} + \xi R_r^{v=f} \right) / NCP, \quad (3.58)$$
$$\forall h \in H, fp \in FP, e \in E, t \in T, k = 1 \mid t \le MNA \vee \neg \exists \sum_{fm,r,y1} \kappa r_{h,fm,r,y1,fp} \eta_{h,h,r1,e}, \xi P_{r,p}^{v=f} \vee \xi R_r^{v=f}, r1 = DEPOT$$

3.5.4.9 Transportation operator margin (MTE)

The capital entities consolidate operator profits proportional to their financial participation in transportation infrastructures which are participated modes $m \in MP$. Eq. (3.59) calculates the logistics operator's transportation cost which is the sum of the fixed, operating and unitary costs. Eq. (3.60) calculates the consolidated profits per entity as the product of the difference between the transportation tariffs and the logistics operator's costs, and the entity capital participation $\eta_{h,l,m,e}$ if existing route or the entity market quota ζ_e if new route.

$$CTD_{h,l,m,t} = \left(CTFD_{h,l,m,t} + CTOD_{h,l,m,t} + \sum_{p \in P} \sum_{e \in E} CTU_{h,l,m,p,e,t} \right) \qquad \forall (h,l,m) \mid \exists \mu_{h,l,m}, t \in T \quad (3.59$$

$$MTE_{h,l,m,e,t} = \left(\sum_{e2} TTE_{h,l,m,e2,t} - CTD_{h,l,m,t} \right) \times \left(\eta_{h,l,m,e} + \zeta_e \right), \quad \forall (h,l,m) \mid \exists \mu_{h,l,m} \wedge m \in MP, e \in E, t \in T, \eta_{h,l,m,e} \vee \zeta_e \quad (3.60)$$

The installation of a transport mode between two nodes implies a fixed cost to the logistics operator for new and non-amortized routes as determined in eq. (3.61). For installed routes, the binary expression $IT_{h,l,m} - \sum_{y \in Y} \kappa t_{\mathrm{h,l,m,y}}$ with the condition $t \in T \wedge \mathrm{t \le MNA \wedge y + t > MNA}$ is true in the non-amortized periods. The route installation cost is the sum of the fixed cost and the distance variable fixed installation cost as given by the second expression.

$$CTFD_{h,l,m,,t} = \left(IT_{h,l,m} - \sum_{y \in Y} \kappa t_{\mathrm{h,l,m,y}} \right) \left(\xi R_m^{\mathrm{v=f}} + \mu_{h,l,m} \times \xi R_m^{v=fv} \right), \quad \forall (h,l,m) \mid \exists \mu_{h,l,m}, t \in T \wedge \mathrm{t \le MNA \wedge y + t > MNA} \quad (3.61)$$

Finally, eq. (3.62) determines the logistics operator fixed operating cost per operating route $OT_{h,l,m}$ which is the sum of the route operating cost $\xi R_{m}^{v=o}$ and the route distance variable operating cost $\mu_{h,l,m} \times \xi R_{m}^{v=ov}$.

$$CTOD_{h,l,m,t} = OT_{h,l,m} \times \left(\xi R_{m}^{v=o} + \mu_{h,l,m} \times \xi R_{m}^{v=ov}\right), \qquad \forall (h,l,m) \mid \exists \mu_{h,l,m}, t \in T \tag{3.62}$$

3.5.4.10 Storage depot operator margin

The depot operating fixed costs are given by eq. (3.63) as an aggregation of depot and resources fixed installation and operating costs and depot unitary costs. Eq. (3.64) determines the capital entity consolidated profits, incorporated from the depot margins which is the product of the difference between the depot tariffs $\sum_{e2 \in E} TDE_{h,e2,t}$ and operator costs $CDD_{h,t}$, and the entity capital participation $\eta_{h,h,r='DEPOT,e}$ if existing depot or the entity market quota ζ_e if new installation.

$$CDD_{h,t} = CDFD_{h,t} + CDOD_{h,t} + \sum_{fp \in FP} \left(CRFD_{h,fp,t} + CROD_{h,fp,t}\right) + \sum_{p \in P} \sum_{e \in E} CDU_{h,p,e,t} \qquad \forall h \in H, t \in T \tag{3.63}$$

$$MDE_{h,e,t} = \left(\sum_{e2 \in E} TDE_{h,e2,t} - CDD_{h,t}\right) \times \left(\eta_{h,h,r1,e} + \zeta_e\right) \quad \forall h \in H, e \in E, t \in T, \eta_{h,h,r,e} \vee \zeta_e, r1 = DEPOT \tag{3.64}$$

Eq. (3.65) calculates the depot fixed cost for the logistics operator where the binary expression $ID_h - \sum_{y \in Y} \kappa d_{h,y}$ amounts to 1 for new depots as well as for existing depots during the non-amortized periods. Parameter $\xi R_{DEPOT}^{v=f}$ defines a depot fixed installation cost.

$$CDFD_{h,t} = \left(ID_h - \sum_{(y+t) > MNA} \kappa d_{h,y}\right) \times \xi R_{r1}^{v=f}, \qquad \forall h \in H, t \in T \wedge t \le MNA, r1 = DEPOT \tag{3.65}$$

Eq. (3.66) provides the logistics operator's fixed operating cost per operating depot and period.

$$CDOD_{h,t} = OD_h \times \xi R_{r1}^{v=o}, \qquad \forall h \in H, t \in T, r1 = DEPOT \tag{3.66}$$

Eq. (3.67) computes the logistics operator's fixed resource cost per depot, product and period, as the product of the number of resources $IR_{h,fm,r,fp} - \sum_{(y+t)>MNA} \kappa r_{h,fm,r,y,fp}$ installed or existing in the non-amortized periods $t \in T \wedge t \le \mathrm{MNA} \wedge y + t > \mathrm{MNA}$ and the resource fixed cost $\xi P_{r,fp}^{v=f} + \xi R_{r}^{v=f}$ defined per resource or product.

$$CRFD_{h,fp,t} = \sum_{fm,r} \left(IR_{h,fm,r,fp} - \sum_{(y+t)>MNA} \kappa r_{h,fm,r,y,fp}\right) \times \left(\xi P_{r,fp}^{v=f} + \xi R_{r}^{v=f}\right), \forall h \in H, fp \in FP, t \in T \wedge t \le MNA, \xi P_{r,fp}^{v=f} \vee \xi R_{r}^{v=f} \tag{3.67}$$

Finally, eq. (3.68) gives the logistics operator's resource operating cost per depot, product and period which is the product of the number of operating resources $OR_{h,fm,r,fp}$ and the resource operating cost $\xi P_{r,p}^{v=o} + \xi R_{r}^{v=o}$ defined per resource or product.

$$CROD_{h,fp,t} = \sum_{fm \in FM, r \in R} OR_{h,fm,r,fp} \left(\xi P_{r,p}^{v=o} + \xi R_{r}^{v=o}\right), \quad \forall h \in H, t \in T, fp \in FP, \xi P_{r,fp}^{v=o} \vee \xi R_{r}^{v=o} \tag{3.68}$$

3.5.5 Model Reduction

Depot location candidates: As location problems are large-scale, a p-median concentration set is constructed as per Rosing and ReVelle,[24] to reduce the problem size. A list of depot candidates is created in descending order of the customer demand within distance RCD divided by the road transport distance. The top NCD depots then comprise the depot concentration set as presented in eq. (3.69).

$$\forall\left(n \in N \wedge descending\ order\ of \sum_{k \in K; \mu_{n,k,m='ROAD'} < RCD} \left(\frac{\delta^{s}_{k,p,e,t}}{\mu_{n,k,m}} \right) \right)$$
$$\left\{ \begin{array}{l} If\left((LCD <= NCD) or \left(\sum_{p} \rho_{n,p} > 0 \right) or \left(\sum_{m,p,y} \kappa^{a}_{m,n,p,y} > 0 \right) \right) \\ \left\{ \begin{array}{ll} J = N; & !*add\ depot \\ LCD = LCD + 1; & \end{array} \right\} \\ else \\ \{; \qquad !*drop\ depot\} \end{array} \right\} \tag{3.69}$$

Secondary route candidates: Similarly, a p-median concentration set of potential secondary routes is constructed for each customer location. The secondary route concentration set is formed as a list of depot routes within RCD distance from each customer location. The heuristic is produced below in (3.70).

$$\forall (m \in M; l \in L; h \in H) \left\{ \begin{array}{l} If \left(m \in \{'RAIL','MARITIME'\} \vee \left(m = 'ROAD' \wedge \mu_{j,k,m} \leq RCD \right) \right) \\ \quad \left\{ \mu_{j,k,m} = \mu_{j,k,m}; \ !*add\ \sec ondary\ route \right\} \\ else \left\{ \mu_{j,k,m} = 0; \qquad !*drop\ \sec ondary\ route \right\} \end{array} \right\} \tag{3.70}$$

Route suppression: Route distances between any two nodes per transportation mode are obtained forefront. Similarly, distance estimates for new pipelines are computed based on GPS coordinates. Constraints (3.71) suppress inexistent routes having zero distances $\mu_{n,o,m} = 0$.

$$OT_{n,o,m} = IT_{n,o,m} = 0, \qquad \forall n \in N, o \in O, m \in M, \wedge \mu_{n,o,m} = 0 \tag{3.71}$$

Transportation stage, mode and product restrictions: The installation of a transportation mode on a route, does not imply that all products use this transport mode. Parameter $\upsilon^{a}_{m,p}$ defines the maximum allowed transport distance per transport stage, mode and product. Road transportation, for instance, is not permitted for primary distribution, while pipeline transportation for non-heated fuel-oil is limited to small distances. Similarly, secondary transportation by pipeline or railway is only permitted for transporting Jet fuel to airports. Either situation can be modelled with distance bounds per product for primary and secondary distribution as in constraints (3.72) and (3.73).

$$QP_{h,j,m,p,e,t} = 0 \qquad \forall h \in H, j \in J, m \in M, p \in P, e \in E, t \in T, \wedge\left(\mu_{h,j,m} > \upsilon_{m,p}^{PRIMARY}\right) \quad (3.72)$$

$$QS_{j,k,m,p,e,t} = 0 \qquad \forall j \in J, k \in K, m \in M, p \in P, e \in E, t \in T, \wedge\left(\mu_{j,k,m} > \upsilon_{m,p}^{SECONDARY}\right) \quad (3.73)$$

Maritime, rail and road distances in kilometres are obtained using the Google API whereas distances for new pipelines are calculated using the Weisstein[21] Big Circle formula given in eq. (3.74). The transport duration in minutes $\mu'_{h,l,m}$ is given by eq. (3.75) where $\mu_{h,l,m}$ is the distance.

$$\mu_{h,l,PIPELINE} = 6371.01\,Cos^{-1}\left(Sin\left(NR \times \Gamma_h^T\right)Sin\left(NR \times \Gamma_l^T\right) + Cos\left(NR \times \Gamma_h^T\right)Cos\left(NR \times \Gamma_l^T\right)Cos\left(NR \times \left(\Gamma_h^G - \Gamma_l^G\right)\right)\right) \quad (3.74)$$

$$\mu'_{h,l,PIPELINE} = \mu_{h,l,PIPELINE} \times MV_m \quad (3.75)$$

Reduction of the model size and execution time are thus achieved through pre-processing for large-scale instances. The response of the MILP model to real-world situations based on the Portuguese PSC is now studied in the following sections presenting results, insights and new research directions.

3.6 The Portuguese Real Case

The MILP presented above is now validated using a real case network based on the downstream PSC in Portugal for which we construct a comprehensive parameter database as detailed in table 3.1. The current PSC network is presented in figure 3.4 where two refineries import crude oil and produce the derived products for the local market consumption. Product may be exported or imported as required. The refined product is then commercialized by four petroleum entities operating in Portugal, which distribute the product through eleven existing storage depots to customers which may be arranged into eighteen districts using their capitals as reference points. Here the GPS coordinates are crucial to identify available land routes and known distances or compute possible new routes. The commercial entity market shares are known per product and these include the smaller companies and hypermarket demands which are also cumulatively supplied by them. The transportation network is divided into two: the primary distribution that delivers product from refinery to depots and the secondary distribution responsible for the delivery of product from depots to the final customer which are airports, petrol stations, factories and utilities. Distant districts may receive product across the frontier from Spain.

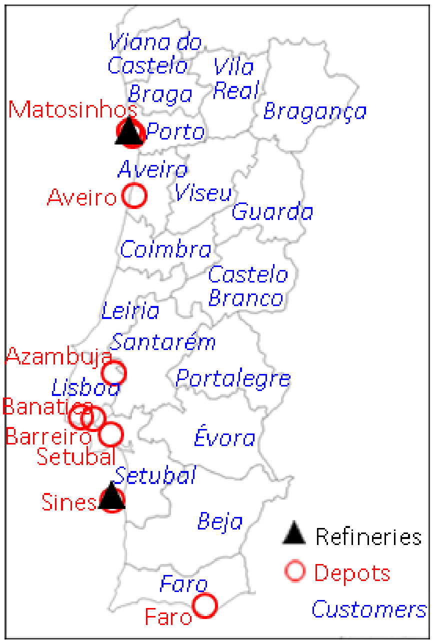


Figure 3.4. The Portuguese PSC

The present study focusses on the distribution of 8 oil products in Portugal (C3, C4, U5, U8, GO, FU, A1, B5), including liquefied petroleum gases (LPG – C3 and C4). Transportation is undertaken by pipeline, maritime or railway for the primary distribution and by road for the secondary distribution. JET A1 is however an exception where pipeline or railway transportation may be used. Crude oil purchases and refinery production is planned in accordance to refinery capacities. Excess production is exported and deficit products are imported to fulfill the market requirements. Particularities relating to the LPG bottle-filling and the archipelagos of Madeira and Azores have not been included in this study.

The MILP parameters are a statistical database, specifically constructed from the quantitative data published by the economics ministry of Portugal through the Direcção Geral de Energia e Geologia (DGEG, 2015), Instituto Nacional de Estatística (INE, 2014) and Autoridade de Concorrência (AdC 2014,2015); and the European performance indicators in the European Commission's Directorate-General for Energy (Eurostat, 2015) and Purvin & Gertz Inc. (2014). While Google Maps application program interfaces (Google Inc., 2014) provide the road, rail and maritime transportation distances, the pipeline distances are calculated using the GPS coordinates and the big circle formula (Weisstein, 2014).

3.7 Real case parameter data

As the current model is an improvement of the individualistic model presented in chapter 2, both chapters use the same test data for the purpose of comparison. The parametric data covers a twelve year period, collected from Portuguese and European statistics, encompassing the period 1999 to 2010, currently being the most recent available. Due to the enhancement of this model, the structure of some parameter tables have evolved while new ones have also been created. Table 3.1 synthesizes the model parameters used in the collaborative MILP which represent some changes when compared to those presented in chapter 2 as well as the new tables B.9, B.12 and B.25 to B.29 created to support the model improvements presented earlier. These are the basis of the results presented hereafter and are provided as Appendix B.

Table 3.1. Model Parameters

Parameter[a]	comment
A.1	product characteristics such as density for mass to volume conversions and vessel dimensions
A.2	demand per product, customer district and period in cubic meters
A.3	market quotas per entity used to calculate demand volumes per entity
A.4	crude processing capacities per day per installed refinery in cubic meters
A.5	refinery typical yields of product mix per unit of crude oil
A.6	entity ownership percentages per refinery, storage depots and transportation infrastructure
A.7	paid amortization periods for current depot infrastructures
A.8	paid amortization periods for current transportation infrastructures
A.9[a]	installed resource units per location, transport mode family, resource, product family and paid amortization periods; replaces tables A9, A10 and A11 from Fernandes *et al.* (2013)
A.12[a]	resource typical capacities in cubic meters given as volume capacity for static resources or flow rate per hour for continuous resources, defined per transport mode family, resource and product family; replaces tables A12, A13 and A14 from Fernandes *et al.* (2013)
A.15	vessel rotations per product
A.16	product independent resource costs typified as fixed, fixvar (fixed variable), operating, opervar (operating variable) and unitary
A.17	product dependent resource costs
A.18	price per product at the exit of each activity stage
A.19	maximum distance allowed per distribution stage, transportation mode and product
A.20	rail distances (in kilometers between refinery, depot and customers)
A.21	pipeline distances (”)
A.22	maritime distances (”)
A.23	road transport distances (”)
A.24	GPS latitude and longitude per refinery, depot and customer location
Parameter[b]	comment
B.9[a]	installed resource units per location, transport mode family, resource, product family and paid amortization periods; replaces tables A.9, A.10 and A.11 from Fernandes *et al.* (2013)
B.12[a]	resource typical capacities in cubic meters given as volume capacity for static resources or flow rate per hour for continuous resources, defined per transport mode

	family, resource and product family; replaces tables A.12, A.13 and A.14 from Fernandes *et al.* (2013)
B.25[a]	product families
B.26[a]	transport mode families
B.27[a]	transfer volume thresholds
B.28[a]	tariff linearization coefficients
B.29[a]	transportation mode capacity characteristics

[a] Table parameters in reference to those given in chapter 2. [b] Table parameters newly created in chapter 3.

The following section presents the performance results of the dynamic and collaborative MILP, comparing it with respect to the earlier static and individualistic model.

3.8 Case-Study Results

The MILP model is implemented using GAMS 24.1.3 64-bit and solved with MILP commercial solver CPLEX 12.5.1 using 4 parallel threads on a dual-core Pentium 2.4 Ghz CPU with 8GB RAM. The stopping criterion is 0 % optimality gap or 24 hours of CPU time.

The MILP models six different possible networks for the Portuguese PSC maximizing profits for each setting. Current, grassroots and retrofit network designs are generated, first modelled as uni-entity networks where all resources and product demand are assumed to be owned by a single entity; and subsequently as multi-entity collaborative networks where the resources owned by various entities are shared. Here, the current network selects eleven existing depots while the grassroots and retrofit networks consider twenty-six possible locations - the current depot locations plus the district capitals. The model simplification only considers node to node affectations below 250 kilometer distances. The project is amortized in 12 years each encompassing 12 month periods of 22 days with 16 hours. A 15% margin is considered per activity, with 5% WACC, where the operating cost is 20% of the installation cost, and each resource is divided into 10 capacity partitions for price linearization. In the current network, only existing depots, capacities and routes are operated, however transfer volumes may be freely assigned. The grassroots provides an optimal network design of depots and transportation routes beginning from scratch. The retrofit design allows the opening of new and expansion and closure of existing network depots, routes and capacities. This study is based on demands and prices over twelve years from 1999 to 2010 (latest available at the time).

Table 3.2 presents the model statistics for the real case example. The number of equations, continuous variables and CPU time for the multi-entity networks are about four times that of the uni-entity networks. CPU time increases from current, to retrofit to grassroots design. The MILP total PSC profits are higher in the uni-entity networks than in the multi-entity counterparts. These improve from the current to grassroots by 2.5% and to retrofit design cumulatively by 7% under both uni-entity and multi-entity collaborative networks.

Table 3.2. Performance results

Network	Design	Equations	Variables	Discrete	Objective	Gap (%)	CPU (s)
Uni-entity	Current	177667	235145	5484	11.047×10^9	0	10
	Grassroots	186494	237133	7472	11.312×10^9	0	44688
	Retrofit	186302	237106	7445	11.834×10^9	0	69705
Multi-entity	Current	586429	830927	19944	11.046×10^9	0	277
	Grassroots	595256	832915	21932	11.426×10^9	1.6	86422
	Retrofit	595064	832888	21905	11.882×10^9	0.43	86515

3.8.1 The PSC network strategic decisions

Figure 3.5 shows the selected depots and the operated primary transportation between refinery and depots in the uni and multi-entity MILP designs for current, grassroots and retrofit networks. The depots Aveiro, Azambuja, Faro, Matosinhos, Setubal and Sines are opened and operated in all designs. Banatica, Barreiro, Leixões, Perafita and Real are operated only in the current networks, hence closed in the grassroots and the retrofit networks. No new depots are opened thereby confirming some existing locations and simultaneously identifying their importance as they are selected in all networks. Excess existing capacity is evidenced for the Portuguese PSC as the less efficient depots are closed.

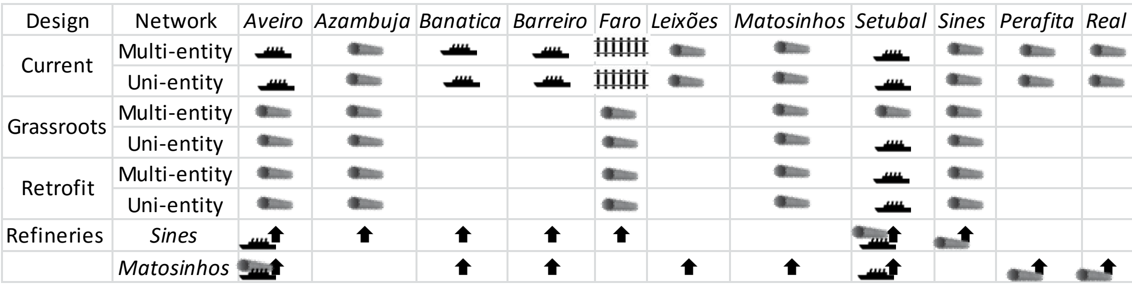


Figure 3.5. PSC depots operated by MILP with transportation routes (rail, maritime, pipeline)

Rail transport is selected between Sines and Faro for the current networks. Maritime transport is selected between Matosinhos or Sines to Aveiro, Banatica, Barreiro and Setubal in both current networks and to Setubal in both retrofits and uni-entity grassroots. Pipelines transfer product from Matosinhos refinery to Leixões, Matosinhos, Perafita, Porto and Real depots in both current networks, and from Sines refinery to Azambuja and Sines in all networks and to Setubal in the multi-entity grassroots. Pipeline transportation is also selected from Matosinhos refinery to Aveiro and Matosinhos, and from Sines refinery to Azambuja and Sines in the grassroots and retrofits. Besides, Jet A1 is supplied by pipeline from Azambuja to Lisboa and Sines to Faro in the two retrofits and uni-entity grassroots and from Matosinhos to Porto in all networks except multi-entity grassroots.

Table 3.3 presents the objective function components, including the total profits, refinery margin, exportation and importation costs, gross retail margin, depot and transportation tariffs, margins and fixed and total costs for each network design for the twelve-year period.

Table 3.3. Value contribution per network in millions of monetary units

Design	Network	Total profits	Refinery margin	Export costs	Retail margin	Transport tariffs	Depot tariffs	Import costs	Transport margin	Depot margin	Resource fixed costs	Depot fixed costs	Transport fixed cost	Depot costs	Transport costs
Current	Multi	11,046	4,912	247	10,435	2,967	1,888	171	200	1,081	21	115	0	807	2,450
	Uni	11,047	4,912	248	10,434	2,877	1,318	171	111	511	21	115	0	807	2,449
Grassroots	Multi	11,246	4,912	223	12,114	3,217	1,888	853	156	556	240	435	184	1,332	2,757
	Uni	11,312	4,912	216	12,262	3,135	1,451	884	36	99	246	435	147	1,352	2,777
Retrofit	Multi	11,831	4,912	216	12,275	3,260	2,012	878	208	1,116	107	115	91	897	2,730
	Uni	11,834	4,912	215	12,296	3,151	1,463	891	94	567	106	115	91	896	2,735

The gross retail margin is the difference between the retail price and the ex-refinery price and retail station cost, excluding the transportation and storage costs which are shown separately and are influenced by the expedited volumes. Non-tangible costs such as the entity publicity and trading costs are not considered. The total profits and contributing margins, resulting from the collaborative design, operation and inventory holding, are significantly improved as compared to our earlier model. Total profits and retail margins are seen to improve from current to grassroots to retrofit designs. Uni-entity profits and retail margins are marginally better than multi-entity profits. Importation and exportations costs are lowest respectively for the current and retrofit designs. Depot and transportation costs increase between the current to retrofit, being the highest in grassroots design due to the amortization of new installations. Transportation tariffs are observed to be higher than depot tariffs. Tariffs and consequently margins are highest in the retrofit designs being maximum for the multi-entity network. In conclusion, retrofit design obtains the best result, at the expense that the grassroot designs require total investment in infrastructures.

Table 3.4 presents the total refined, primary, secondary, exported and imported product and crude volumes, inventories and unsatisfied demand for the twelve-year period. Multi-stage and periodic crude oil and product inventories are generated by the MILP in all designs evidencing that considering their utilization improves the network efficiency.

Table 3.4. Volumes per network in millions of cubic meters

Design	Network	Refined product	Primary distribution	Secondary distribution	Exported product	Imported volumes	Crude inventory	Refinery inventory	Depot inventory	Retail inventory	Unsatisfied demand
Current	Multi	195.331	99.266	100.261	96.961	0.995	3.772	0.317	1.139	18.497	48.834
	Uni	195.331	99.242	100.237	96.985	0.995	3.772	0.447	1.009	18.497	48.858
Grassroots	Multi	195.331	113.216	118.332	87.232	5.116	3.772	0.511	0.722	18.829	30.763
	Uni	195.331	115.939	121.251	84.704	5.312	3.772	0.403	0.442	18.993	27.844
Retrofit	Multi	195.331	116.121	121.432	84.522	5.311	3.772	0.219	1.061	19.014	27.663
	Uni	195.331	116.265	121.621	84.422	5.356	3.772	0.511	0.754	18.975	27.474

Uni-entity networks in general outperform the multi-entity networks. Imported volumes increase from current to grassroots and retrofit designs whereas the depot inventories decrease. With stable production, the exported volumes and unsatisfied demand which are highest in the current network decrease respectively 13% and 45% for both grassroots and retrofit designs with a consequent increase in primary and secondary product volumes. As current refinery capacities are used which have expanded from the initial years when product demand was low, hence the MILP generated export volumes are high.

3.8.2 Resource installation and capacities at depots

The installation of transportation modes at depots implies the installation of resources which may or may not be product specific. Figure 3.6 shows the product independent resources installed per depot per network design. All designs show single maritime docks at Matosinhos and Sines while the current networks show a railway station at Sines. Two Pump16 and interfaces are installed at Sines and Matosinhos in all networks, which contrasts with the current network at Sines where three are installed. Finally, a Pump16 and interface is installed at Azambuja in the grassroots and retrofit networks.

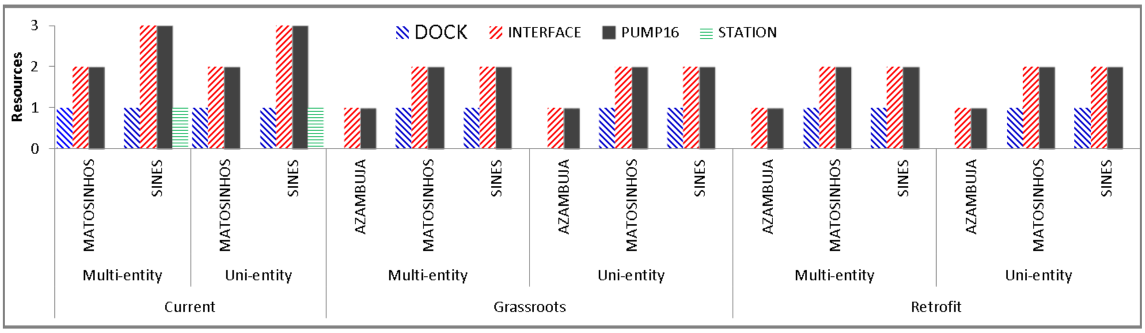


Figure 3.6. Number of product independent resources per network design and depot

Vessels and loading arms are product dependent resources whose operating numbers per depot for the six networks are presented in Figure 3.7. The number of resources to operate in each case is determined by the product transfer volumes, resulting in their installation and operation costs. The number of operating arms per depot and per product, which is larger than that of vessel units in the current networks, is reduced in the retrofit and grassroots network, thereby determining cost savings. In current networks, all installed resources are selected for operation hence uni and multi-entity network operating resources are equal. However, the grassroots and retrofit designs show fewer depots with a higher concentration of operating resources which are greater in multi-entity when compared to uni-entity networks. Curiously the same depots are selected for operation in these four networks but the retrofit designs operate more resources when compared to the grassroots designs. The resulting depot configurations show that the MILP generates depot capacities closer to the client locations as in the case of fuel oil at Setubal, driven by higher costs of secondary distribution. Another positive effect in the MILP designs is that the depots operate more products. Hence, there is a natural shift of product capacities from current depots having fewer products to full-fledged depots offering the majority of products.

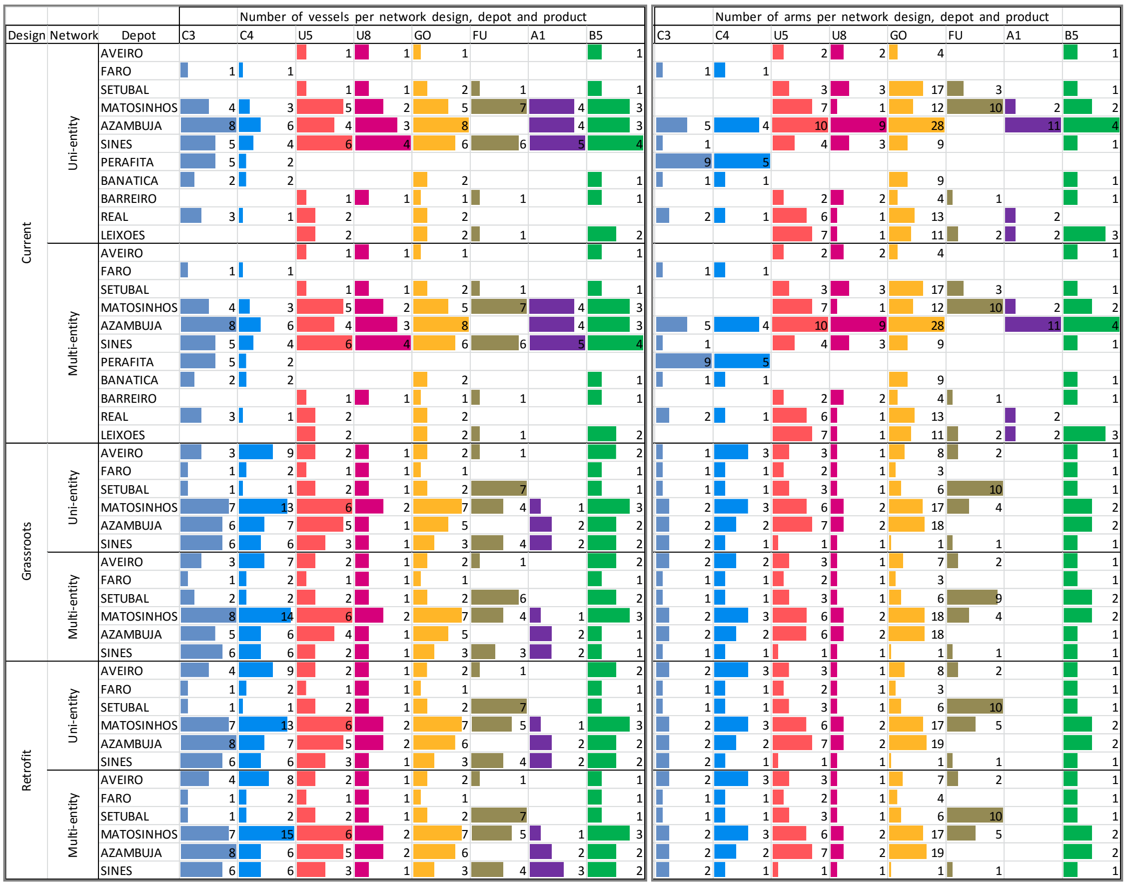


Figure 3.7. Product dependent resources per network design and depot

Figure 3.8 presents the tank trucks operating per product, depot and transport route for the uni-entity retrofit. Significant changes are also observed in truck numbers for the remaining five designs, the reason being the differences in product volumes and the determination of truck numbers per entity. Two shifts of 7 hours are considered per day with 30 minutes expedition and unloading times. The transport duration per trip is also known between any two nodes. The number of tank trucks is observed to be higher for GO, followed by U5 and FU. However, Setubal is an exception where FU supersedes GO. No trucks are present for A1 in the uni-entity retrofit, as pipeline transport is selected between Azambuja and Lisboa.

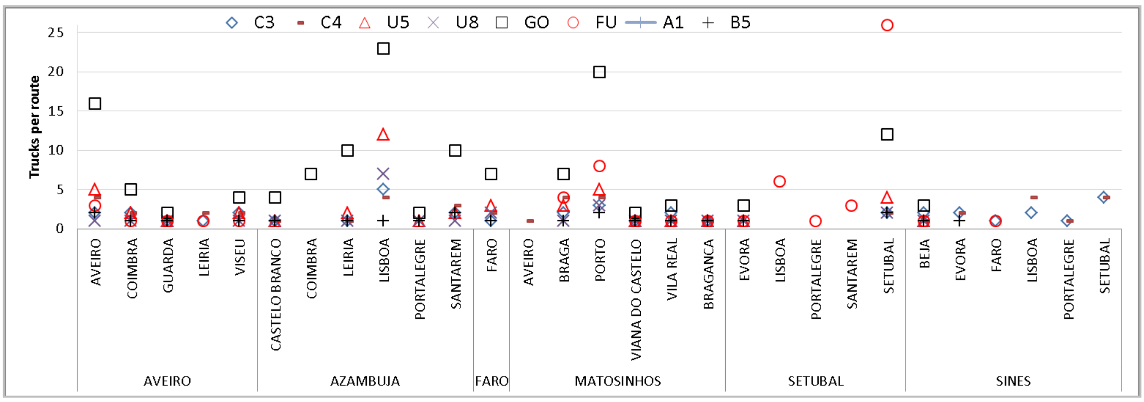


Figure 3.8. Operating trucks per product, depot and route for uni-entity retrofit

3.8.3 Product Volumes and Inventories at depots

Figure 3.9 presents the volumes expedited per depot and entity for the six networks. Aggregates of multi-entity volumes at each depot are observed to approximate the uni-entity network volumes. Entity utilization of currently operating depots is uneven as compared to the grassroots and retrofit designs where these volumes are fairly distributed among depots. Besides, volumes are not restricted to entity participations in depots where entities use non-participated depots in all network designs, hence denoting an improvement in collaboration as compared to our earlier model. While depots Banatica, Barreiro, Leixões, Perafita and Real have low volumes in the current networks, they are closed in the grassroots and retrofit designs. Volumes from Banatica and Barreiro are reallocated to Setubal. Likewise, the volumes from Leixoes, Perafita and Real are re-affected to Matosinhos. The highest volumes are observed at the Azambuja depot followed by Matosinhos, Setubal, Aveiro and Faro. Expedited volumes are comparable between the grassroots and retrofit in the uni-entity, however, they increase considerably in the multi-entity retrofit.

		Multi-entity					Uni-entity
		A	B	C	D	Total	
Current	AVEIRO	3502	1139	1378	705	6723	6723
	AZAMBUJA	17349	8223	7680	3978	37229	37229
	BANATICA	338	97	125	109	669	669
	BARREIRO	770	608	619	160	2157	2135
	FARO	98	47	45	23	212	212
	LEIXOES	203	684	5520	3026	9433	823
	MATOSINHOS	13150	1448	599	951	16148	26222
	PERAFITA	272	224	159	91	747	747
	REAL	3006	6008	1453	79	10546	9058
	SETUBAL	4792	1949	1886	1190	9818	9839
	SINES	2988	1501	1396	696	6580	6580
Grassroots	AVEIRO	5863	3547	3446	1901	14757	16216
	AZAMBUJA	17100	7714	6802	3671	35287	37651
	FARO	2616	1171	1173	586	5546	5631
	MATOSINHOS	18481	7434	7594	3676	37185	36097
	SETUBAL	8740	4421	4937	2302	20400	20277
	SINES	2372	1116	1069	601	5158	5377
Retrofit	AVEIRO	7602	3348	3303	1674	15927	16352
	AZAMBUJA	17580	8631	8159	4252	38622	38685
	FARO	2597	1239	1173	610	5619	5631
	MATOSINHOS	16829	7889	7555	3941	36215	35907
	SETUBAL	9116	4326	4119	2151	19712	19679
	SINES	2456	1118	1113	652	5338	5367

Figure 3.9. Volumes expedited in thousands of cubic meters per depot and entity for each network design.

Figure 3.10 shows the MILP generated inventories per design, depot, product and entity in each period of the study. Considerable differences are observed between current, grassroots and retrofit inventories where the sum of multi-entity inventories (A,B,C,D) approximate the uni-entity inventories. Inventories vary between the network designs, being very low in 1999, 2000, 2006 and 2007, are absent for grassroots and retrofits, and inexistent for 2001, 2005 and 2010. This is due to the hike in product ex-refinery prices, following the world trade center bombings in 2001 and the Iraq war in 2005 which was followed by a drop in prices in the next year, and finally due to disinvestment in inventories in the closing year. Grassroots and retrofit inventories are generated in similar years whereas the current network inventories are spaced. Inventories are generated for all depots for the grassroots and retrofits except for Setubal and some periods. In the current networks, inventories are not generated for Barreiro, Leixoes, Matosinhos, Setubal and Sines, and year 2009. Inventories are higher for Matosinhos and Azambuja and involve more products. Additionally, the inventory volumes are higher for entity A and product GO in consonance with the higher demand.

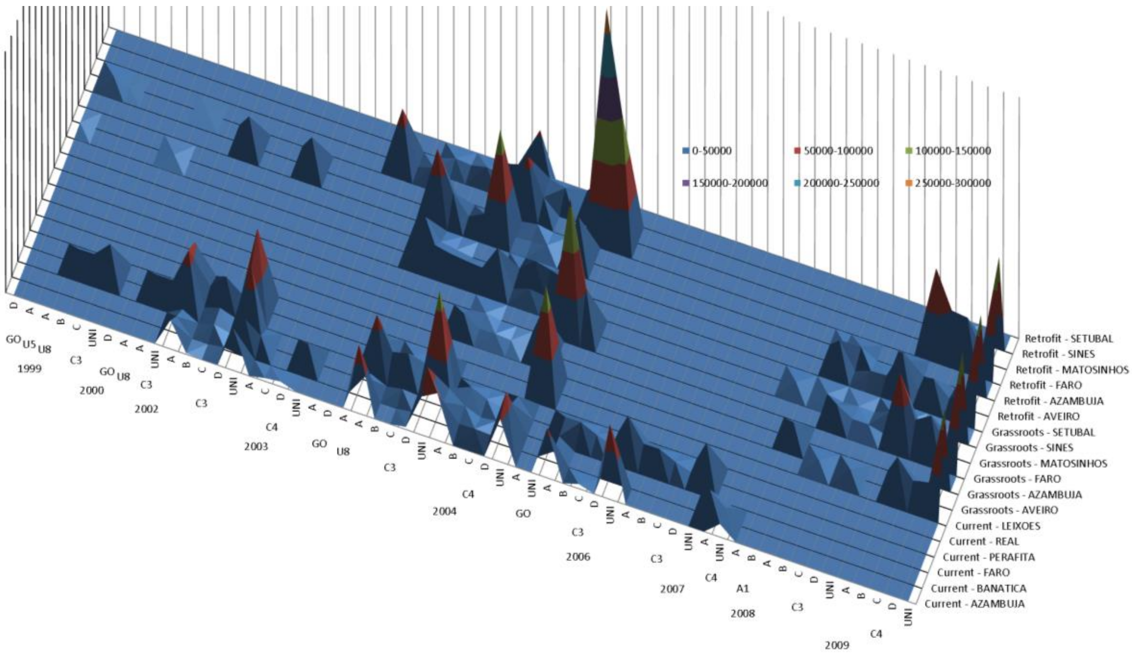


Figure 3.10. Inventories in cubic meters at depots per design, year, product and entity.

3.8.4 Transportation and depot profits

When analysing the obtained results, figure 3.11, it can be seen that the operator and entity profits increase from grassroots to current, being highest for retrofit designs. Uni-entity operator profits are lower when compared to multi-entity. They are highest for the Sines-Azambuja pipeline in the current and retrofit networks as they transport higher volumes and besides are already amortized. The Sines-Setubal maritime profits increase in both retrofits and the grassroots uni-entity networks as volumes increase. The new Azambuja-Lisboa pipeline in the grassroots and retrofit designs and the Sines-Faro pipeline in the uni-entity networks yield negative profits due to the transported volumes and cost structure that take longer to breakeven. However, the multi-entity networks generate profits as the tariffs are higher due to the piecewise linearization. Operator profits and losses are seen to be distributed among the installation owner entities.

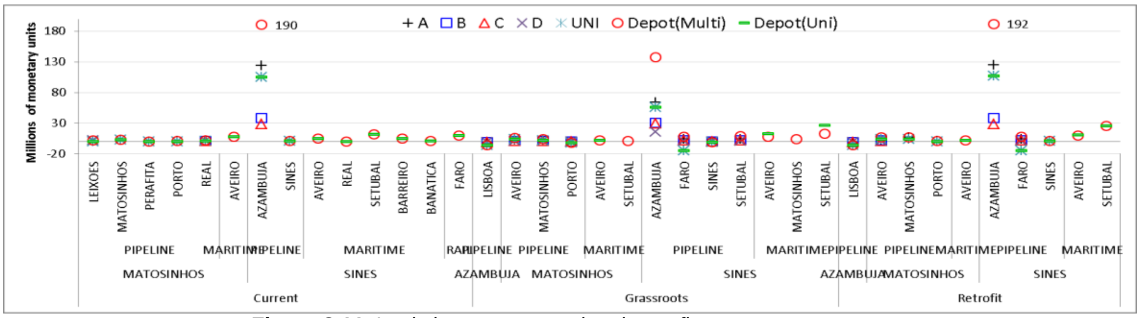


Figure 3.11. Logistics operator and entity profits per transport route

Figure 3.12 presents the depot operator and entity profits per network design. Multi-entity profits are higher than uni-entity profits due to the piece-wise linearization of tariffs which cumulatively have higher pricing thresholds. Individual profits are highest for the major and older depots namely Azambuja, Matosinhos and Sines and tend to be negative in the newer low-volume depots namely Faro, Barreiro and Aveiro, being lower in the uni-entity networks. The profits increase from the grassroots designs to current and are highest for retrofits as most depots are already amortized besides concentration of depots leads to higher volumes. Losses are observed for Faro in the grassroots due to low volumes and unpaid amortizations and for Barreiro in the retrofit design, due to its closure prior to amortization. Entity profits are observed to follow their participation at each depot. Being fully amortized, depot Leixoes shows considerable profit for operator and entity D in the current multi-entity network due to volumes expedited to other entities. However, it is closed in the collaborative grassroots as the operator and entities would have to bear the cost of opening a new depot. It is also closed in the retrofit as the profits of Matosinhos rise substantially with its closure driving the total PSC profits much higher than if Leixoes were to stay open. Although negative profits are observed at some depots, the total PSC profits are improved.

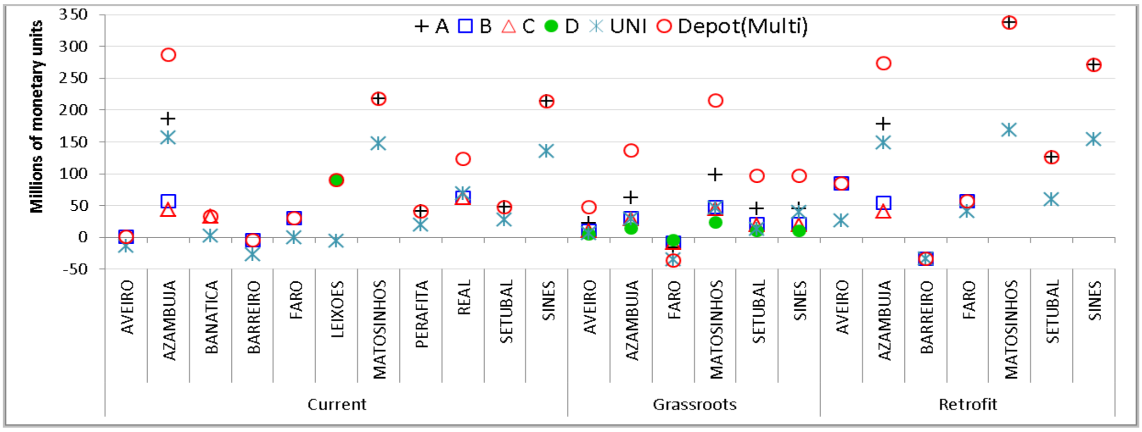
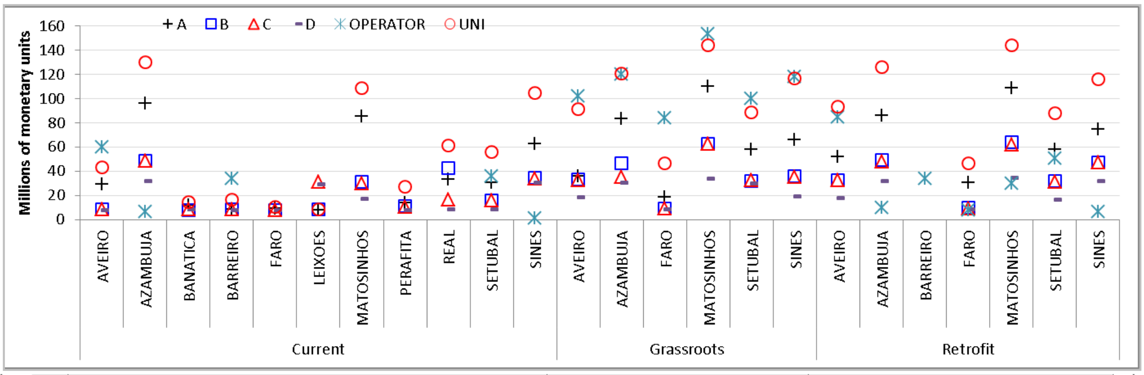


Figure 3.12. Logistics operator and entity profits per depot and network design

3.8.5 Piecewise linearized Costs and Tariffs

Figure 3.13 compares the depot thresholds and average depot fixed costs per entity and network for the twelve year period. The storage and transportation fixed costs vary significantly based on the transported volumes, thereby incorporating economies of scale into the entity tariffs. The entity threshold coefficients should totalize 100% or the depot must be significantly amortized to be sustainable. Currently, the lowest aggregate is observed for Faro contrasting with a 100% aggregate for Azambuja, Matosinhos and Sines in the uni-entity network, and for Setubal and Real in the multi-entity network. Note that operators and not entities bear costs at Barreiro due to its closure. The multi-entity threshold aggregates and consequently entity fixed costs are greater than the uni-entity thresholds and fixed costs. Depots are less numerous and thresholds cumulatively lower in the grassroots and retrofit, proving their efficiency to the current depots. With the exception of Aveiro, Azambuja, Barreiro and Setubal whose infrastructures are not yet amortized, the operator fixed costs are inexistent for the current network. These increase for the retrofits as depot capacities are expanded, and are highest in the grassroots where depots are installed from scratch. Hence, the current network has the biggest difference between operator and aggregate entity costs.



Thresholds (Current)

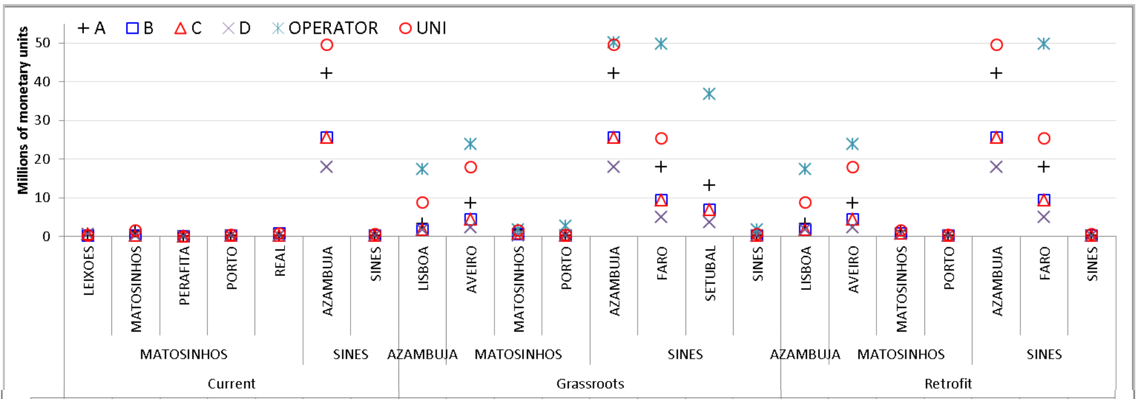
	AVEIRO	AZAMBUJA	BANATICA	BARREIRO	FARO	LEIXOES	MATOSINHOS	PERAFITA	REAL	SETUBAL	SINES
A	0.36	0.91	0.1	0.1	0.1	0.1	0.91	0.1	0.36	0.36	0.64
B	0.1	0.51	0.1	0.1	0.1	0.1	0.36	0.1	0.51	0.19	0.36
C	0.1	0.51	0.1	0.1	0.1	0.36	0.36	0.1	0.19	0.19	0.36
D	0.1	0.36	0.1	0.1	0.1	0.36	0.19	0.1	0.1	0.1	0.36
UNI	0.51	0.99	0.1	0.19	0.1	0.1	0.99	0.19	0.64	0.64	0.96

Thresholds (Grassroots and Retrofit)

	AVEIRO	AZAMBUJA	FARO	MATOSINHOS	SETUBAL	SINES	AVEIRO	AZAMBUJA	BARREIRO	FARO	MATOSINHOS	SETUBAL	SINES
A	0.36	0.84	0.19	0.99	0.64	0.64	0.51	0.84	0	0.36	0.99	0.64	0.75
B	0.36	0.51	0.1	0.64	0.36	0.36	0.36	0.51	0	0.1	0.64	0.36	0.51
C	0.36	0.36	0.1	0.64	0.36	0.36	0.36	0.51	0	0.1	0.64	0.36	0.51
D	0.19	0.36	0.1	0.36	0.36	0.19	0.19	0.36	0	0.1	0.36	0.19	0.36
UNI	0.84	0.99	0.51	0.99	0.91	0.99	0.84	0.99	0	0.51	0.99	0.91	0.99

Figure 3.13. Depot thresholds and fixed cost per entity and logistics operator.

Figure 3.14 presents the twelve-year average transportation route thresholds and the fixed costs per entity and network design. These thresholds are instrumental to affect the logistics operator pipeline fixed costs, to uni and multi-entities tariffs. Fixed costs and entity participation are considered only for pipeline transportation. The lowest thresholds observed for the Matosinhos-Perafita pipeline in current networks and for Matosinhos-Porto pipeline in all networks, imply highest unitary tariffs with implicit lower profits for the PSC. The Matosinhos-Porto pipeline transports Jet to the airport which is still cheaper than transportation by other modes, besides the pipeline is already amortized. Uni-entity thresholds are lower than the aggregate of multi-entity thresholds which is notorious for Leixoes. The thresholds significantly improve from the current network to the grassroots and retrofit designs. Operator fixed costs are inexistent in the current network where all pipelines are amortized, increase in the retrofits with the installation of new pipelines for Azambuja-Lisboa, Matosinhos-Aveiro and Sines-Faro, and are highest for grassroots with the installation of Sines-Azambuja and Sines-Setubal pipelines.



Network	Origin	Destination	A	B	C	D	UNI
Current	MATOSINHOS	LEIXOES	0.1	0.1	0.36	0.36	0.1
Current	MATOSINHOS	MATOSINHOS	0.75	0.1	0.1	0.1	0.99
Current	MATOSINHOS	PERAFITA	0.1	0.1	0.1	0.1	0.1
Current	MATOSINHOS	PORTO	0.1	0.1	0.1	0.1	0.19
Current	MATOSINHOS	REAL	0.19	0.51	0.1	0.1	0.51
Current	SINES	AZAMBUJA	0.84	0.51	0.51	0.36	0.99
Current	SINES	SINES	0.19	0.1	0.1	0.1	0.36
Grassroots	AZAMBUJA	LISBOA	0.19	0.1	0.1	0.1	0.51
Grassroots	MATOSINHOS	AVEIRO	0.36	0.19	0.19	0.1	0.75
Grassroots	MATOSINHOS	MATOSINHOS	0.91	0.36	0.51	0.19	0.99
Grassroots	MATOSINHOS	PORTO	0.1	0.1	0.1	0.1	0.19
Grassroots	SINES	AZAMBUJA	0.84	0.51	0.51	0.36	0.99
Grassroots	SINES	FARO	0.36	0.19	0.19	0.1	0.51
Grassroots	SINES	SETUBAL	0.36	0.19	0.19	0.1	
Grassroots	SINES	SINES	0.19	0.1	0.1	0.1	0.36
Retrofit	AZAMBUJA	LISBOA	0.19	0.1	0.1	0.1	0.51
Retrofit	MATOSINHOS	AVEIRO	0.36	0.19	0.19	0.1	0.75
Retrofit	MATOSINHOS	MATOSINHOS	0.84	0.51	0.51	0.36	0.99
Retrofit	MATOSINHOS	PORTO	0.1	0.1	0.1	0.1	0.19
Retrofit	SINES	AZAMBUJA	0.84	0.51	0.51	0.36	0.99
Retrofit	SINES	FARO	0.36	0.19	0.19	0.1	0.51
Retrofit	SINES	SINES	0.19	0.1	0.1	0.1	0.36

Figure 3.14. Pipeline thresholds and fixed cost per entity and logistics operator.

The average transportation tariffs per transportation mode, network design and operated route are shown in Figure 3.15. The transportation tariffs by pipeline are significantly lower to those of maritime and road transport. The road tariffs are based on the distance, being the same for all entities and products. The maritime and rail transportation tariffs are directly proportional to the distances, however vary according to the product density, being higher for FU when compared to low density products like C3. Rail tariff to Faro is higher than the pipeline tariff, hence only selected in the current network. The pipeline tariffs depend on the distance, the product and the transported volumes. The Sines-Azambuja, Matosinhos-Aveiro and Matosinhos-Porto pipeline tariffs are highest in the current network followed by grassroots and lowest in retrofit where the increase in volumes provides the economies of scale. The transportation tariffs are based on the investment costs which vary between entities based on the transfer volumes. The tariffs could be calculated based on unamortized costs, which would be significantly lower for the owner entities. Although not plotted, tariffs are inversely proportional to volumes, being lowest for the uni-entity, followed by entity A, B, C and finally D.

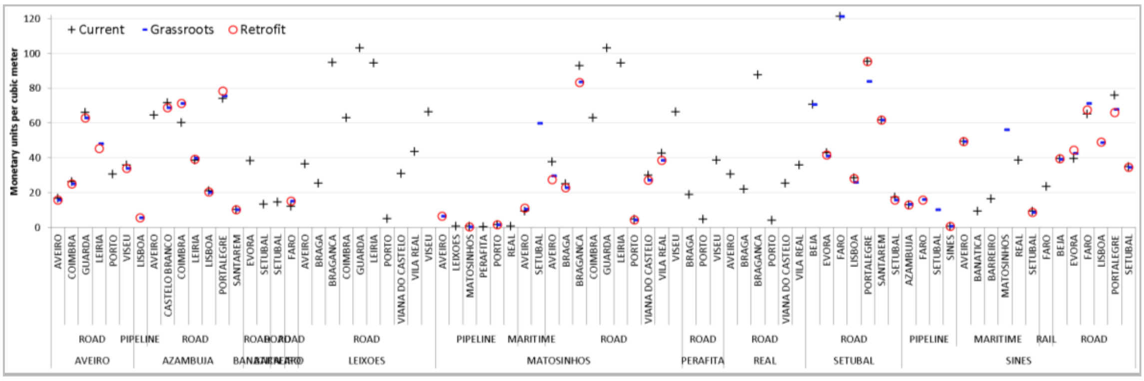


Figure 3.15. Average transportation tariffs per network design

Figure 3.16 shows the depot tariffs per product, entity and network design. Sines and Matosinhos show the lowest tariffs, followed by Azambuja, Real, Setubal, Aveiro, Banatica, Perafita and finally Faro. Barreiro and Leixões show excessive tariffs. For all products, the depot tariffs are highest in current networks, significantly lower in the grassroots and lower in the retrofits. Faro and Setubal show high tariffs for most products.

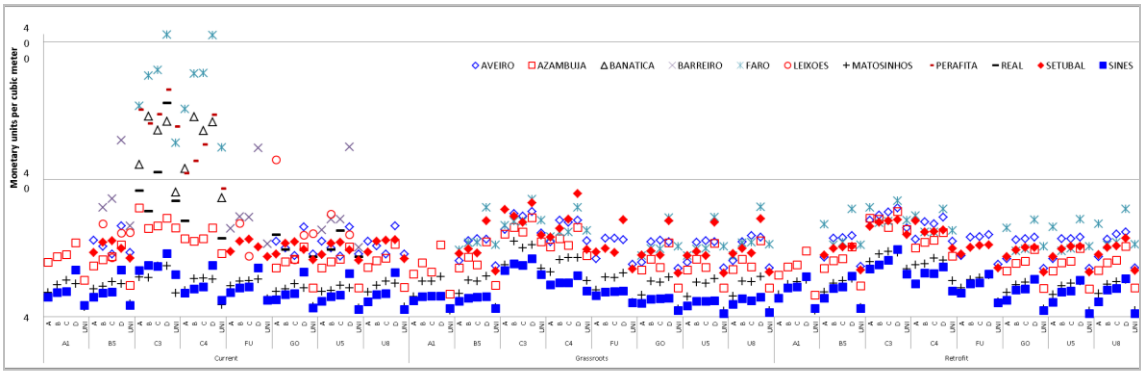


Figure 3.16. Depot tariffs per network design, product and entity.

Figure 3.17 presents for year 2006 the cost per cubic meter of product along the PSC, specifically at the ex-refinery, ante-depot, ex-depot, ex-retail and retail stages for the multi-entity retrofit design. We choose year 2006 for simplicity as the price evolution is stable, however these charts change considerably between years driven by price variations showing high profitability in some years and losses in others. As referred earlier, publicity and trading costs are not included. As expected, the product cost is lowest at the refineries and increases as it moves forward through the various stages of the PSC. In this particular year, ex-retail costs approximated the retail prices for GO, B5, U5, U8 and FU, whereas A1, C3 and C4 had the ex-retail costs much lower to the retail price, thereby identifying a higher commercial margin for these products. Product retail cost is higher at locations such as Castelo Branco, Portalegre, Viseu, Guarda and Bragança which are distant locations. At times and based on the cost parameters, the distribution costs to these locations push the ex-retail costs above the retail prices, hence these locations observe high unsatisfied volumes. The ex-retail costs are also high when product is transported first by maritime and later by road as in Aveiro and Coimbra where we note that the ante-depot and ex-depot costs for Aveiro are higher than those for Setubal. The later compares in costs to Azambuja, being lower than Faro and significantly higher than those at Sines and Matosinhos.

As can been seen in the results presented, the current MILP minimizes costs and maximizes revenues thereby maximizing profits of the PSC networks through collaborative and strategic/tactical planning with inventories for the current, grassroots and retrofit designs. Multi-entity profits are seen to approximate the uni-entity profits, which is a major benefit, made possible by the collaborative design with inventories and piecewise linearization of entity tariffs. The results and test data are based on the cost benchmarks of the Mediterranean region. The accuracy of the results could be improved by using company negotiated costs, however, such data is confidential.

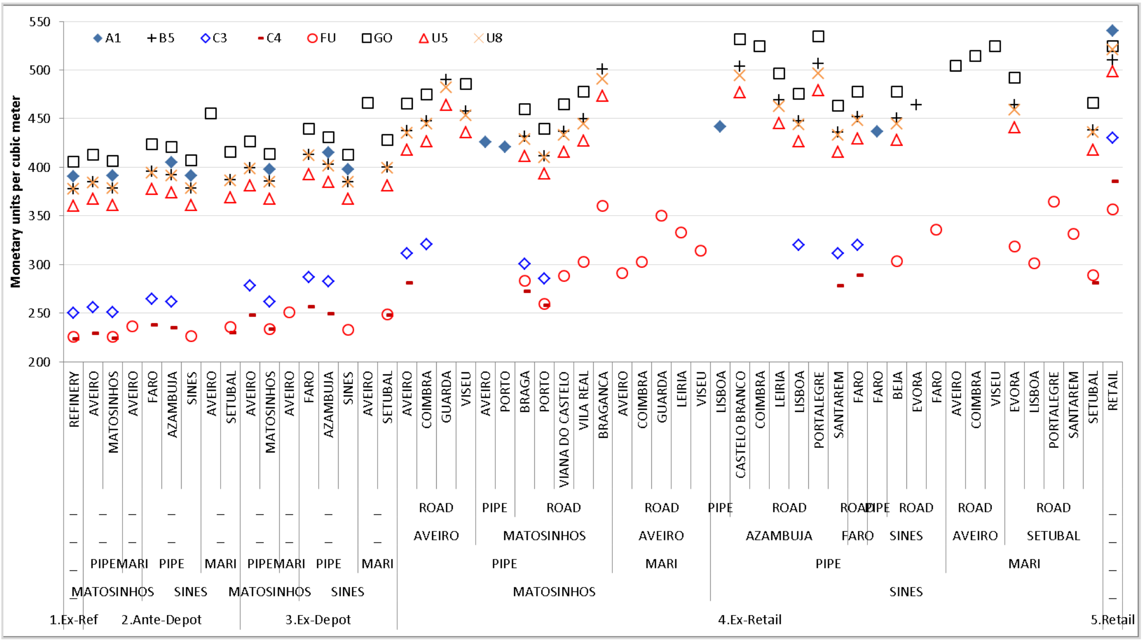


Figure 3.17. Product cost determined for year 2006 at each stage of the multi-entity retrofit.

3.9 Conclusions and future research

In this chapter, we present a MILP for strategic and tactical planning of collaborative downstream PSC networks, considering a piecewise linearization function for company tariff differentiation and multi-stage dynamic inventories. The MILP decides the current, grassroots and retrofit designs for the uni-entity and multi-entity networks, determining depot locations, transport modes, routes, resource capacities and network affectations, crude oil and petroleum products' importation, exportation, production, storage and transportation volumes, and various stage dynamic inventories. Efficient network designs with higher total PSC profits are achieved for the Portuguese PSC, where the model generated grassroots and retrofit designs for the uni-entity and multi-entity networks supersede the current networks. The profits improve when compared to those in Fernandes *et al.* (2013) where individualistic operation was considered.

While big countries like Spain construct uni-entity PSC networks which are built and managed by a single entity, smaller countries like Portugal have preferred multi-entity networks where petroleum entities are in competition for the smaller market. However, the logistics costs and consequently tariffs in multi-entity networks with individualistic strategies, are higher (while overall profits are lower) than those in uni-entity networks. Collaborative multi-entity designs generated by the current MILP, however, approximate the results of the more efficient uni-entity designs.

Several improvements have been achieved along this study compared to the earlier individualistic and static model presented in (Fernandes *et al.* 2013): i) collaborative design and planning significantly improve the PSC performance. ii) the various stage dynamic inventories improve the PSC profits by optimizing the procurement and exportation costs by taking advantage of the crude oil and petroleum derivative price variations. iii) current entity participation percentages in existing installations were a barrier for flexible utilization of competitor installations. By using the piecewise linearization function for entity price differentiation, the current MILP permits collaborative utilization of competitor installation capacity by determining entity prices based on economies of scale. Hence, considerable improvements are achieved in costs, tariffs and profits for the PSC members. Future research will proceed with the study of results by company specialists, suggesting future model improvements, including cross-country sales, taxation policies, risk and uncertainty management, as also identifying cuts to improve the model performance.

CHAPTER 4. Downstream Petroleum Supply Chain Design, Planning and Risk Reduction with Uncertainty

4.1 Abstract

The current chapter addresses the PSC design and planning and improves the collaborative design and tactical planning mixed integer linear program (MILP), presented in chapter 3, by presenting two developments: i) a stochastic program (SP) for PSC design and planning under demand uncertainty that maximizes the expected net present value (ENPV) of a multi-entity multi-product PSC network; ii) a multi-objective MILP that simultaneously maximizes the ENPV while minimizing the risk measures such as the mean variance, the variability index, the downside risk, the value-at-risk and the conditional value-at-risk (CVaR) under demand uncertainty.

The MILP determines the strategic decisions related to the installation, sizing and operation of infra-structures, the fair price costs and tariffs; and the tactical decisions concerning periodic depot and route product affectations and inventory levels. Pareto-optimal solutions are obtained for the multi-entity retrofit network design of the real-case Portuguese PSC network, considering three scenarios: the optimistic, realistic, and pessimistic scenarios of customer demand. The resulting designs, decisions, expected profits, and risk measures are discussed thereby obtaining relevant insights and research directions.

4.2 Introduction

The PSC attracts much attention as it shapes the living standards and prices of nearly all goods (World Economic Forum, 2012). In recent years, the fabric of this industry has changed dramatically. As crude oil prices doubled since 2009, many global companies invested in the pre-salt crude oil production where higher production costs were sustained by the increased prices. In recent years, however, the technology improvement accompanied by cost reduction in hydraulic fracking has made shale oil development economically viable. Investors hence moved into shale oil production in Northern America and many other countries. However, the recent fall in oil prices, which sharply reverted to the 2009 prices, has changed the economic equation and consequently the viability of these investments. Even shale oil whose production costs lowered significantly following the introduction of new technology is being hit by the collapse of oil prices.

While world economies struggle amidst financial crunches and volatile crude oil prices, the strain on the reduction of the petroleum products' costs has increased for companies to stay competitive and sustain sales. Market condition adjustments have created volatility among the petroleum prices, margins and product demand. Consequently, inefficiencies resulting from crude oil price fluctuations that are transferred down to the final product prices and customer demand misbalances that scale up the distribution, inventory, production and procurement stages have adversely affected PSC profitability. This has created cost, price, and demand volatility in the PSC, where companies and scholars try to improve the PSC efficiencies and manage uncertainties as seen in recent developments.

In this context there is a need to look into the presence of uncertainty and risks in PSC, namely on the downstream PSC the focus of this thesis. To do so is important to look into the Supply Chain (SC) literature, which has undergone considerable developments. Here, literature shows three levels of planning: strategic or long-term above-year planning, tactical or mid-term up to a year planning, and operational or short-term day-to-day planning. Uncertainties are identified related to demand, price, supply yields, lead times, supply capacity, supply cost, new technologies, production capacities, information, interest rates, taxes, breakdowns, disasters, terrorism and financial crisis. Resilient strategies such as demand shift across time, markets and products; supplier have been identified to manage these risks (Cardoso *et al.* (2014) .

Within the PSC and being an investment intensive and highly structured process industry, has long been under focus for supply chain optimization. Considerable research has focused refinery and the distribution network planning, more related to process design and operational planning, however, more lately, some work also on network design. Recently, integrated downstream PSC design and planning under uncertainty has been identified for research in many works. Interesting directions for the strategic, tactical and operational planning levels under several types of uncertainties were identified. While upstream optimization has focused decisions related to the selection and design of prospection, exploration and production sites, the downstream PSC modelling has focused two distinct areas: i) refinery process design and operational planning and ii) design and planning of the distribution network. This research has been extensively reviewed in Fernandes *et al.* (2011) with respect to the supply chain and PSC risk management, An *et al.* (2011) with planning models applied to the biofuel industry, Leiras *et al.* (2011) with respect to upstream, midstream, and downstream PSC planning models, and Shah *et al.* (2011) focusing the oil refinery scheduling and planning.

In a recent review, Sahebi *et al*. (2014) while referring to the PSC as a flourishing research area, identify several unaddressed research directions, such as: full vertical integration of strategic and tactical decisions; full horizontal integration of the complex crude oil supply chain; dealing with nonlinear models essential for the formulation of refinery operations; development of efficient solution techniques for multi objective modelling as a result of global factors including international taxation and transfer prices as also environmental impacts. Future research to study uncertainty with multi-stage stochastic models rather than with two-stage problems has also been identified as a key area for investigation. In the following sections, are specifically reviewed the uncertainty and risk management methodologies and models with relevance and appeal to the Downstream PSC. Lately, Lima *et al.* (2016) reviewed the strategic, tactical, operational downstream PSC planning literature, identifying the decision levels, uncertainty features, formulation approach, sustainability and objective functions, entity collaboration. They identify opportunities for developing more stochastic and robust optimization decision-support tools to integrate decision levels and the PSC characteristics in order to simultaneously deal with uncertainty, risk, resilience, sustainability and collaboration between entities.

In continuation to the developments in chapters 2 and 3, and with the objective of introducing the topic of uncertainty in these models, we now focus our attention to the works found in the literature relating specifically to the treatment of uncertainty and the management of risk, which are reviewed in the following two subsections.

4.2.1 Stochastic Modelling for Uncertainty

Uncertainty is a particularly important issue that arises when dealing with complex networks. Stochastic programing has been extensively used in production planning, location, transportation, logistics, finance, energy, and engineering design (Sahinidis, 2004). Despite the progress in two-stage stochastic programs, the authors identify multistage and stochastic integer programming as conceptually and computationally challenging. Birge and Louveaux (2010) discuss several SP methods such as two-stage recourse, multi-stage stochastic and stochastic integer programming methods; and methods for scenario generation such as Monte Carlo, approximating expectations and multistage approximation sampling methods. More important, they propose some solution evaluation measures such as the expected value of perfect information (EVPI), value of stochastic solution (VSS), expected value of the wait-and see problem (WS), expected recourse problem solution value (RP) and expectation of the expected value problem (EEV) to evaluate the quality of the stochastic solutions.

In the SC design under uncertainty problem, Azaron *et al.* (2008) propose a stochastic model to minimize the total operation cost and the variance or the financial risk, which is the probability of exceeding the budget. An interesting application is the hydroelectric generation scheduling, where Garcia-González *et al.* (2007) present a MILP formulation using lower bounds for profit and CVaR. Later, Guillén *et al.* (2005) propose a two-stage stochastic multi-objective MILP to simultaneously maximize ENPV and demand satisfaction, and minimize financial risk. Pareto stochastic curves are obtained for the SC trade-off design configurations. More recently, Cardoso *et al.* (2013) propose a stochastic MILP to maximize the ENPV for the strategic planning of supply chains with integration of reverse logistics. The model’s decisions are the installation, sizing and location of plants, warehouses and processes besides the inventory levels and the forward and reverse flows to retailers.

4.2.2 Multi-objective Risk Optimization

A research stream emerged following the development of risk measures. Risk management has significant appeal in the financial and other industrial sectors, which has motivated the development of various risk measures, which include: the mean variance in Mulvey *et al.* (1995), the variability index or upper partial mean in Ahmed and Sahinidis (1998), the probabilistic financial risk in Barbaro and Bagajewicz (2004), the downside risk in Eppen and Martin (1989) and You *et al.* (2009), the risk premium in Applequist *et al.* (2000), the Value-at-

Risk (VaR) and the Conditional Value-at-Risk (CVaR) in Rockafellar and Uryasev (2000, 2002), and the real-options valuation portfolio framework in Gupta and Maranas (2003).

At this point it is important to discuss the fundamental properties of risk measures. According to Artzner *et al.* (1999), risk measures are classified using a particularly interesting and important characteristic known as coherence. They identify four necessary and desirable properties that define whether a risk measure is coherent: i) translation invariance where the risk of the sum of a scalar and a random variable is equal to the sum of the scalar and the risk of the variable; ii) subadditivity where the risk of making two possible investments is lower than the sum of their individual risks; iii) positive homogeneity where the product of the risk of a random variable and a scalar is equal to the product of the scalar and the risk of the variable; iv) monotonicity where a variable with lower returns implies lower risk than another variable with higher returns.

An important reference for modelling production, cost, demand and price uncertainty for electrical markets is Conejo *et al.* (2013), who recur to multiobjective optimization in order to maximize the net profits and simultaneously minimize a risk measure, either directly as a second objective or indirectly as an upper bound constraint. The variance, variability index, probabilistic financial risk, downside risk, risk premium, VaR and CVaR were precisely the risk measures modelled. Conejo *et al.* (2010) identified that all above risk measures except CVaR are not coherent. The variance reduces the probability of a profit different from the expected, however, it penalizes both the higher as well as the lower profit scenarios. The shortfall probability and the downside risk are linear risk measures where the first is the probability of the profit being lower than a profit value target, and the latter is the expected value of the profit being lower than the target. Although easy to implement, they require an arbitrary target, as they are not expressed in profit units, and consequently, may result in “fat tail” profit distributions. The VaR does not satisfy the subadditivity property and hence may discourage diversification, as different levels of losses are not distinguished. Instability and numerical difficulties are observed when the loss distribution is not “normal”, resulting in ‘‘fat tails’’ or empirical discreteness. All these drawbacks are not present in CVaR, which is coherent and can be expressed using a linear formulation, and hence has been extensively used.

Associated with the PSC, Barbaro and Bagajewicz (2004) develop two-stage stochastic formulations to manage financial risk with planning under uncertainty. Cumulative risk curves, which are used to compare the risk versus profitability trade-offs, show a close relationship between the downside risk and the financial risk. Soon after, Aseeri and Bagajewicz (2004) develop the upside potential or opportunity value measure as an alternative to the VaR. The risk curves are used to analyse the solutions, obtained using SAA and chance constraints. Later, Pongsakdi *et al.* (2006) present a stochastic model for refinery crude purchases and derivatives production planning, which maximizes profits. The solutions obtained using SAA show lower losses in expected profit as measured by the upside potential. Following these works, Lakhanawat and Bagajewicz (2008) model the demand and cost uncertainties, using a non-linear program (NLP) for refinery operations planning, in order to determine the final product prices and the crude purchase and production quantities that maximize profits. The stochastic model incorporates the upper-bound risk curve, opportunity value, and VaR, which shows higher expected gross margin and lower risks, when compared to the deterministic solutions.

In an interesting development, Mavrotas (2009), propose an augmented ε-constraint method, to simultaneously optimize two objective functions, which was later improved in Mavrotas and Florios (2013) to optimize more than two objective functions, so as to determine the trade-offs between two or more objective functions. This method has been successfully implemented in many works, one such being Cardoso *et al.* (2013) for a closed-loop supply chain, where they simultaneously maximize the NPV and minimize a risk measure.

The remainder sections present the stochastic MILP modelling framework and formulation, the multi-objective formulation, the PSC real case scenario construction, the MILP computational results for the stochastic and multi-objective models, the ongoing research and future outlook, and finally conclusions.

4.3 The Stochastic MILP modelling framework

The PSC comprises of a network of petroleum companies, refineries, distribution depots, transportation modes and routes that purchase crude oil, produce or import petroleum products and export or commercialize and distribute the petroleum products to retail customer districts. The mission of the petroleum companies is to design, construct and manage PSC networks that maximize the profits and reduce risks under uncertainty for the entire PSC. Considering multi-entity networks, entities must collaborate to balance participations and install and operate optimal depot and transport route locations, storage and transport capacities, determine efficient product transfer volumes, service tariffs and cost synergies, thereby improving the network efficiency and making the PSC member participation cost-effective for all entities.

The downstream PSC strategic and tactical planning problem, was first modelled as a static MILP in chapter 2, to obtain multi-entity, multi-echelon PSC network designs with transfer volumes for existing, grassroots and retrofit networks and later as a dynamic collaborative MILP in chapter 3, to provide entity collaboration, multistage inventories, piecewise cost linearization and entity/product service tariff differentiation.

The contributions of the current research are:

1) Development of a stochastic PSC strategic and tactical planning MILP which is an extension of the deterministic MILP model presented in chapter 3.
2) Incorporation of storage depot level product importation and stochastic scenarios for demand uncertainty.
3) The stochastic model is extended to a multi-objective model for simultaneously maximizing the ENPV and minimizing one of the risk measures.

The modelling framework for the PSC Strategic and Tactical planning MILPs is shown in figure 4.2. A detailed description of the mathematical formulation of the deterministic PSC planning MILPs and the model assumptions can be found in chapters 2 and 3.

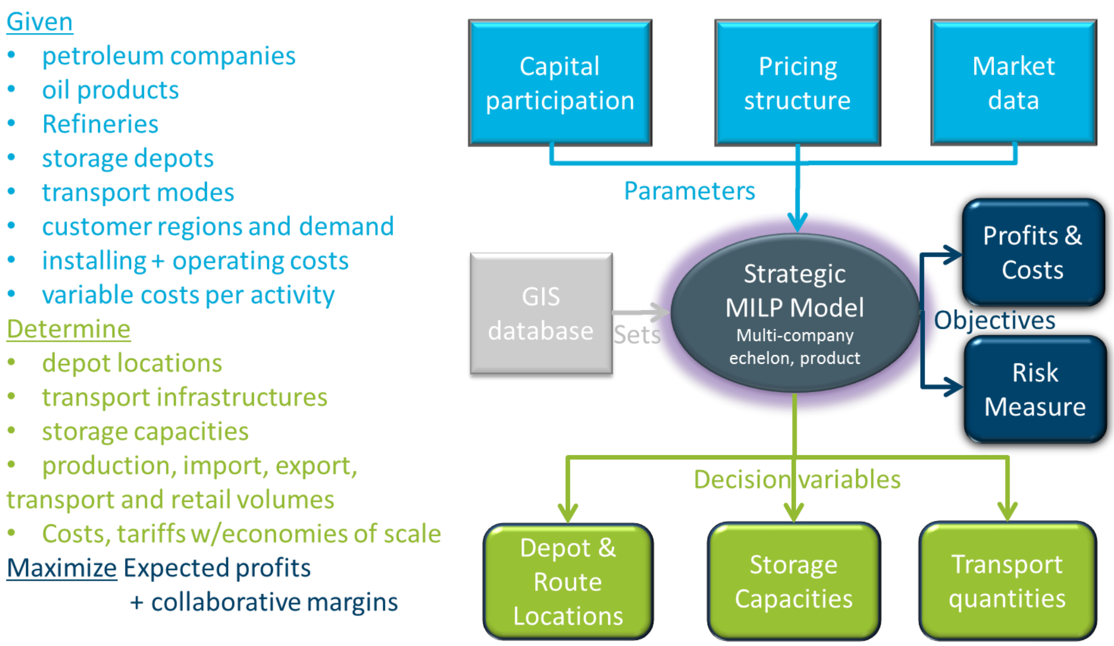


Figure 4.2. The Downstream PSC Design considerations

The current chapter extends two additional works which were published with the objective of modelling demand uncertainties within this planning context. A two-stage SP model presented in Fernandes *et al.* (2015) is proposed in the current section for the strategic and tactical planning under demand uncertainty, considering optimistic, realistic, and pessimistic scenarios. The ENPV is maximized for the Portuguese PSC real case, whereby are presented the test results and evaluated the solution quality using performance indicators. Numerical difficulties are identified as a result of the multi-dimension scenarios and the size of the model. Simultaneous modelling of costs, prices and demand uncertainty using decomposition techniques and robust optimization are identified for future research, see chapter 5 of this thesis. Thus as presented in Fernandes *et al.* (2015) in the current chapter, section 4, we integrate the two-stage SP model for the PSC strategic and tactical planning under demand uncertainty with the augmented ε-constraint method, for simultaneously maximization of the ENPV and minimization of the CVaR risk measure. By considering the Portuguese PSC, the stochastic model developed, generates Pareto-optimal network design solutions by varying the demand scenarios. On one side we consider the maximization of the ENPV while on the other side we minimize the CVaR. The developments of the mathematical formulations are now presented. The stochastic and the multi-objective MILPs use the compact node-variable mathematical formulation which performs better than the scenario-variable formulation as identified in Conejo *et al.* (2010) In contrast, if it was intended to use decomposition methods, then the preferred approach would be the scenario-variable formulation, in order to exploit the scenario structure of the model variables. The stochastic MILP formulation is now presented in the following sections.

4.3.1 Nomenclature

The main characteristics of the PSC can be identified or defined using the nomenclature found in the subsequent sections, namely sets, subsets and indices, and parameters.

4.3.1.1 Sets / Indices

$a \in A$	set of petroleum supply chain activity stages
$c \in C$	set of product characteristics, C={ DENSITY, TYPICAL_M3}
$e, e1 \in E$	set of petroleum entities/companies E={E1, E2, ..., EN }
$fc \in FC$	set of trip attributes, FC={CAPACITY, WORK, DAYS, BEFORE, AFTER, WAYS}
$g \in G$	set of GPS coordinates, G={ LATITUDE, LONGITUDE }
$n, o \in N$	set of all nodes
$p \in P$	set of products P={ P1, P2,..PN }
$r, r1 \in R$	set of resources including services and equipment
$t \in T$	set of time periods, T={1,2,..N}
$s \in S$	set of scenarios, S={1,2,..N}
$u \in U$	set of volume thresholds U={1,2,..N} for piecewise price linearization
$v \in V$	set of costs, V={TARIFF,FIXED,FIXVAR,OPERATING,OPERVAR, UNITARY}
$y \in Y$	set of amortization periods, Y={ 1,2,…,N }

4.3.1.2 Subsets / Indices

$i \in I \subseteq N$	set of refinery nodes
$IL \subseteq I$	set of local refinery nodes which are allowed to export
$j \in J \subseteq N$	set of depot nodes
$k \in K \subseteq N$	set of customer nodes
$m, m1 \in M \subseteq R$	set of transport modes, M={ PIPELINE, MARITIME, RAIL, ROAD }
$MC \subseteq M$	set of continuous flow transport modes MC={ PIPELINE }
$MP \subseteq M$	set of participated transport modes MP={ PIPELINE }

4.3.1.3 Subset unions / Indices

$fm \in FM = M \bigcup \{ANY\}$	set of transport mode families, FM={PIPELINE,MARITIME,RAIL,ROAD ,ANY}

$fp \in FP = P \bigcup \{ANY\}$ set of products families FP={ P1, P2,..PN, ANY }

$h \in H = I \bigcup J$ set of refinery and depot nodes

$l \in L = J \bigcup K$ set of depot and customer nodes

4.3.1.4 Parameters

$SC0_i$ beginning inventory of crude at refinery *i*

$SD0_{j,p,e}$ beginning inventory at depot *j*, for product *p*, entity *e*

$SL0_{k,p,e}$ beginning inventory at customer *k*, for product *p*, entity *e*

$SR0_{i,p,e}$ beginning inventory at refinery *i*, for product *p*, entity *e*

BN big number for number of resources

HPP hours per time period

MD_m distance multiplier per transportation mode to account for route curves,

MNA maximum number of amortization periods

MV_m medium velocity per transportation mode *m*

$FM_{fm,m}$ association of transportation mode family *fm* with transportation mode *m*

$FP_{fp,p}$ association of product family *fp* with product *p*

$FR_{m,fc}$ trip parameters per transportation mode *m* and trip characteristic *fc*

NCD number of depots in concentration set

NCP number of capacity partitions per resource available to entity

NFF fill factor in percentage

NPS number of periods multiplied by number of scenarios

NR degree to radian coefficient for distance calculations

OCC operating cost coefficient

RCD radius for concentration set for demand at depot

TPP time aggregation in period, TPP=1 / 12 for monthly / yearly data

α^a operator margin percentage per activity *a*

β^a_{Min} minimum safety stock percentage per activity *a*

β^a_{Max} maximum safety stock percentage per activity *a*

γ	working average cost of capital (WACC) coefficient for inventory costs
$\Gamma_{n,g}$	GPS (global pointing system) coordinates *g* for each location node *n*
$\delta_{k,p,e,t,s}$	customer demand in m3 at customer node *k*, product *p*, entity *e*, period *t*, scenario *s*
$\varepsilon_{i,p}$	fraction yield of product *p* per unit measure of crude oil at refinery *i*
$\varphi^{a}_{p,t}$	price of product *p* after activity *a* (crude, ex-refinery, retail) at time *t*
$\eta_{n,o,r,e}$	capital participation percentage by entity *e* in resource *r* between nodes *n,o*; consider *r* = *m* in case of transport mode and *n*=*o* in case of depots
$\kappa d_{n,y}$	Completed amortization periods *y* per existing depot at location *n*
$\kappa r_{n,fm,r,y,fp}$	current resource units per transport mode family *fm*, product family *fp*, resource *r*, refinery or depot *n*, and completed amortization periods *y*
$\kappa t_{n,o,m,y}$	Completed amortization periods *y* for current transport infrastructures on route *n,o,m*; road routes are considered as existing and fully amortized
$\mu_{n,o,m}$	transport distance between any two nodes (*n,o*) including refineries, depots or customer zones, using transport mode *m*
$\mu'_{n,o,m}$	transport duration between any two nodes (*n,o*) including refineries, depots or customer zones, using transport mode *m*
$\theta r_{fm,r,fp}$	typical capacity of resource *r* for transport mode family *fm* and product family *fp*
ρ_i	crude processing capacity at refinery *i* in m3 per day
τc_u	piecewise linearization function coefficients per step *u*
$\tau t_{h,u}$	piecewise linearization function thresholds per depot *h* and step *u*
$\upsilon^{a}_{m,p}$	maximum permitted distance for activity *a* by transportation mode *m* for the petroleum product *p*; example: fuel-oil transportation by pipeline
ω^{c}_{p}	characteristic *c* per product *p*, e.g. density
$\omega m_{m,c,p}$	characteristic *c* per transport mode *m* per product *p*
ξR^{v}_{r}	cost component *v* for resource *r* independent of product, example pipeline
$\xi P^{v}_{r,p}$	cost component *v* for resource *r* dependent of product *p*, example vessel
ζ_e	market quota per entity *e*

4.3.2 Model Decisions

The decision variables are approached distinctly as the first stage decisions which are independent of scenarios; and the second-stage decisions which are scenario dependent and determined after the uncertainty is revealed. The model's decisions presented below are divided into six groups of variables, wherein the first four groups are the strategic decisions and the last two are tactical decisions.

- Binary variables ID_j for the depots to install, OD_j for the depots to operate, $IT_{h,l,m}$ for the transport routes to install, and $OT_{h,l,m}$ for the transport routes to operate.
- Binary variables $ZD_{j,e,u}$ determine the operating depot thresholds per entity, whereas $ZT_{h,l,m,e,u}$ determine the operating transport route thresholds per entity, in order to impute the piecewise linearized costs into the respective product entity service tariffs.
- Integer variables $IR_{j,fm,r,fp}$ and $OR_{j,fm,r,fp}$ for the depot resource types and capacity (number) to install and operate per depot, transport mode family and product family.
- Integer variables $ZR_{j,fm,r,fp,e}$ are the number of operating resource partitions to impute into product service tariffs, which are determined per depot, transport mode family, resource, product family and entity.
- Non-negative variables prefixed by *Q* determine the multi-stage transfer volumes.
- Non-negative variables prefixed by *S* determine the multi-stage inventory volumes.

The variables are now presented in deeper detail.

4.3.2.1 Binary Strategic Decision Variables

$$ID_j = \begin{cases} 1, & \text{if depot } j \text{ is to be installed} \\ 0, & \text{otherwise} \end{cases}$$

$$IT_{h,l,m} = \begin{cases} 1, & \text{if route using transport mode } m \text{ is to be installed between node } h \text{ and node } l \\ 0, & \text{otherwise} \end{cases}$$

$$OD_j = \begin{cases} 1, & \text{if depot } j \text{ is to be operated} \\ 0, & \text{otherwise} \end{cases}$$

$$OT_{h,l,m} = \begin{cases} 1, & \text{if route using transport mode } m \text{ is to be operated between node } h \text{ and node } l \\ 0, & \text{otherwise} \end{cases}$$

$$ZD_{j,e,u} = \begin{cases} 1, & \text{if } \pi t_{j,u-1} < QP_{j,l,m,p,e,t} + QS_{j,l,m,p,e,t} < \pi t_{j,u} \\ 0, & \text{otherwise} \end{cases}$$

$$ZT_{h,l,m,e,u} = \begin{cases} 1, & \text{if } \pi t_{h,u-1} < QP_{h,l,m,p,e,t} + QS_{h,l,m,p,e,t} < \pi t_{h,u} \\ 0, & \text{otherwise} \end{cases}$$

4.3.2.2 Integer Strategic Decision Variables

$IR_{j,fm,r,fp}$ number of resources *r* to install per depot *j* per transport mode family *fm* for product family *fp*

$OR_{j,fm,r,fp}$ number of resources *r* to operate per depot *j* per transport mode family *fm* for product family *fp*

$ZR_{j,fm,r,fp,e}$ number of resource partitions to impute installation and operating costs per entity *e*, resource *r*, depot *j*, transport mode family *fm* and product family *fp*

4.3.2.3 Non-negative Tactical Decision Variables

$QCI_{i,t,s}$ Crude volume imported by refinery *i*, in period *t* and scenario *s*

$QCP_{i,t,s}$ crude volume processed by refinery *i*, in period *t* and scenario *s*

$QE_{i,p,t,s}$ exported volume by refinery *i* of product *p* in period *t* and scenario *s*

$QI_{j,p,e,t,s}$ imported volume by depot *j* of product *p* and entity *e* in period *t* and scenario *s*

$QP_{h,j,m,p,e,t,s}$ primary volumes of product *p* transported by mode *m* from refinery/depot *h* to depot *j* by entity *e* in period *t* and scenario *s*

$QR_{i,p,t,s}$ production volume per refinery *i* of product *p* in period *t* and scenario *s*

$QS_{j,k,m,p,e,t,s}$ secondary volumes of product *p* transported by mode *m* from depot *j* to customer *k* by entity *e* in period *t* and scenario *s*

$QU_{k,p,e,t,s}$ unsatisfied demand per customer *k*, for product *p*, entity *e*, end of period *t*, scenario *s*

$SC_{i,t,s}$ crude inventory at refinery *i* at end of period *t* and scenario *s*

$SD_{j,p,e,t,s}$ inventory at depot *j*, for product *p*, entity *e* at end of period *t* and scenario *s*

$SL_{k,p,e,t,s}$ inventory at customer *k*, for product *p*, entity *e* at period *t* and scenario *s*

$SR_{i,p,e,t,s}$ inventory at refinery *i*, for product *p*, entity *e* at end of period *t* and scenario *s*

4.3.2.4 Continuous Objective Function Variables

The objective function terms per PSC stage and location or transportation route, product, entity, time periods, and scenario, are determined as continuous and non-negative variables as defined below. These include the PSC profits, the multi-stage revenues the multi-stage costs and the multi-stage tariffs that are defined per stage, location or route. These terms are derived using auxiliary equations that are part of the formulation.

$\mathrm{Profit}_{t,s}$ PSC profits in period *t* and scenario *s*

$MDE_{j,e,t,s}$ margin per depot *j*, entity *e* in period *t* and scenario *s*

$ML_{k,p,e,t,s}$ margin per retail customer *k*, product *p*, entity *e* in period *t* and scenario *s*

$MR_{i,e,t,s}$ margin per refinery *i*, entity *e* in period *t* and scenario *s*

$MTE_{h,l,m,e,t,s}$ transportation margin between nodes *h,l* for mode *m*, entity *e*, period *t* and scenario *s*

4.3.2.5 Non-negative Objective Function Variables

$CDD_{h,t,s}$ depot costs at depot *h* in period *t* and scenario *s*

$CDFD_{h,t}$ depot fixed costs for depot *h* in period *t*

$CDFE_{h,e,t}$ depot fixed costs to entity *e* for depot *h* in period *t*

$CDOD_{h,t}$ depot operating costs for depot *h* in period *t*

$CDOE_{h,e,t}$ depot operating costs to entity *e* for depot *h* in period *t*

$CDU_{h,p,e,t,s}$ unitary costs at depot *h* for product *p*, for entity *e* in period *t* and scenario *s*

$CE_{i,p,e,t,s}$ exportation costs per refinery *i*, product *p*, entity *e* in period *t* and scenario *s*

$CI_{j,p,e,t,s}$ importation costs per depot *j*, product *p*, entity *e* in period *t* and scenario *s*

$CNC_{i,p,e,t,s}$ inventory costs for crude at refinery *i*, product *p*, entity *e*, period *t*, scenario *s*

$CND_{j,p,e,t,s}$ inventory costs at depot *j*, product *p*, and entity *e* in period *t* and scenario *s*

$CNL_{k,p,e,t,s}$ inventory costs at customer *k*, product *p*, and entity *e* in period *t*, scenario *s*

$CNR_{i,p,e,t,s}$ inventory costs at refinery *i*, product *p*, and entity *e* in period *t* and scenario *s*

$CRFD_{h,fp,t}$ resources fixed costs for depot *h*, resource *fp* in period *t*

$CRFE_{h,fp,e,t}$ resources fixed costs to entity *e* for depot *h*, resource *fp* in period *t*

$CROD_{h,fp,t}$ resources operating costs for depot *h*, resource *fp* in period *t*

$CROE_{h,fp,e,t}$ resources operating costs to entity *e* for depot *h* resource *fp* in period *t*

$CTD_{h,l,m,t,s}$ transportation costs between nodes *h,l* for mode *m* in period *t* and scenario *s*

$CTFD_{h,l,m,t}$ transportation fixed costs between nodes *h,l* for mode *m* in period *t*

$CTFE_{h,l,m,e,t}$ transportation fixed costs to entity *e* between nodes *h,l* for mode *m* in period *t*

$CTOD_{h,l,m,t}$ transportation operating costs between nodes *h,l* for mode *m* in period *t*

$CTOE_{h,l,m,e,t}$ transportation operating costs to entity *e* on route *h,l,m* and period *t*

$CTU_{h,l,m,p,e,t,s}$ transportation unitary costs on route *h,l,m,* product *p*, period *t* and scenario *s*

$TDE_{h,e,t,s}$	depot tariffs at depot *h* for entity *e* in period *t* and scenario *s*
$TTE_{h,l,m,e,t,s}$	transportation tariff on route *h,l,m* for entity *e*, period *t* and scenario *s*

4.3.3 Objective Function

The stochastic model maximizes the Expected Net Present Value (ENPV) while considering uncertain demand scenarios. The ENPV obtained in eq. (1), is the sum of the multiplication of each scenario NPV_s and its probability of occurrence ψ_s. In eq. (2) the NPV_s for each scenario is calculated as the summation of the discounted profits obtained in each time period, using an arbitrary interest rate irr.

$$Max(ENPV) = \sum_s \psi_s \times NPV_s \tag{4.1}$$

$$NPV_s = \sum_t \mathrm{Pr}ofit_{t,s} \big/ (1+irr)^t \tag{4.2}$$

The multi-echelon PSC profit per scenario and time period $\mathrm{Pr}ofit_{t,s}$, provided in eq. (3), aggregates: 1) the refinery profit $MR_{i,e,t,s}$ less the export $CE_{i,p,e,t,s}$ and import $CI_{j,p,e,t,s}$ costs; 2) minus the inventory costs at the crude $CNC_{i,p,e,t,s}$, refinery $CNR_{i,p,e,t,s}$, depot $CND_{j,p,e,t,s}$ and retail $CNL_{k,p,e,t,s}$ stages; 3) plus the gross retail margin $ML_{k,p,e,t,s}$ less the depot $TDE_{h,e,t,s}$ and transportation $TTE_{h,l,m,e,t,s}$ tariffs and the shortage $CS_{k,p,e,t,s}$ costs; 4) plus the participation in depot $MDE_{h,e,t,s}$ and transportation $MTE_{h,l,m,e,t,s}$ profits the later calculated only for possible transport links $(h,l,m) \mid \exists \mu_{h.l.m}$.

$$\mathrm{Pr}ofit_{t,s} = Max \sum_{e \in E} \begin{pmatrix} \sum_{i \in I} MR_{i,e,t,s} - \sum_{i \in I} \sum_{p \in p} CE_{i,p,e,t,s} - \sum_{j \in J} \sum_{p \in p} CI_{j,p,e,t,s} \\ - \sum_{p \in P} \left(\sum_{i \in I} CNC_{i,p,e,t,s} + \sum_{i \in I} CNR_{i,p,e,t,s} + \sum_{j \in J} CND_{j,p,e,t,s} + \sum_{k \in K} CNL_{k,p,e,t,s} \right) \\ + \sum_{k \in K} \sum_{p \in P} ML_{k,p,e,t,s} - \sum_{h \in H} TDE_{h,e,t,s} - \sum_{(h,l,m) \mid \exists \mu_{h,l,m}} TTE_{h,l,m,e,t,s} - \sum_{k \in K} \sum_{p \in P} CS_{k,p,e,t,s} \\ + \sum_{h \in H} MDE_{h,e,t,s} + \sum_{(h,l,m) \mid \exists \mu_{h.l.m}} MTE_{h,l,m,e,t,s} \end{pmatrix} \tag{4.3}$$

Each of the function terms are obtained using the sub-function expressions that are detailed in the current and following subsections (3.3.1-3.2.4). The objective function terms are all determined per time period and scenario, however, some of the expressions, especially relating to the fixed costs are determined per location, entity, product and time period. These decisions do not change between scenarios.

All these costs, tariffs, revenues and profits are determined individually per entity *e*, per product *p*, per PSC stage location (refinery *i*, depot storage *j*, transportation *m* and retail *k*), per time period *t* and per scenario *s* for the shared network. The stochastic design and planning MILP considers market product prices, fixed refining, retail markups, and detailed benchmarked costs, which are used in the piecewise linearization function in order to determine individual activity fair-price tariffs, per entity, so as to reduce the costs and increase entity profits.

$$MR_{i,e,t,s} = \left(\sum_{p \in P} QR_{i,p,t,s} \left(\varphi^{a1}_{p,t} - \xi P^{v=t}_{r1,p} \right) - QCI_{i,t,s} \times \varphi^{a2}_{p2,t} \right) \eta^{r1}_{i,i,e}, \forall i \in I, e \in E, t \in T, s \in S, a1 = r1 = REFINERY, a2 = p2 = CRUDE \quad (4.4)$$

$$CE_{i,p,e,t,s} = QE_{i,p,t,s}\, \xi P^{v=t}_{r1,p}\, \eta^{r2}_{i,i,e}, \quad \forall i \in I, p \in P, e \in E, t \in T, s \in S, r1 = EXPORT, r2 = REFINERY \quad (4.5)$$

$$CI_{h,p,e,t,s} = QI_{h,p,e,t,s} \times \left(\varphi^{a1}_{p,t} + \xi P^{v=t}_{r1,p} \right), \quad \forall h \in H, p \in P, e \in E, t \in T, s \in S, a1 = REFINING, r1 = IMPORT \quad (4.6)$$

$$CNC_{i,p,e,t,s} = \gamma \times SC_{i,t,s} \times \varphi^{a1}_{p1,t} \times \varepsilon_{i,p} \times \eta^{r1}_{i,e}, \quad \forall i \in I, p \in P, e \in E, t \in T, s \in S, a1 = p1 = CRUDE, r1 = REFINERY \quad (4.7)$$

$$CNR_{i,p,e,t,s} = \gamma \times SR_{i,p,t,s} \times \varphi^{a1}_{p,t} \times \eta^{r1}_{i,e}, \quad \forall i \in I, p \in P, e \in E, t \in T, s \in S, a1 = r1 = REFINERY \quad (4.8)$$

$$CND_{j,p,e,t,s} = \gamma \times SD_{j,p,e,t,s} \times \varphi^{a1}_{p,t}, \quad \forall j \in J, p \in P, e \in E, t \in T, s \in S, a1 = REFINERY \quad (4.9)$$

$$CNL_{k,p,e,t,s} = \gamma \times SL_{k,p,e,t,s} \times \varphi^{a1}_{p,t}, \quad \forall k \in K, p \in P, e \in E, t \in T, s \in S, a1 = RETAIL \quad (4.10)$$

$$ML_{k,p,e,t,s} = \left(\delta_{k,p,e,t,s} - QU_{k,p,e,t,s} \right) \left(\varphi^{a1}_{p,t} - \xi P^{v=t}_{r1,p} \right) - \sum_{(j,m) | \exists \mu_{j,k,m}} QS_{j,k,m,p,e,t,s} \times \varphi^{a2}_{p,t}, \quad (4.11)$$
$$\forall k \in K, p \in P, e \in E, t \in T, s \in S, a1 = r1 = RETAIL, a2 = REFINERY$$

$$CS_{k,p,e,t,s} = QU_{k,p,e,t,s} \times \xi P^{v=t}_{r1,p}, \quad \forall k \in K, p \in P, e \in E, t \in T, s \in S, r1 = UNMET \quad (4.12)$$

4.3.3.1 Scenario-wise Multi-entity Transportation route tariffs

$$TTE_{h,l,m,e,t,s} = \left(1 + \alpha^{a1} \right) \left(CTFE_{h,l,m,e,t} + CTOE_{h,l,m,e,t} + \sum_{p \in P} CTU_{h,l,m,p,e,t,s} \right), \forall h \in H, l \in L, m \in M \wedge (h,l,m) | \exists \mu_{h,l,m}, e \in E, t \in T, s \in S, a1 = TRANSPORT \quad (4.13)$$

$$CTFE_{h,l,m,e,t} = \sum_{u \in U} \left(ZT_{h,l,m,e,u}\, \tau c_u \right) \left(\xi R^{v=f}_m + \mu_{h,l,m} \times \xi R^{v=fv}_m \right), \quad \forall (h,l,m) | \exists \mu_{h,l,m}, e \in E, t \in T \wedge t \le MNA \quad (4.14)$$

$$CTOE_{h,l,m,e,t} = \sum_{u \in U} ZT_{h,l,m,e,u}\, \tau c_u \left(\xi R^{v=o}_m + \mu_{h,l,m} \times \xi R^{v=ov}_m \right), \quad \forall (h,l,m) | \exists \mu_{h,l,m}, e \in E, t \in T \quad (4.15)$$

$$CTU_{h,l,m,p,e,t,s} = \left(QP_{h,l,m,p,e,t,s} + QS_{h,l,m,p,e,t,s} \right) \times \mu_{h,l,m} \times \xi P^{v=u}_{m,p}, \quad \forall (h,l,m) | \exists \mu_{h,l,m}, p \in P, e \in E, t \in T, s \in S \quad (4.16)$$

4.3.3.2 Scenario-wise Multi-entity Storage Depot tariffs

$$TDE_{h,e,t,s} = \left(1 + \alpha^{a1} \right) \left(CDFE_{h,e,t} + CDOE_{h,e,t} + \sum_{fp \in FP} \left(CRFE_{h,fp,e,t} + CROE_{h,fp,e,t} \right) + \sum_{p \in P} CDU_{h,p,e,t,s} \right), \forall h \in H, e \in E, t \in T, s \in S, a1 = STORAGE \quad (4.17)$$

$$CDFE_{h,e,t} = \sum_{u \in U} ZD_{h,e,u} \times \tau c_u \times \xi R^{v=f}_{r1}, \quad \forall h \in H, e \in E, t \in T, t \le MNA, r1 = DEPOT \quad (4.18)$$

$$CDOE_{h,e,t} = \sum_{u \in U} ZD_{h,e} \times \tau c_u \times \xi R^{v=o}_{r1}, \quad \forall h \in H, e \in E, t \in T, r1 = DEPOT \quad (4.19)$$

$$CRFE_{h,fp,e,t} = \sum_{r \in R} \sum_{fm \in FM} ZR_{m,h,r,p=fp,e} \left(\xi P^{v=f}_{r,p=fp} + \xi R^{v=f}_r \right) / NCP, \forall h \in H, fp \in FP, e \in E, t \in T, t \le MNA, \xi P^{v=f}_{r,p} \vee \xi R^{v=f}_r \quad (4.20)$$

$$CROE_{h,fp,e,t} = \sum_{r \in R} \sum_{fm \in FM} \left(ZR_{h,fm,r,fp,e} / NCP \right) \times \left(\xi P^{v=o}_{r,p=fp} + \xi R^{v=o}_r \right), \quad \forall h \in H, fp \in FP, e \in E, t \in T, \xi P^{v=o}_{r,p} \vee \xi R^{v=o}_r \quad (4.21)$$

$$CDU_{h,p,e,t,s} = \sum_{l\in L}\sum_{m\in M}\left(QP_{h,l,m,p,e,t,s} + QS_{h,l,m,p,e,t,s}\right)\times \xi P^{v=u}_{r1,p}, \qquad \forall h\in H, p\in P, e\in E, t\in T, s\in S, r1 = DEPOT \tag{4.22}$$

4.3.3.3 Scenario-wise Transportation operator Margins

$$CTFE_{h,l,m,e,t} = \sum_{u\in U}\left(ZT_{h,l,m,e,u}\,\tau c_u\left(1-\eta_{h,l,m,e}\sum_{y\in Y}\kappa t_{h,l,m,y}\right)\right)\left(\xi R^{v=f}_m + \mu_{h,l,m}\times \xi R^{v=fv}_m\right),$$
$$\forall (h,l,m)\,|\,\exists \mu_{h,l,m}, e\in E, t\in T \wedge \left(t \le MNA \vee \left(\neg \eta_{h,l,m,e} \wedge y+t > MNA\right)\right) \tag{4.23}$$

$$CDFE_{h,e,t} = \sum_{u\in U}\left(ZD_{h,e,u}\,\tau c_u\left(1-\eta_{h,h,r1,e}\sum_{(y+t)>MNA}\kappa d_{h,y}\right)\right)\xi R^{v=f}_{r1}, \forall h\in H, e\in E, t\in T \wedge \left(t\le MNA \vee \neg \eta_{h,h,r1,e}\right), r1 = DEPOT \tag{4.24}$$

$$CRFE_{h,fp,e,t} = \sum_{fm,r}\left(k\times ZR_{h,fm,r,p,e}\right)\left(\xi P^{v=f}_{r,p} + \xi R^{v=f}_r\right)/NCP,$$
$$\forall h\in H, fp\in FP, e\in E, t\in T, k=1\,|\, t\le MNA \vee \neg\exists \sum_{fm,r,y1}\kappa r_{h,fm,r,y1,fp}\,\eta_{h,h,r1,e}, \xi P^{v=f}_{r,p} \vee \xi R^{v=f}_r, r1 = DEPOT \tag{4.25}$$

$$CTD_{h,l,m,t,s} = \left(CTFD_{h,l,m,t} + CTOD_{h,l,m,t} + \sum_{p\in P}\sum_{e\in E} CTU_{h,l,m,p,e,t,s}\right) \qquad \forall (h,l,m)\,|\,\exists \mu_{h,l,m}, t\in T, s\in S \tag{4.26}$$

$$MTE_{h,l,m,e,t,s} = \left(\sum_{e2\in E} TTE_{h,l,m,e2,t,s} - CTD_{h,l,m,t,s}\right)\left(\eta_{h,l,m,e} + \zeta_e\right), \forall (h,l,m)\,|\,\exists \mu_{h,l,m} \wedge m\in MP, e\in E, t\in T, s\in S, \eta_{h,l,m,e} \vee \zeta_e \tag{4.27}$$

$$CTFD_{h,l,m,,t} = \left(IT_{h,l,m} - \sum_{y\in Y}\kappa t_{h,l,m,y}\right)\left(\xi R^{v=f}_m + \mu_{h,l,m}\times \xi R^{v=fv}_m\right), \forall (h,l,m)\,|\,\exists \mu_{h,l,m}, t\in T \wedge t\le MNA \wedge y+t > MNA \tag{4.28}$$

$$CTOD_{h,l,m,t} = OT_{h,l,m}\times\left(\xi R^{v=o}_m + \mu_{h,l,m}\times \xi R^{v=ov}_m\right), \qquad \forall (h,l,m)\,|\,\exists \mu_{h,l,m}, t\in T \tag{4.29}$$

4.3.3.4 Scenario-wise Depot operator Margins

$$CDD_{h,t,s} = CDFD_{h,t} + CDOD_{h,t} + \sum_{fp\in FP}\left(CRFD_{h,fp,t} + CROD_{h,fp,t}\right) + \sum_{p\in P}\sum_{e\in E} CDU_{h,p,e,t,s}\,\forall h\in H, t\in T, s\in S \tag{4.30}$$

$$MDE_{h,e,t,s} = \left(\sum_{e2\in E} TDE_{h,e2,t,s} - CDD_{h,t,s}\right)\times\left(\eta_{h,h,r1,e} + \zeta_e\right), \forall h\in H, e\in E, t\in T, s\in S, \eta_{h,h,r,e} \vee \zeta_e, r1 = DEPOT \tag{4.31}$$

$$CDFD_{h,t} = \left(ID_h - \sum_{(y+t)>MNA}\kappa d_{h,y}\right)\times \xi R^{v=f}_{r1}, \qquad \forall h\in H, t\in T \wedge t\le MNA, r1 = DEPOT \tag{4.32}$$

$$CDOD_{h,t} = OD_h \times \xi R^{v=o}_{r1}, \qquad \forall h\in H, t\in T, r1 = DEPOT \tag{4.33}$$

$$CRFD_{h,fp,t} = \sum_{fm\in FM}\sum_{r\in R}\left(IR_{h,fm,r,fp} - \sum_{(y+t)>MNA}\kappa r_{h,fm,r,y,fp}\right)\left(\xi P^{v=f}_{r,fp} + \xi R^{v=f}_r\right), \forall h\in H, fp\in FP, t\in T \wedge t\le MNA, \xi P^{v=f}_{r,fp} \vee \xi R^{v=f}_r \tag{4.34}$$

$$CROD_{h,fp,t} = \sum_{fm\in FM}\sum_{r\in R} OR_{h,fm,r,fp}\left(\xi P^{v=o}_{r,p} + \xi R^{v=o}_r\right), \qquad \forall h\in H, t\in T, fp\in FP, \xi P^{v=o}_{r,fp} \vee \xi R^{v=o}_r \tag{4.35}$$

4.3.4 CONSTRAINTS

The PSC system is governed by four groups of constraints. The second and third groups are the scenario independent first stage constraints. These determine respectively the transportation route as well as the depot installation and operation decisions. The installation and operation of the resource capacities per product and entity, may only exist in installed and operating storage depots and network infrastructures, and are enforced by the network volumes determined below. The first and fourth groups are the scenario dependent second stage constraints. The first group includes the scenario dependent volume balances at the crude, refinery, storage depot and the retail stages. Here, the first and second time period crude volume balances, at each refinery, the sum of the previous crude inventory $SC_{i,t-1,s}$ and the imported crude volumes $QCI_{i,t,s}$ minus the processed crude volumes $QCP_{i,t,s}$ determine the transiting crude inventory $SC_{i,t,s}$. Similar volume balance constraints are defined for the other stages. Besides, the retail volume balance constraints also determine the unsatisfied demand $QU_{k,p,e,t,s}$. The fourth group includes the minimum and maximum bounds for the multi-stage inventories and the transfer volumes.

4.3.4.1 Multi-stage Volume Balance constraints

$$SC0_{i,s} + QCI_{i,t,s} = QCP_{i,t,s} + SC_{i,t,s}, \quad \forall i \in I, t \in T \wedge t \le 1, s \in S \tag{4.36}$$

$$SC_{i,t-1,s} + QCI_{i,t,s} = QCP_{i,t,s} + SC_{i,t,s}, \quad \forall i \in I, t \in T \wedge t > 1, s \in S \tag{4.37}$$

$$QCP_{i,t,s} \le \rho_i, \quad \forall i \in I, t \in T, s \in S \tag{4.38}$$

$$QR_{i,p,t,s} = QCP_{i,t,s} \times \varepsilon_{i,p}, \quad \forall i \in I, p \in P, t \in T, s \in S \tag{4.39}$$

$$SR0_{i,p,s} + QR_{i,p,t,s} = \sum_{(j,m):\exists \mu_{i,j,m}, e \in E} QP_{i,j,m,p,e,t,s} + QE_{i \in IL,p,t,s} + SR_{i,p,t,s}, \forall i \in I, p \in P, t \in T \wedge t \le 1, s \in S \tag{4.40}$$

$$SR_{i,p,t-1,s} + QR_{i,p,t,s} = \sum_{(j,m)|\exists \mu_{i,j,m}, e \in E} QP_{i,j,m,p,e,t,s} + QE_{i \in IL,p,t,s} + SR_{i,p,t,s}, \forall i \in I, p \in P, t \in T \wedge t > 1, s \in S \tag{4.41}$$

$$SD0_{j,p,e,s} + \sum_{(h \in H \cap I,m)|\exists \mu_{h,j,m}} QP_{h,j,m,p,e,t,s} + QI_{j,p,e,t,s} = \sum_{(h \in H \setminus I,m)|\exists \mu_{j,h,m}} QP_{j,h,m,p,e,t,s} + \sum_{(k,m)|\exists \mu_{j,k,m}} QS_{j,k,m,p,e,t,s} + SD_{j,p,e,t,s}, \quad \forall j \in H \setminus I, p \in P, e \in E, t \in T, t \le 1, s \in S \tag{4.42}$$

$$SD_{j,p,e,t-1,s} + \sum_{(h \in H \cap I,m)|\exists \mu_{h,j,m}} QP_{h,j,m,p,e,t,s} + QI_{j,p,e,t,s} = \sum_{(h \in H \setminus I,m)|\exists \mu_{j,h,m}} QP_{j,h,m,p,e,t,s} + \sum_{(k,m)|\exists \mu_{j,k,m}} QS_{j,k,m,p,e,t,s} + SD_{j,p,e,t,s}, \quad \forall j \in H \setminus I, p \in P, e \in E, t \in T, t > 1, s \in S \tag{4.43}$$

$$SL0_{k,p,e,s} + \sum_{(j,m)|\exists \mu_{j,k,m}} QS_{j,k,m,p,e,t,s} = \delta_{k,p,e,t,s} - QU_{k,p,e,t,s} + SL_{k,p,e,t,s}, \forall k \in K, p \in P, e \in E, t \in T \wedge t \le 1, s \in S \tag{4.44}$$

$$SL_{k,p,e,t-1,s} + \sum_{(j,m)|\exists \mu_{j,k,m}} QS_{j,k,m,p,e,t,s} = \delta_{k,p,e,t,s} - QU_{k,p,e,t,s} + SL_{k,p,e,t,s}, \forall k \in K, p \in P, e \in E, t \in T \wedge t > 1, s \in S \tag{4.45}$$

4.3.4.2 Multi-entity Transportation Route Installation and Operation

$$\sum_{u\in U}\pi t_{h,u-1} ZT_{h,l,m,e,u} \le \sum_{t\in T}\sum_{s\in S}\sum_{p\in P}\left(QP_{h,l,m,p,e,t,s} + QS_{h,l,m,p,e,t,s}\right)\Big/ NPS \le \sum_{u\in U}\pi t_{h,u} ZT_{h,l,m,e,u},$$
$$\forall (h,l,m)\,|\,\exists\,\mu_{h,l,m} \wedge \exists\,\xi r_{v\in\{\text{FIXED,FIXVAR,OPERATING,OPERVAR}\},m},\ e\in E \quad (4.46)$$

$$\sum_{u\in U} ZT_{h,l,m,e,u} \le OT_{h,l,m},\ \forall (h,l,m)\,|\,\exists\,\mu_{h,l,m} \wedge \exists\,\xi r_{v\in\{\text{FIXED,FIXVAR,OPERATING,OPERVAR}\},m},\ e\in E \quad (4.47)$$

$$OT_{h,l,m} \le IT_{h,l,m}, \qquad \forall (h,l,m)\,|\,\exists\,\mu_{h,l,m} \quad (4.48)$$

$$OT_{h,l,m} \le \sum_{fm=m,r,fp} OR_{h,fm,r,fp}, \qquad \forall (h,l,m)\,|\,\exists\,\mu_{h,l,m} \quad (4.49)$$

$$IT_{h,l,m} \le \sum_{fm=m,r,fp} IR_{h,fm,r,fp}, \qquad \forall (h,l,m)\,|\,\exists\,\mu_{h,l,m} \quad (4.50)$$

$$\kappa t_{h,l,m,y} \le IT_{h,l,m}, \qquad \forall (h,l,m)\,|\,\exists\,\mu_{h,l,m},\ y\in Y \quad (4.51)$$

4.3.4.3 Multi-entity Depot Installation, Operation and Capacities

$$\left(ZR_{h,fm,r,fp,e} - 1\right)\times \theta r_{fm,r,fp} \times \left((1-k)\times HPP + k\times TPP\right)/NCP \le$$
$$\sum_{t\in T}\sum_{s\in S}\sum_{p\in FP}\sum_{m\in FM}\left(\left(\sum_{j\in\mu_{h,j,m}} QP_{h,j,m,p,e,t,s} + \sum_{k\in\mu_{h,k,m}} QS_{h,k,m,p,e,t,s}\right)\left((1-k) + \frac{k}{\omega m_{m,p}}\right) + SD_{h,p,e,t,s}\,k\right)\Big/ NPS \quad (4.52)$$
$$\le ZR_{h,fm,r,fp,e}\,\theta r_{fm,r,fp}\left((1-k)HPP + k\times TPP\right)/NCP,\ \forall h\in H, e\in E, k=1, (fm,r,fp)\,|\,\exists\,\theta r_{fm,r,fp} \wedge fm = ANY$$

$$\sum_{y\in Y} \kappa r_{h,fm,r,y,fp} \le IR_{h,fm,r,fp}, \qquad \forall h\in H,\ fm\in FM,\ r\in R,\ fp\in FP,\ (fm,r,fp)\,|\,\exists\,\theta r_{fm,r,fp} \quad (4.53)$$

$$\sum_{e\in E} ZR_{h,fm,r,fp,e} \le OR_{h,fm,r,fp}\times NCP, \qquad \forall h\in H,\ fm\in FM,\ r\in R,\ fp\in FP,\ (fm,r,fp)\,|\,\exists\,\theta r_{fm,r,fp} \quad (4.54)$$

$$OR_{h,fm,r,fp} \le OD_h \times NB, \qquad \forall h\in H,\ fm\in FM,\ r\in R,\ fp\in FP,\ (fm,r,fp)\,|\,\exists\,\theta r_{fm,r,fp} \quad (4.55)$$

$$IR_{h,fm,r,fp} \le ID_h \times NB, \qquad \forall h\in H,\ fm\in FM,\ r\in R,\ fp\in FP,\ (fm,r,fp)\,|\,\exists\,\theta r_{fm,r,fp} \quad (4.56)$$

$$OR_{h,fm,r,fp} \le IR_{h,fm,r,fp}, \qquad \forall h\in H,\ fm\in FM,\ r\in R,\ fp\in FP,\ (fm,r,fp)\,|\,\exists\,\theta r_{fm,r,fp} \quad (4.57)$$

$$\sum_{u\in U}\pi t_{h,u-1} ZD_{h,e,u} \le \sum_{fm,r,fp} ZR_{h,fm,r,fp,e}\,\theta r_{fm,r,fp}\,TPP/NCP \le \sum_{u\in U}\pi t_{h,u} ZD_{h,e,u},\ \forall h\in H, e\in E, r='VESSEL' \quad (4.58)$$

$$\sum_{u\in U} ZD_{h,e,u} \le OD_h, \qquad \forall h\in H, e\in E \quad (4.59)$$

$$OD_h \le ID_h, \qquad \forall h\in H \quad (4.60)$$

4.3.4.4 Multi-stage Inventory Bounds

$$SC_{i,t,s} \ge \rho_i \times \beta_{Min}^{CRUDE}, \qquad \forall i\in I, t\in T, s\in S \quad (4.61)$$

$$SC_{i,t,s} \le \rho_i \times \beta_{Max}^{CRUDE}, \qquad \forall i\in I, t\in T, s\in S \quad (4.62)$$

$$QCP_{i,t,s} \ge \rho_i \times \beta_{Min}^{REFINING}, \qquad \forall i\in I, t\in T, s\in S \quad (4.63)$$

$$SR_{i,p,t,s} \le \rho_i \times \varepsilon_{i,p} \times \beta_{Max}^{REFINING}, \qquad \forall i\in I, p\in P, t\in T, t>1 \wedge t\le T-1, s\in S \quad (4.64)$$

$$SL_{k,p,e,t,s} \le \delta_{k,p,e,t+1,s} \times \beta_{Max}^{RETAIL}, \qquad \forall k \in K, p \in P, e \in E, t \in T, t > 1 \wedge t < T, s \in S \tag{4.65}$$

$$OT_{n,o,m} = IT_{n,o,m} = 0, \qquad \forall n \in N, o \in O, m \in M, \wedge \mu_{n,o,m} = 0 \tag{4.66}$$

$$QP_{h,j,m,p,e,t,s} = 0, \qquad \forall h \in H, j \in J, m \in M, p \in P, e \in E, t \in T, s \in S, \wedge \left(\mu_{h,j,m} > \upsilon_{m,p}^{PRIMARY}\right) \tag{4.67}$$

$$QS_{j,k,m,p,e,t,s} = 0, \qquad \forall j \in J, k \in K, m \in M, p \in P, e \in E, t \in T, s \in S, \wedge \left(\mu_{j,k,m} > \upsilon_{m,p}^{SECONDARY}\right) \tag{4.68}$$

The stochastic MILP presented thus far, maximizes a single objective considering uncertain demand, the ENPV, which may be referred to as the risk-neutral objective. The stochastic scenario dependent MILP uses the compact node-variable formulation as explained in Conejo *et al.* (2010) Test results for this stochastic MILP which maximizes the ENPV are shown in section 8. Whereas in the scenario-variable formulation, all of the scenarios are defined in each time period, in the case of the node-variable formulation, all the scenarios are defined only in the last time period, and in the preceding time periods are defined only the parent node scenarios as in the nodes of a the scenario tree. A parameter table is used to configure the scenario and time period associations.

4.4 Multi-objective Uncertainty Modelling

In the approach presented in this section, the PSC network is modelled by considering a risk measure objective. In its basic form, the network may be modelled to optimize a specific objective, for example to maximize the ENPV or to minimize a risk measure. The maximization of the ENPV is straightforward and has already been presented in the earlier sections. The minimization of the financial risk, however, is new and can be modelled through the adoption of the risk measures that have been discussed in the literature review. In the current chapter, we model specifically four risk measures: 1) the mean variance of the NPV; 2) the variability index that are related with the variability of the NPV; 3) the downside risk that measures the NPV being inferior to a specific target; and 4) the conditional value-at-risk that measures the expected value of the profit being inferior to a specific target. A comparison of these risk measures is shown in figure 4.3.

Objective	Risk Measure	Characteristics
Reduce the variability of the NPV	Variance	Symetric risk measure
		Includes quadratic terms
	Variability Index	Linear programming problem
		Only penalizes results lower than the expected
Achieve a NPV higher than target value	Downside risk	Only focus on the target
		Requires the decisor to choose a target
	CVaR	Reveals fat tails that may appear in the NPV distribution
		Coherent risk measure

Figure 4.3. Comparison of Risk Measures

We now take the methodology presented in Conejo *et al.* (2010), and the improved augmented ε–constraint method algorithm presented in Mavrotas and Florios (2013) in order to simultaneously maximize the ENPV and minimize a risk measure. The mathematical formulations of the specifically modelled risk measures of the mean variance, variability index, downside risk, and CVaR, are now presented.

4.4.1 Variance

The mean variance of the scenario profit is the average of the squared differences from the expected value as shown in equation (69).

$$Variance = \sum_s \psi_s \times \left(ENPV - NPV_s\right)^2 \tag{4.69}$$

This includes quadratic terms that make the model hard to solve for large scale problems. Thus, a linearized formulation was derived in equations (70) and (71) based on the linearization proposed by Pan and Nagi (2010).

$$ENPV - NPV_s = Q_s^+ - Q_s^- \qquad \forall s \in S \tag{4.70}$$

$$Variance = \sum_s \psi_s \times (Q_s^+ + Q_s^-) \tag{4.71}$$

$$Q_s^+, Q_s^- \in R_0^+ \tag{4.72}$$

Here, two new non-negative variables are defined, Q_s^+, representing the amount by which the NPV_s of each scenario is greater than the expected value and Q_s^-, that is the amount by which the expected value is greater than the NPV_s of each scenario. This is first modelled in eq.(70), and subsequently, the equation (71) calculates the mean variance as the product of the summation of the values of these new variables for each scenario, and the probability of occurrence of the respective scenario.

4.4.2 Variability index

While the variability index is similar to the variance, it only takes into account the positive deviation between the ENPV and the scenario NPV_s, as modelled in the following equations.

$$VarIndex = \sum_s \psi_s \times \Delta VI_s \tag{4.73}$$

$$\Delta VI_s \geq ENPV - NPV_s \qquad \Delta VI_s \geq 0 \wedge \forall s \in S \tag{4.74}$$

$$\Delta VI_s \in R_0^+ \tag{4.75}$$

The objective function for this risk measure $VarIndex$ is the product of the summation of the variability index of each scenario and its probability, as given in equation (73). In equation (74) the variability index per scenario is defined as a positive continuous variable ΔVI_s which is the difference between the ENPV and NPV_s for each scenario.

4.4.3 Downside risk

The downside risk, measures the positive deviation between a given NPV target (Ω) defined by the decision maker and the NPV_s obtained for each scenario.

$$DRisk = \sum_s \psi_s \times \Delta D_s \qquad (4.76)$$

$$\Delta D_s \geq \Omega - NPV_s \qquad s \in S \qquad (4.77)$$

$$\Delta D_s \in R_0^+ \qquad (4.78)$$

The value of the downside risk, hence, is the summation of the product of the positive deviation variable determined per scenario and the probability of occurrence of each scenario, as calculated in equation (76). The new variable ΔD_s introduced in equation (77), defines the positive deviation between the target profit Ω and the scenario's NPV_s. Should the scenario NPV_s be greater than the established target, ΔD_s will be equal to 0, otherwise it takes the value of the deviation between ENPV and NPV_s.

4.4.4 Conditional value-at-risk (CVaR)

The CVaR is the likelihood that an ENPV will be lower than the NPV given by the VaR with a specific confidence level α and can be expressed using a linear formulation. For continuous demand distributions, CVaR is defined as the expected profit when lower than VaR (Rockafellar and Uryasev, 2000). For discrete distributions it is defined as the weighted average between VaR and the expected profit lower than VaR. Rockafellar and Uryasev (2002) defined the following linear approximation of CVaR for discrete distributions:

$$CVaR_a = VaR + 1/(1-\alpha) \times \sum_s (\psi_s \times \Delta V_s) \qquad (4.79)$$

$$\Delta V_s \geq VaR - NPV_s \qquad s \in S \qquad (4.80)$$

$$\Delta V_s \in R_0^+ \qquad (4.81)$$

The CVaR is given in equation (79), where α represents the confidence level, between 0 and 1, defined by the decision maker. A positive variable ΔV_s, which measures the positive difference between the VaR and the NPV_s obtained in each scenario, is calculated in equation (80). A probability $(1-\alpha)$ references the part of the profit distribution that is desired to be managed in order to secure the risk. Here, the VaR is the $(1-\alpha)$-quantile of the profit distribution, whereas the CVaR is the expected value of the profit distribution below that $(1-\alpha)$-quantile.

4.4.5 The augmented ε-constraint algorithm

The stochastic model is integrated with the improved augmented ε-constraint algorithm from Mavrotas and Florios (2013) to determine the Pareto-optimal trade-offs solutions, which are derived in eq. (82), considering the two objective functions f_1 and f_2. The first function is the ENPV and the latter is one of the risk measures presented above. The method begins by first optimizing each objective function individually, thereby obtaining the optimal values Z_1^* and Z_2^*. Then, we add a constraint to fix f_1 to Z_1^*, and optimize f_2 wherein the optimal solution of the second optimization is f_{2Max} . A similar procedure is followed to obtain f_{1Max} where is fixed f_2. This is the general procedure, however if the objective functions are conflicting objectives, that is one maximization and the other minimization, which is our case, then the second objective function is also maximized by multiplying its expression by -1. Hence, the extreme points of the Pareto-optimal curve are determined, which correspond to (Z_1^*, f_{2Max}) and (f_{1Max} , Z_2^*,). The remaining points of the curve are obtained by solving the problem (82) where f_{2Max} is the upper bound of f_2, *slack* is a non-negative surplus variable; and *r* is given by the expression f_{2Max} - Z_2^*, which is the range of objective function f_2, and *k* is the number of efficient solutions to determine between 1 and n_k.

$$Max\left(f_1 + 10^{-6}\left(slack/r\right)\right) \qquad \mid f_2 + slack = f_{2Max} - k\left(r/n_k\right) \tag{4.82}$$

As these are two-stage stochastic models, the strategic distribution network is determined as the first stage decisions independent of the scenarios, whereas, the multi-stage inventories and product flows are determined at the second stage decision variables, which can change in each scenario. Integrating the two levels of decisions is therefore crucial to obtain the optimal solutions, by properly linking the information between the two decision levels. As the PSC reveals high demand uncertainty, it is especially important to develop an adequate integration approach and an appropriate scenario tree.

The formulations and methodology presented above, allow for simultaneous maximization of the ENPV and the minimization of a risk measure. In the following section, is presented a real case example, which will be used to test first the stochastic model presented, which maximizes the ENPV, followed by the multi-objective risk model that maximizes simultaneously the ENPV and a risk measure.

4.5 A Real Case PSC Network

Figure 4.4 presents the Portuguese downstream PSC wherein are shown the two national refineries R1 and R2 along with their MILP determined production throughputs for the petroleum products produced. These are located at Matosinhos and Sines. The companies purchase the oil products from the local refineries or import them from external sources at a premium of insurance and freight. Currently, four major petroleum companies, including national and multinational companies, operate the PSC market in Portugal. Their demands are known and include that of smaller companies and hypermarket chains as they are also supplied by them. The product is then transported using the primary distribution to eleven storage depots, D1-D11, where it is stored and consequently transported, using the secondary distribution channels, to the retail stations. Figure 4.4 shows the depot capacities, accounting for the number of typical capacity tanks, for each of the 8 products (C3, C4, U5, U8, GO, FU, A1, B5), which are considered in this study. Companies may also import product at depots paying the cost of product, insurance and freight charges from a regional refinery cluster.

Note that the PSC retail segment is organized in customer zones corresponding to the eighteen districts of Portugal. The retail stations are aggregated into customer zones corresponding to the eighteen districts of Portugal. The model generated satisfied demand, considering the current network, is shown in the figure. The transportation is undertaken by pipeline, maritime or railway for the primary distribution and by road for the secondary distribution. JET A1 is however an exception where pipeline or railway transportation may be used. The district capitals' GPS coordinates, obtained using the Google maps application program interfaces, are used in the locations, land routes and distance computations. Particularities relating to the LPG bottle-filling and the archipelagos of Madeira and Azores have not been included in this study.

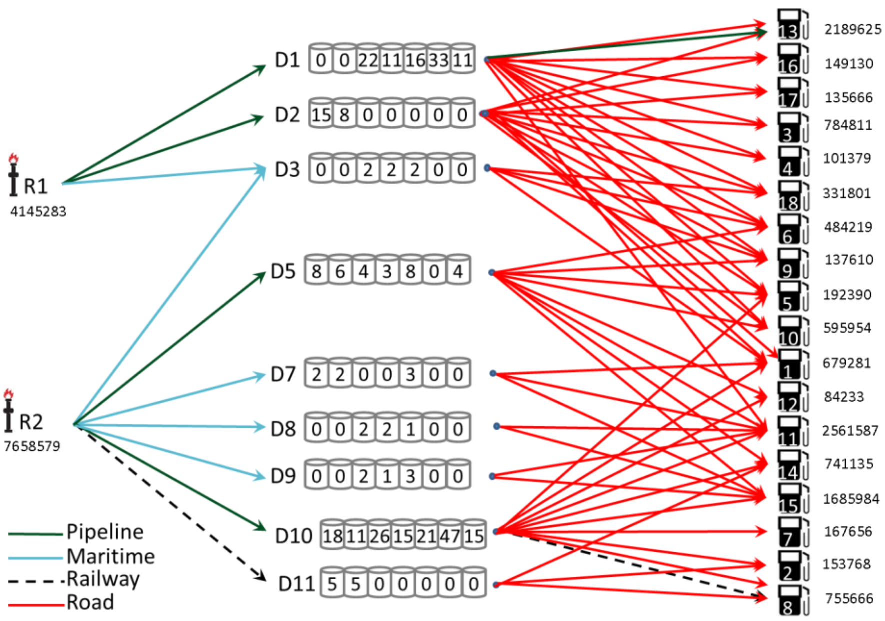


Figure 4.4. A Real Case PSC network

The stochastic and multi-objective MILPs for the collaborative, strategic and tactical planning of the PSC, may consider an existing network, a retrofit network or a grassroots network. Hence the network initial conditions for the infrastructure may begin with the existing depots and transport infrastructures with known entity participations which may not change (Current), may begin with the current and are allowed to change (Retrofit), or need to be constructed from scratch (Grassroots).

4.5.1 Scenario Construction

The stochastic MILP presented earlier is now tested using the real example based on the Portuguese downstream PSC network, whose real case data has been presented in chapters 3 and 4. To test the stochastic MILP, the customer demand scenarios are required per entity, product and customer zone. Figure 4.5 presents the scenario tree for the demand uncertainty which is constructed based on the market demand evolution in recent years, which has increased for some products and decreased for others. After the recent economic crisis, the market evolution is expected to be favourable, so the scenarios are constructed considering an optimistic growth of 10%, a realistic growth of 5% and a pessimistic growth of 0%. These scenario probabilities, considered for the stochastic MILP were respectively 0.25, 0.5 and 0.25, whereas for the multi-objective MILP were considered the probabilities 0.35, 0.5 and 0.15. The product demand per period and scenario $\delta_{p,t,s}$ is given by expression $\delta_{p,t-1,s} \times (1+dp) \times (1+\tau s)$ where dp is the product trend per period and τs is the market tendency per scenario. This provides 9 scenarios over the three periods with cumulative probabilities as shown in figure 4.5.

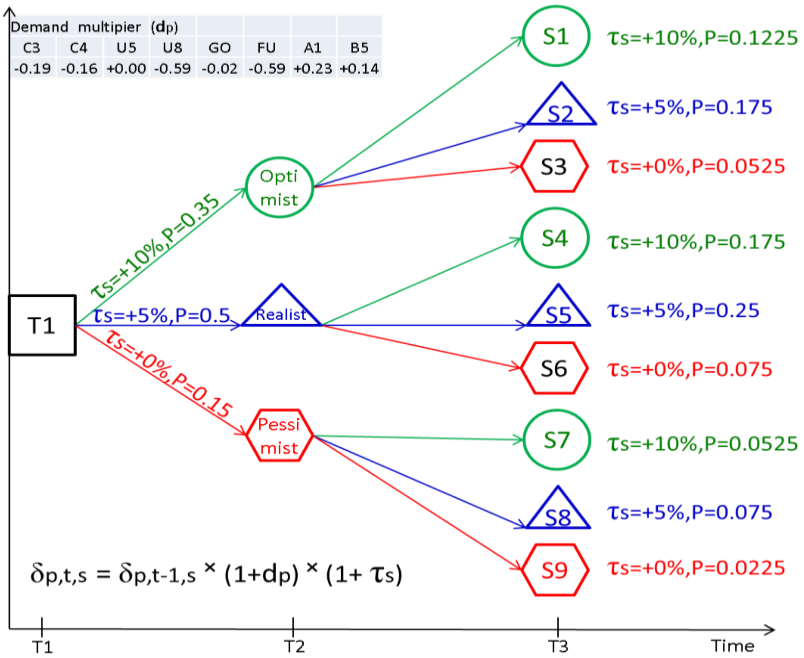


Figure 4.5. Demand scenario tree construction

The stochastic MILP performance will be now compared for various settings including risk neutral ENPV as well as optimizing for minimizing one risk measure and finally simultaneous maximizing ENPV and minimizing risk. The MILP is tested for current, grassroots and retrofit designs and for the single-entity and multi-entity networks.

4.5.2 Implementation of the Stochastic MILP

The stochastic MILP is implemented using GAMS 24.4.3 64-bit and solved with MILP commercial solver CPLEX 12.6 on an Intel Xeon 2 GHz CPU with 16GB RAM and 16 parallel threads. The stopping criterion is 0.1% optimality gap or 24 hours of CPU time. The MILP is tested for a real example based on the Portuguese PSC data that was presented in Fernandes *et al.* (2014). In the current study, the demands, costs and prices are averaged into three five-year periods. To evaluate the stochastic MILP, a grassroots design is modelled for the Portuguese PSC considering demand uncertainty. Besides the strategic and tactical decisions, the stochastic solution values are evaluated, following which is executed the multi-objective stochastic model integrated with the augmented ε-constraint algorithm.

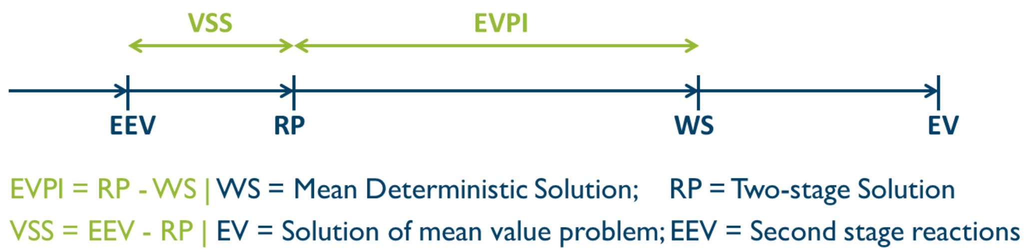


Figure 4.6. Solution methodology and comparison

Figure 4.6 illustrates the methodology that was presented in Birge and Louveaux (2010), and that is used to obtain the quality of the stochastic solutions, and the relationships between the stochastic solution values and the respective evaluation measures. The WS solution is the expected value obtained after solving each scenario using the deterministic model. The RP solution is the expected value obtained after solving the recourse problem. The EEV solution is the expected value obtained after fixing the first-stage variables to the deterministic solution for the average scenario. Hence to obtain the VSS measure, we solve the deterministic model where all stochastic parameters are replaced by their expectations and after that we evaluate that solution (which we fix) against all the scenarios, computing its average performance. We then compute VSS as the difference between the optimal stochastic solution and the average performance of the deterministic solution. To compute the EVPI, each scenario is solved individually and then computed the average of the individual optimal solutions. This would be the average performance in case the perfect information was available (perfect foresight). The EVPI is then computed as the difference between the average performance with perfect information and the optimal stochastic solution.

4.5.2.1 Stochastic MILP solutions

As shown in Figure 4.7, the first period has a single demand scenario with probability 1; three second period scenarios S1-S3 with 0%, 5% and 10% demand growth and probability 0.25, 0.5

and 0.25; and nine third period scenarios S11-S33 with cumulative growth 0%, 5% and 10% and conditional probability 0.25, 0.5 and 0.25 based on the second period scenarios.

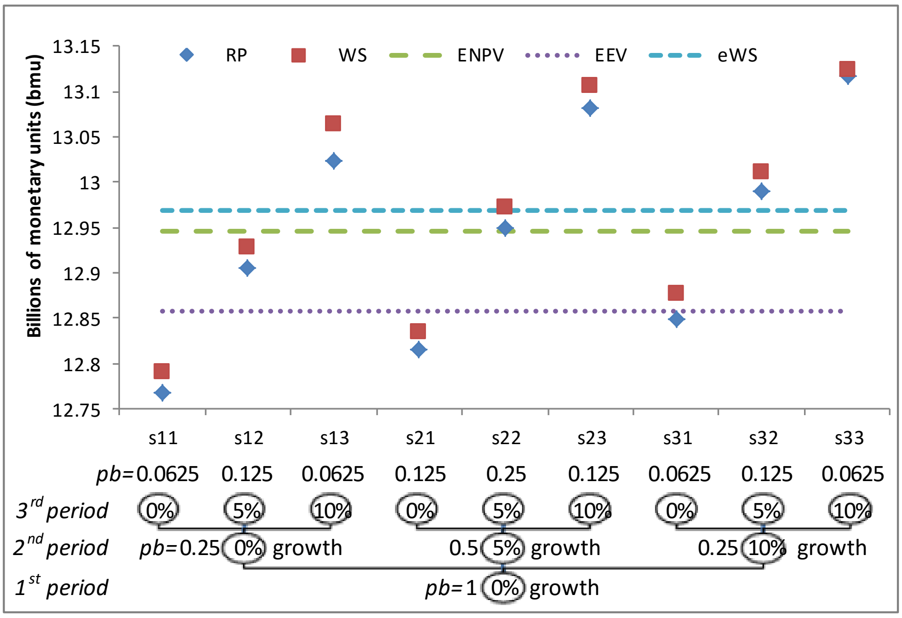


Figure 4.7. NPV results for varying demand scenarios

The above methodology is used model the grassroots uni-entity network for the Portuguese PSC, and compared the wait and see (WS), the recourse problem (RP) and the evaluation of the expected value (EEV) solutions. It is seen that the NPV varies between 1.276,8 and 1.311,7 billions of monetary units (bmu), for the recourse solution and between 1.279 up to 1.312,4 bmu for the wait and see solution. The ENPVs obtained for the EEV, RP and WS problems are respectively 1.286, 1.295 and 1.297 bmu. On the network design Aveiro, Matosinhos, Azambuja, Sines and Barreiro are the depots selected in the EEV and RP solutions whereas Lisboa is additionally selected in the WS solution.

4.5.2.2 Quality of Stochastic Solutions

Figure 4.8 compares the ENPV obtained for the WS, RP and EEV solutions. Here, the EVPI is the difference between the RP and EEV solutions which is the value a decision maker is willing to pay for perfect information. The VSS is the difference between the WS and RP solutions which measures the value of the stochastic solution over the deterministic equivalent. Based on the data used, significantly low improvement is observed with the stochastic solution, as the VSS measure shows 88.24 mmu and the EVPI obtained is 22.93 mmu. The explanation for these small values is because the major contributor to the ENPV is the refinery margin which has been the least affected by the domestic demand variations. The equidistance of the scenarios' probabilities and the growth rates in comparison to the average scenario S22 is yet another reason. This suggests a detailed study of the demand history, in order to construct more realistic market scenarios so as to improve the quality of the stochastic solutions.

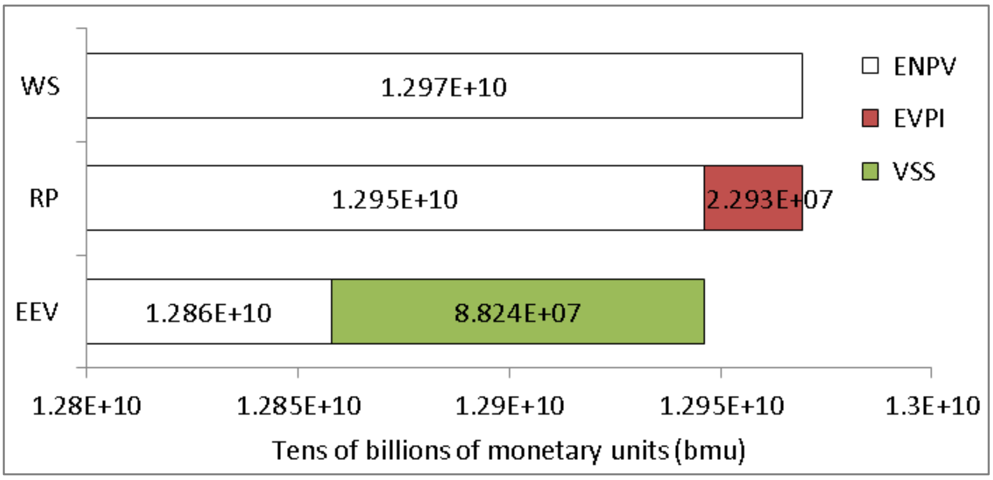


Figure 4.8. Quality of the stochastic solution

4.5.2.3 Performance Statistics

Table 4.1 presents the model statistics for the execution of the WS, RP and EEV problems. The stochastic MILP is executed nine times for each scenario to obtain the WS solutions and is solved in 70280 seconds, the RP is solved in 86408 seconds and the EEV is solved in 29340 seconds. With only 3 periods and 9 scenarios, the problem is quite large as seen by the number of equations and variables. To increase the scenarios to correctly capture the stochastic information of the problem, it will be necessary to deploy decomposition methods. However, given the dimension of the problem and as the distribution of the various uncertainties is not precisely known, robust optimizing methods are seen to be more suitable to such a problem. This option was hence selected to continue the PhD studies, as will be presented in Chapter 7.

Table 4.1.Model Statistics

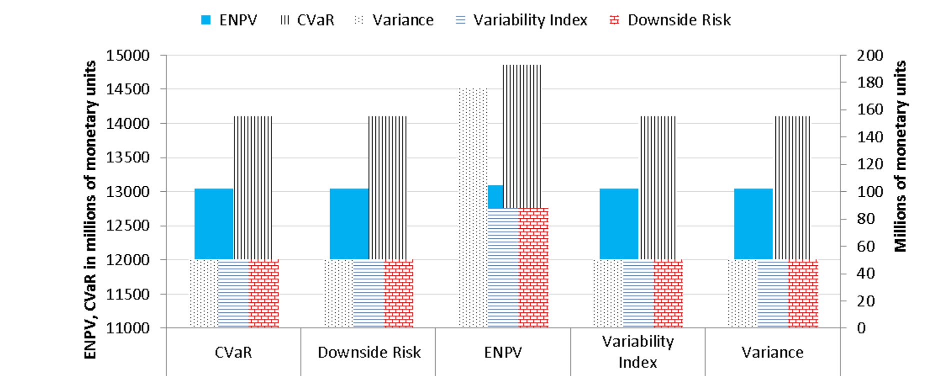
Model	Problem	Equations	Total Variables	Binary Variables	Gap (%)	CPU(s)
Stochastic MILP	WS x 9	30149	101841	4259	0.00	1005 – 27799
	RP	134785	415685	4259	0.67	86408
	EEV	88610	368053	3971	0.00	27799 + 1541

4.5.3 Multi-objective Stochastic MILP results

In the previous section, were presented the implementation results for the stochastic MILP, wherein were optimized the real case network's total ENPV and evaluated the quality of the solutions using the stochastic solution indicators. In the current section, are presented the results for the multi-objective stochastic MILP. The four risk measures implemented are first optimized individually and observed the effect of this optimization on the remaining objectives. The multi-objective stochastic MILP is then executed within the augmented ε-constraint algorithm framework to obtain the Pareto-optimal solutions that simultaneously maximize the ENPV and minimize a risk measure for a set of grid points on the efficient frontier. The ENPV cumulative probability distribution curves, while optimizing each risk measure individually, are also presented.

4.5.3.1 Optimization of a Single Objective

The single-entity current network design is first optimized for a single objective which is the maximization of the ENPV or minimization of a single risk measure. While specifically optimizing one of the following objectives, ENPV, Variance, Variability index, Downside risk, or CVaR are also simultaneously computed (but not optimized) the other measures, as can be seen in figure 4.9. The ENPV and CVaR are plotted on the primary axis while the Variance, Variability index and Downside risk are plotted on the secondary axis. It is observed that all the risk measures are at their highest values when the ENPV is maximized, which is when the ENPV is at 13.09 bmu. Although this method can be used to observe the impact of the optimization of one objective on the other objectives, it has not been tested for each and every objective with varying levels of the desire profit target. In all the risk measures optimized here, the target profit was fixed to the ENPV, reason why there are not significant differences between the objectives when optimizing for the variance, variability index, downside risk, and CVaR.



	CVaR	Downside Risk	ENPV	Variability Index	Variance
ENPV	13041317448	13041342441	13091774082	13041342441	13041321815
CVaR	14100907000	14100407000	14846035000	14100407000	14100819000
Variance	50456633.2	50431640.2	175426141	50431640.2	50452265.6
Variability Index	50456633.2	50431640.2	87713069.6	50431639.6	50452265.7
Downside Risk	50456633.2	50431639.6	87713069.6	50431640.2	50452265.7

Figure 4.9. Risk measures while optimizing single objective

4.5.3.2 Minimization of the Variance

The stochastic MILP integrates with the augmented ε–constraint method algorithm proposed by Mavrotas and Florios (2013), thereby permitting the study of the model's response for multiple scenarios and also the study of solution trade-offs while simultaneously maximizing the ENPV and minimizing a risk measure for the PSC design under study.

Figure 4.10 presents the Pareto-optimal solutions when ENPV is maximized and the variance of the net profit minimized. The variance is calibrated for 9 grid points which are generated ranging from the lowest variance of 0.05 up to a maximum variance of 0.17 bmu. It is seen that the ENPV is fairly stable between a minimum of 13.04 for the lowest variance and a maximum of 13.09 bmu, which has the highest variance. Figure 4.11 presents the cumulative probability distribution for the variance minimization problem, which generates NPVs ranging from 12.4 bmu up to 13.09 bmu. The NPVs can be compared against the ENPV of 13.04 bmu and observed the probability of the NPV being higher than the ENPV is 0.525. Here, the maximum NPV that could be attained is 13.09 bmu.

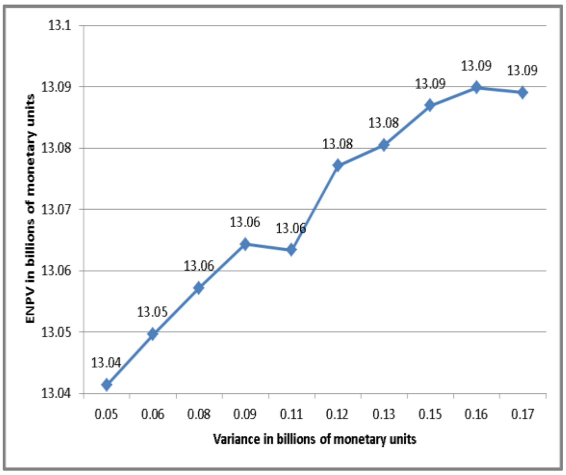


Figure 4.10. Pareto-optimal solutions for maximizing ENPV and minimizing Variance.

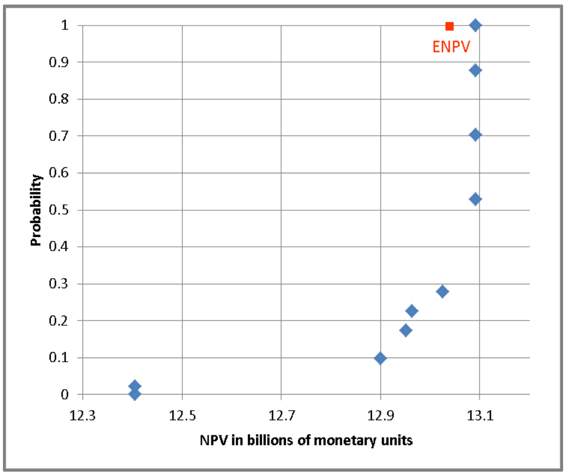


Figure 4.11. Cumulative probability distribution for Variance minimization.

4.5.3.3 Minimization of Downside Risk

Figure 4.12 presents the Pareto-optimal solutions when ENPV is maximized and the downside risk of the net profit minimized. The downside risk is calibrated for 9 grid points which are generated ranging from the lowest downside risk of 0.05 up to a maximum downside risk of 0.09 bmu. It is seen that the ENPV is fairly stable between a minimum of 13.04 for the lowest downside risk and a maximum of 13.09 bmu which has the highest downside risk. Figure 4.13 presents the cumulative probability distribution for the downside risk minimization problem, which generates NPVs ranging from 12.4 bmu upto 13.09 bmu which may be compared with the ENPV of 13.04 bmu. It is observed that the NPV has the probability of only 0.29 of being lower than the ENPV. The maximum attainable NPV is 13.09 bmu.

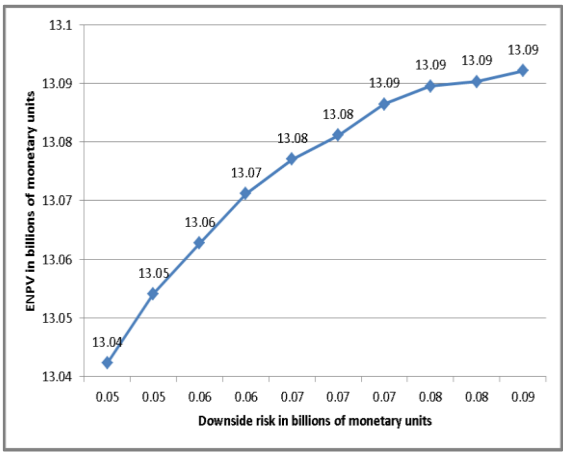


Figure 4.12. Pareto-optimal solutions for maximizing ENPV and minimizing downside risk

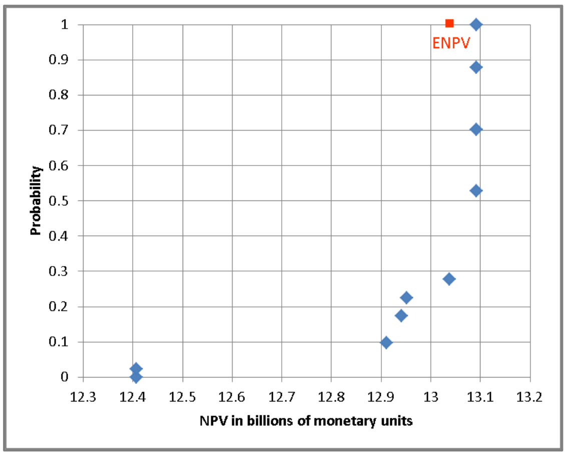


Figure 4.13. Cumulative probability distribution for Downside risk minimization.

The above results show the relation between the various measures when optimizing for maximizing net profits or for minimizing a risk measure. It is also observed that the MILP and methodology adequately maximizes profits and simultaneously manages risk for the downstream PSC design and planning problem with uncertainty. Other risk measures may be incorporated in order that the MILP may generate better and resilient PSC networks.

4.5.3.4 Minimization of CVaR

The multi-objective MILP is now executed, considering the single-entity, retrofit network design, for maximization of the ENPV. The base scenario probabilities considered here, are 0.35, 0.5 and 0.15 for the optimistic, realistic and pessimistic scenarios respectively. Figure 4.14 presents the cumulative probability distribution for this ENPV maximization problem. The PSC designs for the 9 scenarios generate NPVs between 12.23 to maximum 12.89 bmu. The cumulative probability distribution curve shows that the probability for the MILP generated network design generating NPVs higher than the ENPV value of 12.64 bmu, is 0.5275.

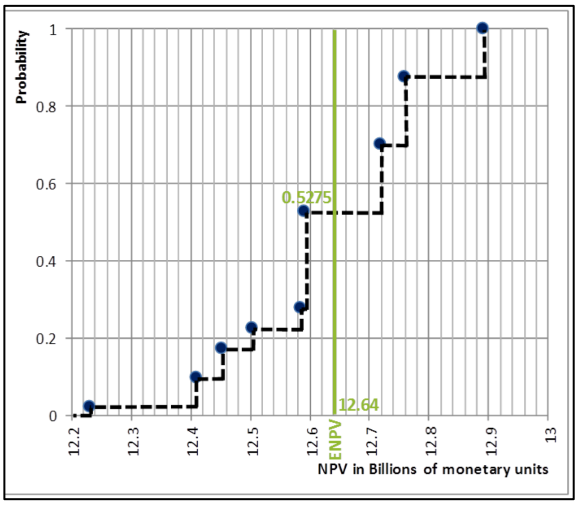


Figure 4.14. Cumulative probability distribution for ENPV maximization.

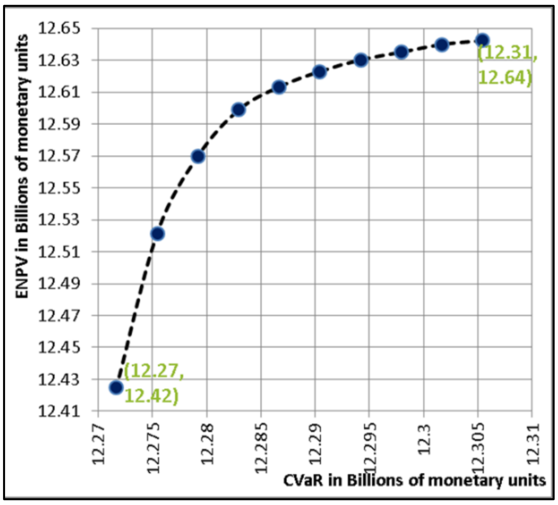


Figure 4.15. Pareto-optimal solutions ENPV/CVaR

The multi-objective MILP is now executed for the single-entity retrofit network design. Figure 4.15 presents the Pareto-optimal solutions for the Portuguese PSC when simultaneously maximized the ENPV and minimized the CVaR. By fixing the VaR to 12.64 bmu which is the result of the max(ENPV) problem and using a confidence level of 95%, are obtained the minimum CVaR of 12.27 bmu and maximum CVaR of 12.31 bmu. The CVaR range is then calibrated into 10 equidistant grid points and solved the max(ENPV) problem for these CVaR grid points deriving the ENPV values which vary between a minimum of 12.42 up to a maximum of 12.64 bmu. It can be seen that the min(CVaR) problem implies a reduction of the ENPV and the Max(ENPV) problem yields a higher CVaR risk as seen in the Pareto-optimal curve. Based on these results, it can be concluded that the MILP and augmented ε-constraint methodology adequately models the downstream PSC design and planning under uncertainty.

4.5.3.5 The Portuguese PSC Retrofit Designs

In figure 4.16, are presented the uni-entity retrofit designs, the first considering the ENPV maximization objective and the second considering the CVaR minimization objective. The red pins show the differences among the strategic decisions of the two designs. In the Max(eNPV) problem, the Azambuja-Lisboa pipeline has been additionally installed, specifically for product A1. Considering the Min(CVaR) problem, the model installs instead a new pipeline from Sines to Lisboa, including a storage depot at Lisboa for the products U5, GO, B5 and A1, wherein no depot storage is installed for A1, which means that here is considered the secondary transportation. In this design, and with respect to Azambuja, which is an existing installation, a reduction of the installed capacity for the products U5, GO, A1 and B5 is observed.

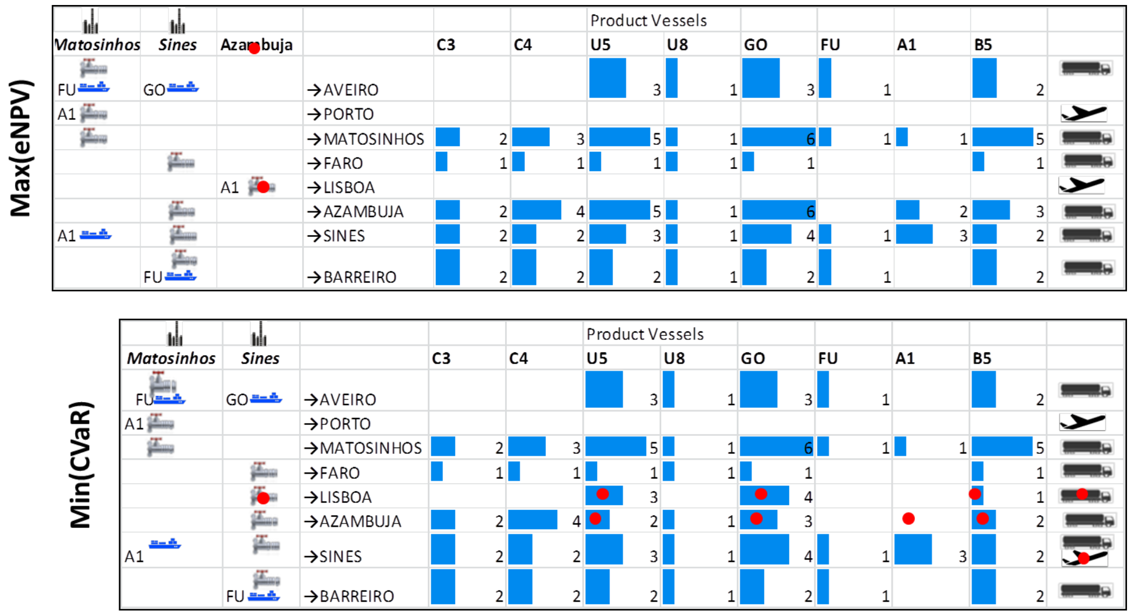


Figure 4.16. Portuguese Downstream PSC retrofit designs

The multi-objective methodology validated above with the real case, may be used to minimize variance, variability index, downside risk, CVaR or other risk measures. While variance is the summation of the modulus of the differences between ENPV and NPV per scenario multiplied by its probability, the variability index only considers the positive differences. The downside risk is similar to the variability index, however the reference is not ENPV but a NPV target value defined by the decision maker. The bad news for the variance is that negative differences will nullify positive differences whereas NPVs higher than the target value will be discarded for the variability index and downside risk measures. As the CVaR considers both the expected profit and the profit variability where the trade-off between the two is modelled using a weighting parameter, it is a coherent risk measure, and therefore has been extensively used.

The multi-objective MILP maximizes the ENPV and minimizes a risk measure, thereby generating robust network designs, which achieve higher NPVs in the presence of demand uncertainty. The network is more flexible and adjustments are made at the tactical level for each demand scenario. The cumulative probability distribution plots is an interesting form of comparing the scenario NPVs and the risk measure variance, variability index, downside risk, and CVaR. Given two conflicting objective functions, the ε-constraint method optimized first one objective function, divided its solution space into equidistant grid points, introduced a constraint to fix the first objective function at each grid point, and obtained the Pareto-optimal efficient solutions that optimize both the objective functions. Hence, the decision maker may choose one of many efficient solutions based on his/her personal preferences.

The decision maker's risk attitude is of much relevance, as it ultimately determines the appropriate risk measure to be used. Basically, a risk averse decision maker who prefers the network design that guarantees scenario NPVs higher than a minimum value even if this means a lower ENPV, would use the downside risk. If he/she prefers less variability of the NPV, then the variability index would be more suitable. However, if the decision maker has a risk-seeking attitude, then he/she would prefer the design that offers scenario NPVs aimed at a higher target, in which case the CVaR is considered to be the better measure.

4.5.3.6 Performance Statistics

Table 4.2 presents the model statistics for the PSC example. The MILP is executed four times to obtain the payoff table: 1) the maximum ENPV; 2) the minimum CVaR; 3) the minimum ENPV which is max(ENPV) when z2*= min(CVaR); 4) the maximum CVaR which is min(CVaR) when z1*= max(ENPV). These are the extreme points of the Pareto-optimal curve. The MILP is then executed 10 times to obtain the efficient solutions of max(ENPV) for 10 equidistant CVaR values derived from the payoff table. Each run considered 9 scenarios with execution times varying between 8402 and 86400 seconds. The model statistics demonstrate the problem complexity and the computational load for solving such problems even when considering only 9 scenarios.

Table 4.2 – Model Statistics

	Total Variables	Binary Variables	Equations	Gap (%)	CPU(s)
ENPV	433397	4591	101501	0.1	59556
CVaR	433414	4591	101515	0.1 - 3.5	8402 – 86400

4.6 Ongoing Research and Outlook

In the stochastic models, the strategic distribution network is determined independent of the scenarios, whereas the multi-stage inventories and product flows are determined per scenario. An integrated approach is crucial for the PSC to obtain optimal decisions by properly linking the information between the two decision levels. This is especially relevant as the PSC has high demand uncertainty. However, as the scenario tree approach is used, this integration and uncertainty treatment inevitably increases the problem complexity. As the number of scenarios increase in order to adequately represent the stochastic phenomenon, the problem quickly becomes intractable and unviable especially when considered various uncertainties.

For the stochastic models, simultaneous modelling of the costs, prices and demand uncertainties was also implemented, however, as the PSC planning under uncertainty is a large scale problem, the stochastic MILPs quickly ran into time performance difficulties when considered the uncertainties simultaneously, as the number of scenarios multiplied from 9 to over 125, even after some reduction. Decomposition strategies and robust optimization methodologies have therefore been proposed for their simultaneously modelling in the PSC.

The augmented ε-constraint methodology was implemented specifically to reduce the numerical issues with uncertainty. However, other strategies could also be used to manage the complexity of these problems. Specifically, the Sample Average Approximation (SAA) and Monte Carlo simulation based approaches have been proposed in Shapiro and Homem-de Mello (1998) for the two-stage stochastic programing problems. These consider a controlled size of scenarios, in order to derive the optimal solution approximations, thereby increasing the model efficiency, the quality and the precision of the solutions. Another approach that could be used is the application of the stochastic Benders and the Lagrangean decomposition procedures which were recently proposed in Oliveira *et al.* (2013, 2014). The model structure would require to be changed from the compact node-variable mathematical formulation to the scenario-variable formulation in order to be decomposable and exploit the scenario structure.

The two-stage stochastic MILP developed, maximized the ENPV considering uncertain demand scenarios with known probabilities, where the first stage are here and now decisions and the second stage are wait and see decisions. The study of the performance indicators VSS, EVPI, and EEV showed low values for the stochastic solutions. Hence, fresh efforts are underway, with the Extended Pearson-Tukey Method from Clemen and Reilly (2003) and recently available data, to construct new uncertain scenarios and probabilities, based on the demand growth percentiles per localization, entity and product. The 15 year historical data is being used to obtain the future crude oil costs, product prices and the customer demand forecasted scenarios. This could be corrected by calculating the mean absolute deviation (MAD), mean square error (MSE) and/or mean absolute percent error (MAPE).

4.7 Conclusions

The current chapter proposes a stochastic mixed integer linear program (MILP) and a multi-objective MILP for the strategic and tactical planning of a collaborative downstream PSC networks considering demand uncertainty. This work is an extension of the deterministic models presented in Chapter 3 and 4 of this thesis. The first stochastic model maximizes the expected net present value (ENPV) considering uncertain demand scenarios. The second is a multi-objective model that incorporates the augmented ε-constraint method, for simultaneously maximizing the ENPV and minimizing a popular risk measures such as the variance, variability index, downside risk, value-at risk and the conditional value-at-risk (CVaR) in order to manage the demand uncertainty.

The models are validated for the planning of the real-case Portuguese PSC, as a single and multi-entity network, and current, grassroots and retrofit designs, by considering the optimistic, realistic and pessimist demand scenarios for three 5 year periods. Computational results are first presented for the stochastic MILP, comparing the stochastic RP solution to the EEV and WS solutions, and determining the EVPI and VSS measures. It is identified an opportunity to study and capture the market variations and develop realistic scenarios in order to obtain better solutions. Pareto-optimal solutions are then obtained for the single entity current network and the single entity retrofit network designs, wherein are compared the network designs and the expected profits. As the expected profits are not significantly affected when the risk measure is simultaneously minimized, network designs with lower risk are obtained.

To increase the protection for uncertainty, the risk is minimized, thereby increasing the PSC's flexibility, however, this increases the PSC's complexity and as a consequence decreases the profits. Using the resilience concepts, the deterministic grassroots network is found to have the lowest node and flow complexity, the highest density and a small number of critical nodes, which means that the network is less flexible and could become infeasible or suboptimal in the presence of uncertainty. The MILP solutions could be therefore improved, by considering the resilience indicators in the objective function or constraints.

The objective of developing the stochastic MILPs was to model the costs, prices and demand uncertainties for the PSC planning problem, however, time performance difficulties have been identified. Hence, the study of decomposition strategies and alternative robust optimization methodologies are proposed for simultaneous model of these uncertainties.

CHAPTER 5. Strategic and Tactical Planning Robust Optimization of Petroleum Downstream Supply Chains under Uncertainty

5.1 Abstract

Petroleum Supply Chains (PSC) are networks of critical infrastructures whose efficient design and planning are crucial to the sustainability of national economies. In this chapter such a problem is addressed for the collaborative design and tactical planning of the PSC, and is proposed a mixed integer linear program (MILP) considering a robust optimization (RO) methodology to deal with cost, price, and demand uncertainty. The decisions are: strategic such as the installation, expansion, operation or closure of depot locations, capacities and multi-modal routes; tactical such as the multi-stage inventories and volumes per entity and product, at the importation, production, exportation, storage, transportation, and retail stages; and the strategic-tactical fair price tariffs at the storage depots and transportation links. The real-case of the Portuguese PSC network is studied, including costs, prices, and demand from the past twelve years, in order to model uncertainty and generate the optimal collaborative current, grassroots, and retrofit network designs. The RO model generates robust and trade-off solutions for the Portuguese PSC, which are presented and discussed in detail, identifying insights and opportunities for future development.

5.2 Introduction

Petroleum supply chains (PSC) are strategic and high-value supply chains, whose efficiency is crucial to virtually all business sectors. Being a long supply chain, it is a complex network of infrastructure and processes that have a combination of precedencies in order to guarantee the quality and operating conditions of the petroleum products. This investment intensive industry comprises of a sequence of activities that source, transform, and deliver petroleum products to the end consumers, in the transportation, heating, residential, industrial, commercial, and electric utility sectors of the economy. Such supply chain comprises of two major areas (see figure 5.1), the first being the upstream, which involves crude oils' prospection, exploration, production, and transportation. The second is the downstream, which involves the petroleum products' refining, primary distribution, depot storage, secondary distribution, and the retail commercialization.

Given the importance of the petroleum sector, several European countries like Portugal (DR, 2015) and Spain (CLH, 2015) have long foreseen the need for national petroleum policies, and have developed strategic plans to achieve long term goals. An illustrative view of the current paradigm of the Downstream Petroleum industry is synthesized in figure 5.2.

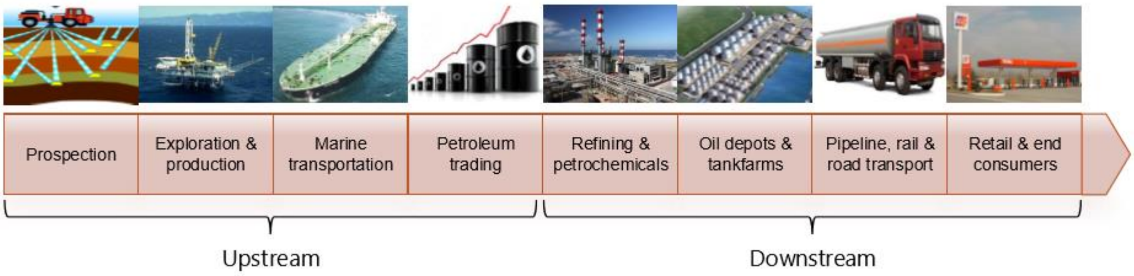


Figure 5.1. The Upstream and Downstream Petroleum Supply Chains

The challenges stem from the increased competition, unbundling of services, huge investments, reduced profits, and alternative energy competition; and the crude costs, product prices and demand uncertainties; as well as the regulatory changes, increased taxation, emission reductions, political, financial instabilities, law reforms, and fair price controls. Nevertheless, significant opportunities come from the rise in the oil products' demand due to fleet renewals, increased trucking and aviation, improved operating conditions, new investments in inter-modal facilities, and the availability of geographic information systems. The handling of these challenges and opportunities create very large scale problems where advanced PSC planning models are decisive to improve the PSC efficiencies through the collaboration of entities, the integration of planning levels and multi-echelons, efficient inventories, timely procurement, and risk management methodologies.

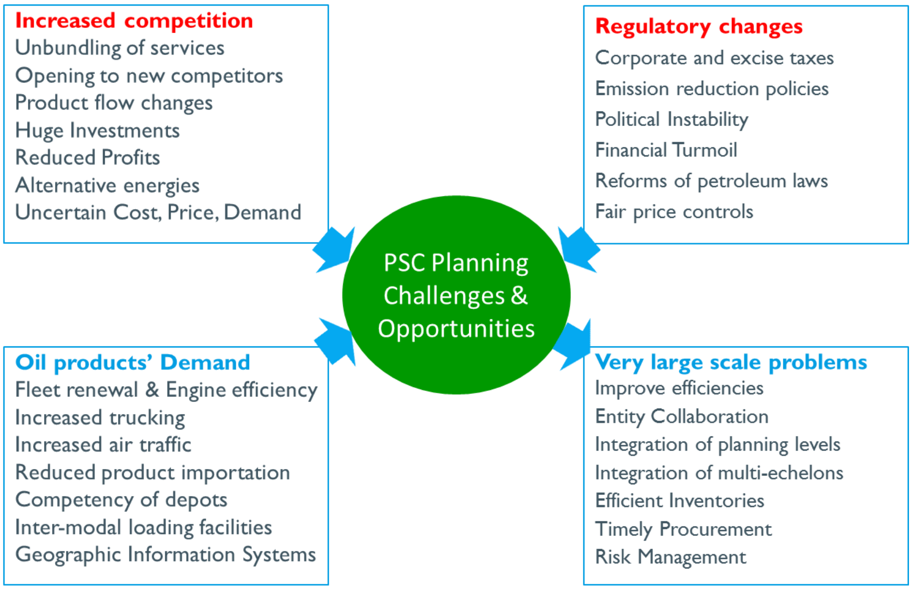


Figure 5.2. The Downstream Petroleum Industry Paradigm

The combination of the above attributes have tremendous influence on the variability of the crude oil costs, the petroleum products' prices, and customer demand, and consequently determine the sustainability or profitability of the PSC investments. This very high volatility and uncertainty in the crude costs, product prices and customer demand has created an unprecedented need, and therefore opportunity, for the design and planning of collaborative integrated networks, which are crucial for the improvement of the PSC efficiencies and profits.

5.3 Literature Review

The PSC literature, related to the upstream PSC optimization, has focused the selection and design decisions for the crude oil prospection, exploration, and production, whereas the downstream PSC has focused two fundamental activities related to petroleum products: i) the refinery process design, the unload, and upload scheduling; and ii) the design and planning of PSC networks. Recently, three reviews studied the PSC design and planning models. Leiras *et al.* (2011) categorize and review the research on upstream, midstream, and downstream PSC planning models, whereas, Shah *et al.* (2011) review the oil refinery scheduling and planning literature. The developments in PSC are also reviewed in Sahebi *et al.* (2014), identifying it as a flourishing research area with many unaddressed opportunities such as: vertical integration of strategic and tactical decisions; horizontal integration of the complex crude oil supply chain; nonlinear modelling essential for the formulation of refinery operations; development of efficient solution techniques for multi objective modelling concerning global factors, including international taxation, transfer prices, and environmental impacts. Multi-stage stochastic modelling has been identified as a key area of investigation to deal with uncertainty.

When considered refinery planning, several works are found to address diverse problems. Neiro and Pinto (2004) address the refinery operational planning problem through the development of a mixed integer non-linear program (MINLP). The stream flow rates, properties, operational variables, inventories, and facility assignments are determined. The production units, tanks, and pipelines are considered as aggregate nodes, which interconnect refineries, terminals, and pipelines. They use decomposition methods to increase the number of time periods and scenarios. Kim *et al.* (2008) propose a supply network and market planning MILP, which determines the refinery production rates, and the transportation routes from the distribution centers to markets. Profits are maximized, these being the differences between the revenues and the depot and transportation fixed and variable costs. Al-Othman *et al.* (2008) propose a two-stage stochastic program (SP) for the operational planning of crude oil production, processing, and distribution at refineries in order to maximize the resource utilization and profits. The crude production plans are the first-stage decisions while the refinery and petrochemicals production, shipment, storage, lost demand and backlog volumes are the second-stage decisions. Guyonnet *et al.* (2009) propose the integration of the crude oil unloading, refinery production planning, and the product distribution models, where the unsatisfied demands are the interfaces which are penalized, resulting in the optimization of the overall PSC profits.

With respect to pipeline transportation, Cafaro and Cerdá (2004) present a multiproduct pipeline scheduling MILP, for transportation between an oil refineries and depots. By adopting a continuous time and volume representation, they reduce the model size and the computational time to solve the model, and consequently increase the length of the planning horizon and the flexibility of the slug pumping, when compared to discrete representations. Cafaro *et al.* (2011) later improve the continuous-time MILP to reduce the energy consumption and the pump maintenance costs. Heuristic rules are incorporated, to select the receiving terminals, minimize the total restart flows, and reduce the single-delivery pumping runs. Relvas *et al.* (2006,2007) integrate a multiproduct pipeline scheduling and rescheduling MILP with the depot inventory management. Product sequences, pumping schedules, and batch volumes are determined while considering the customer demand, quality control and operating constraints. Boschetto *et al.* (2010) propose a hierarchical approach, integrating a MILP model with a set of heuristic modules to address a real-world pipeline network. More recently, Relvas *et al.* (2007) present a single holistic model, which compared to their earlier models, obtains faster solutions for smaller time frames without the use of decomposition approaches.

Operational planning of downstream PSCs was first modelled in Escudero *et al.* (1999), using a two-stage stochastic scenario analysis and a deterministic equivalent MILP. Later, Dempster *et al.* (2000) develop deterministic and stochastic models for the PSC tactical and operational planning to minimize overall costs. MirHassani (2008) propose an operational planning MILP for a capacitated transportation network of existing refineries and depots. While satisfying the customer demand, are determined the inventory shortages and excesses, for minimizing the total inventory and transportation costs. More recently, Guajardo *et al.* (2013) propose a tactical planning model to minimize the production, transportation, inventory planning, hubs and depot's operational costs. Vendors' contributions are maximized for distribution planning and decentralized sales.

Strategic planning of downstream PSCs is introduced in Oliveira *et al.* (2013,2014) by presenting two two-stage stochastic MILPs, where the first uses Lagrangian decomposition and the second the Benders (L-Shaped) decomposition. The investment and logistics costs are minimized, while determining the portfolio and timings of network capacity expansions, and inventory decisions with uncertain demand. Fiorencio *et al.* (2014) proposes, for NPV maximization, a deterministic MILP decision support system (DSS) that determines the investment portfolio and installation times of vessel, and transportation capacity expansions. Fernandes *et al.* (2013,2014) propose strategic and tactical planning MILPs that maximize the network profits for the individualistic and collaborative designs of uni and multi-entity current, grassroots, and retrofit PSC networks. They determine the installation, operation, and expansion of depot locations, resource capacities, transport modes, and routes; the multi-stage import, export, production, storage, transportation flows and inventories of crude oil and petroleum products; and the piecewise linearized economy of scale costs and tariffs per location, entity and product.

Uncertainty has emerged to be a crucial aspect, as identified in the above works and as expressed in the studies of Deloitte (2010), and E2Open (2015), amongst others, with respect to today's industrial reality. A clear need has emerged to build efficient and robust PSCs, that take into account the modern reality and problems of current PSCs while exploring collaborative models under an uncertain context. Recently, Fernandes *et al.* (2015) proposed a two-stage stochastic programming (SP) model for the PSC strategic and tactical planning under demand uncertainty. The augmented ε-constraint method was used to simultaneously optimize opposing objectives: maximize the expected net present value (ENPV); and minimize the conditional value-at-risk (CVaR) of the NPVs, wherein test results for the Portuguese PSC are presented. One of the major problems observed is the multi-dimensional scenarios' complexity, which results in performance difficulties due to the size of the model. Robust optimization was identified for future research of simultaneous modelling of cost, price, and demand uncertainty.

Based on the downstream PSC literature and the difficulties therein, it is identified the need to develop collaborative PSC design and planning models and integrate crude oil costs, product prices, and customer demand uncertainties. As discussed, two-stage stochastic programs have revealed some difficulties in modelling uncertainty due to augmented complexity. On the other hand, a tractable Robust Optimization (RO) methodology (Ben-Tal and Nemirovski (1998,2000), Bertsimas and Sim (2003)), has become increasingly popular in operational research and supply chain management to handle uncertainty without using scenarios. The application of robust optimization in general is not new to planning problems and even to some areas of the PSC, more specifically related to the uncertainty management in refinery scheduling and operational planning problems. However, to the best of our knowledge, RO has not been developed for a problem with the characteristics and the dimension of the problem under study, which is a large scale real world downstream PSC strategic and tactical planning problem. Moreover, most of the works published for the PSC are confined to what the authors call as a "robust stochastic optimization" approach (Mulvey *et al.* 1995, Kouvelis and Yu, 1997), which consists of a scenario-based approach with a goal programming formulation to control model and solution robustness.

Given the difficulties of the scenario based RO and two-staged stochastic models, the non-scenario based RO is seen with revived interest, as the PSC planning under uncertainty is a large-scale problem. The impetus to implement RO for the downstream PSC strategic and tactical planning problems, has motivated our study of the RO methodologies, which in the recent years have undergone significant developments, and hence, are the focus of the following section.

5.4 Developments in Robust Optimization (RO)

Over the past 10 years, RO has been consolidating as a powerful methodology to handle uncertainty in optimization problems. RO assumes that uncertainty parameters are random variables, which belong to a given uncertainty set, which in turn describes how uncertainties span over their nominal or deterministic values. RO is sometimes recognized as being more appealing than stochastic programming approaches (SP) because robust counterparts remain computationally tractable for various types of uncertainty sets, for example, box and polyhedral. Also, the full knowledge of the underlying distributions of the random variables is not necessary in RO, but historical data may be useful to derive more accurate uncertainty sets. On the other hand, SP has the disadvantage of being dependent on the knowledge of the probability distributions, as well as on the type and number of scenarios.

The goal of RO is to determine solutions that are immune to data uncertainty so that they are always feasible and at the same time near-optimal in spite of the uncertainty. In this context, Soyster (1973) was apparently the first author who discussed a relatively general methodology to incorporate data variation within inexact linear programming (LP) problems, by using the idea of set-inclusive constraints. For this reason, his work is viewed as the first steps in RO. However, as the Soyster (1973) model basically consists of replacing data uncertainty by their respective worst-case values, the resulting solution is overly conservative, i.e., it offers the highest protection, but at the expense of a big deterioration in the optimal value.

Some years later, Ben-Tal and Nemirovski (1998,2000) proposed a less conservative approach to provide trade-offs between solution robustness and cost in LP problems and general convex programming. Relatively immunized or robust solutions are generated by quantifying the level of infeasibility of the nominal solution and also the impact on the optimal value. This formulation considers uncertainty sets that are ellipsoids, thus leading to nonlinear robust counterparts, which can be computationally intractable depending on the size of the instance.

To overcome the aforementioned drawbacks, Bertsimas and Sim (2003) developed a tractable model where the degree of conservatism for each constraint may be adjusted according to the tolerance of the decision-maker. They considered that it is unlikely for all the parameters, exposed to uncertainty, to reach their worst-case value simultaneously, and therefore derived the concept of *budget of uncertainty* to bound the overall cumulative amount of variation regarding the nominal values. The authors also illustrated a high potential of their methodology for discrete problems, by showing that the corresponding robust counterparts do not change the complexity of the original problem when polyhedral uncertainty sets are adopted. In a further paper, Bertsimas and Sim (2004) derived probabilistic guarantees for the violation of a given constraint when the proposed budget of uncertainty is smaller than the total amount of coefficients that may change. Hence, the decision maker can intuitively assign appropriate budget parameters to each constraint based on the acceptable probability of violation. Bertsimas and Thiele (2006) extended the previous ideas to model RHS-uncertainty in general supply chain problems.

Several papers have employed these RO ideas in diverse contexts and applications; see Beyer and Sendhoff (2007) and the references therein for a general overview of the applications in the field of RO and an outlook of research perspectives. The authors identified that RO could be the rule in engineering design and optimization problems, however, boundaries need to be expanded, specifically with respect to direct search techniques, expected performance indices, multi-criteria optimization, and robustness constraints' coupling. Recently, Bertsimas *et al.* (2011) surveyed the theoretical and applied research in RO, with a focus on its modelling power and its wide application in the fields of portfolio optimization, statistics, learning, estimation, supply chain management, and various areas of engineering. Their review on RO developments identified research directions related to adaptable RO tractability for multi-stage decision-making, characterization of when robustness is cheap or expensive, and RO with a data-driven perspective. In an interesting work, Alem and Morabito (2012) implemented a RO approach for the production planning of typical furniture companies by using polyhedral uncertainty sets, and individually and simultaneously modelling cost uncertainties in the objective function and the demand uncertainty in the constraints. The authors showed that beyond a certain level of the budget of uncertainty, there is no further deterioration of the objective function values and the probability of constraint violation tends to zero.

With respect to the petrochemical industry, Lin *et al.* (2004) propose RO for the design, synthesis and scheduling of multipurpose batch plants, considering bounded and symmetric uncertainty in objective function coefficients, and in the left and right hand side parameters of constraints. Uncertainties related to processing times of tasks, product demands, and prices of raw materials and products are addressed. Janak *et al.* (2007) later extend this work for uncertainty with known probability distributions. These works demonstrate the generation of reliable schedules, trade-offs between conflicting objectives, and potential for solving large real-world problems.

Within the context of PSCs, Ribas *et al.* (2010) proposed stochastic optimization approaches, for the expansion of conversion units, transportation infrastructures, and operational planning at refineries. Considering crude oil production, products demand, and market prices uncertainties, they propose three formulations: i) a two-stage stochastic model, ii) a robust min–max regret model, and iii) a max–min model. Test results for a large scale PSC in Brazil, show significant financial differences, reduction and postponement in investments with lower demand for products. Al-Qahtani and Elkamel (2010), and Leiras *et al.* (2010) extend the deterministic MILP presented in Al-Qahtani and Elkamel (2008), for refinery production planning and expansion decisions, to handle the crude oil costs, product prices and demand uncertainties. The first uses the idea of model and solution robustness from Mulvey *et al.* (1995), identifying complexity issues related with scenarios. The second uses the non-scenario RO approach from Bertsimas and Sim (2003), obtaining economic trade-offs and production flexibility. The methodology is identified to have potential for strategic planning of the PSC.

In the bio product supply chain, McLean and Li (2013) propose an affinely adjustable robust formulation, for strategic planning under uncertainty. They show that uncertain equality constraints can be reformulated into tractable deterministic constraints without any assumptions about the uncertainty region, when uncertainties are bounded by the infinity norm. Tong *et al.* (2014) also apply RO for the optimal design and planning of a biofuel supply chain in order to minimize the unit cost of final products. They simultaneously consider the biofuel SC design, the biomass integration strategy selection, and production planning with demand and supply uncertainties.

The context of uncertainty modelling is focussed in Adida and Perakis (2010), comparing the robust optimization with the stochastic methodologies. Stochastic models are identified to be complex and good provided that the probability distributions are correct, whereas, RO leads to tractable formulations, allowing efficient control of the trade-offs between performance and conservativeness even with limited information. Based on the findings in the literature and earlier work with large-scale PSC design and planning problems, where numerous numerical difficulties are observed even with few scenarios, we propose to use the Bertsimas and Sim[34] methodology for the PSC design and planning problem.

In the view of these limitations, and the intrinsic complexity of the problem described in the following section, we propose to use RO instead of the traditional two-stage stochastic programming approach, motivated by its underlying strengths, namely: the absence of an explicit probabilistic description of the data; the availability of limited information (mean value and its range) about the distributions of the uncertainties; avoiding estimation and specification of a large number of scenarios and their probabilities or expected recourse costs; the possibility of modelling various sources of uncertainty; and the need for less conservative solutions for the PSC design and planning problem. The RO implementation is based on the approach used in Alem and Morabito (2012), and the RO procedure presented in Gorissen *et al.* (2015) This method first solves the optimal nominal problem and determines the uncertain parameters; and then iteratively determines the uncertainty sets, determines and adjusts the budgets of uncertainty, formulates the robust counterpart, solves the adjustable robust counterpart, verifies the quality of the robust solution, and if the solution is too conservative repeats the iterative steps.

The contributions of this chapter are as follows:

1) We extend the PSC strategic and tactical planning MILP presented in Fernandes *et al.* (2014), by incorporating petroleum products' importation at depot level by all petroleum entities, restricting bridge crossings to a maximum number of journeys per route per period, simplifying its notation and formulation, improving its performance and modularity for the implementation of RO.
2) We develop for the PSC planning problem, three numerically tractable robust formulations, that incorporate crude oil costs, ex-refinery product prices and customer demand uncertainties, which are first individually modelled and then simultaneously without assuming specific distributions.
3) We improve our real case PSC database consisting of geographical information; fixed, variable and volume based costs for depots, typical resources and transportation infrastructures; crude oil costs, product prices, and customer demand based on our research on industry data and benchmarks.
4) We perform an extensive numerical study to access the performance of the different models based on the real case Downstream PSC network in Portugal.

5) We present the results and insights into the relationships between the budgets of uncertainty, profits, strategic decisions of installation of depots, pipelines and resource capacities and the tactical decisions related to crude oil and product inventories as well as the product transfer, importation, and exportation volumes, and the strategic-tactical decisions of the service costs and tariffs per entity.

The research developments are now presented, beginning with the problem statement, the deterministic model and its reformulation to a robust model, a real case study, the construction of the uncertainty sets, followed by the implementation results, conclusions, and proposals for future research.

5.5 Problem Statement

As can be seen in figure 5.1, the downstream PSC begins with the refineries, which buy crude oil and produce petroleum products. These are either sold to petroleum companies for national consumption, or exported at wholesale market prices, thereby collecting a refining margin. Refineries support the cost, insurance, and freight (CIF) charges in case of exportation. The petroleum companies transport the product from refineries to storage depots using the primary distribution, which is generally undertaken by pipeline, maritime or railway. They also import products by paying the wholesale price and the CIF costs for transportation up to their depots. The storage depots hold the product on behalf of the petroleum companies, before transporting them to retail clients using the secondary distribution, normally using tank trucks on road. The secondary distribution may also be undertaken by pipelines or railways, when huge volumes are transported to specific destinations such as airports. The retail stage commercializes the product to the final consumer at petrol stations, factories, airports, and hospitals, thereby retaining a retail profit margin.

The downstream PSCs are a critical part of the global PSC networks, and therefore, are affected by many types of uncertainties, specifically related to crude oil costs, ex-refinery product prices, customer demand, supply delays, labour costs, technology, process yields, and facility disruptions. Among these, the first four are the most common, and therefore, these have the major impacts whereas the last three types are more infrequent. As the mission of the petroleum companies is to construct and manage efficient and robust PSC networks, our goal in this work is to develop a strategic and tactical planning model for the profit maximization of the entire PSC while considering crude costs, product prices, and customer demand uncertainties. An integrated view of the problem description and planning decisions is presented in figure 5.3 and characterized thereafter.

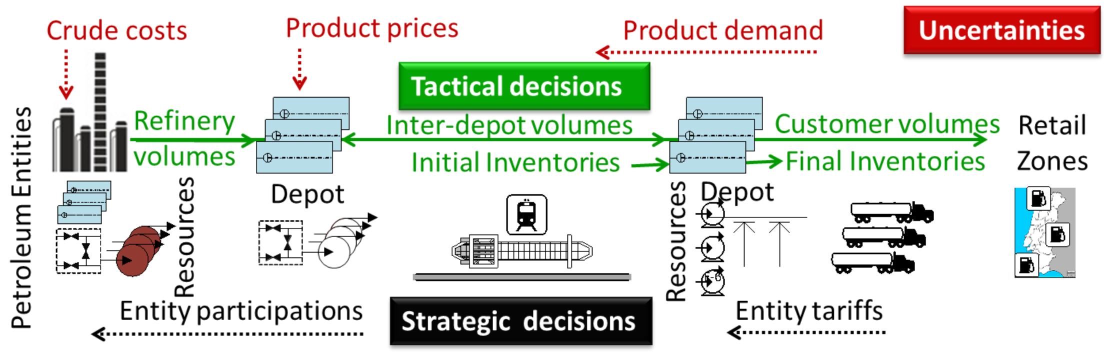


Figure 5.3. The Downstream PSC Strategic and Tactical planning problem

The aim is to develop a tractable robust optimization MILP for the collaborative strategic and tactical planning of the PSC under uncertainty, whereby:

- Given a set of petroleum companies that produce or import a set of petroleum products at a known set of refineries, and commercialize them to a set of customer regions;
- Determine the current, grassroots, and/or retrofit PSC network designs for the multi-echelon, multi-product, and multi-modal transportation PSC network;
- Strategic decisions of installation, operation, expansion, or closure of storage depots, resource capacities, and multi-modal routes;
- Strategic-tactical decisions of entity investment participations and the economy of scale costs and tariffs per location, route, entity and product;
- Tactical decisions of time-dependent and multi-stage (production, storage, transportation, importation, exportation, and customer fulfillment) volumes and inventories for crude oil and derived products;
- So as to maximize the total network profit, comprising the oil company direct profits and their consolidated revenues from participated PSC member infrastructures: refineries, depots, transporters, and retailers;
- Subject to the production, depot, transportation, resource capacity, inventory, and demand restrictions;
- Considering uni-entity and multi-entity networks; pipeline transportation as product flow per time period restricted to the pipeline physical capacities; and ship, truck, and wagon transportation as discrete product flow blocks, where the total transported volume for the permitted products and between origin-destination pairs, is restricted to the modal fleet capacity per time period;

Such that the solution is robust and feasible at the uncertainty level defined by the decision maker for the crude costs, product prices, and customer demand uncertainties.

The development of the PSC planning MILP was motivated by the pounding need in Portugal in recent years following the awareness of the necessity to restructure the Portuguese PSC and the absence of a benchmarking and decision support tool to assist the different decision makers, specifically at the level of the PSC regulators, PSC operators as well as petroleum companies (main producer and other major companies that share the physical network).

One important aspect is that the petroleum companies are full or partial owners of the refineries, storage depots, transportation infrastructures and retail stations, and therefore incorporate a relevant share of their profits. The MILP is therefore implemented so as to be configurable, in order to permit designs and analysis of the PSC network based on the profile of the decision-maker. Hence, the decision maker may analyze parts of the network, or the network in the perspective of an individual company, or as an integrated collaborative network, where the petroleum companies earn revenues by commercializing products and also aggregate profits from their participations in refineries, depots and transportation infrastructures. These plants have the autonomy to purchase and manage the crude oils and petroleum products; nevertheless, they serve the needs and the objective of improving the consolidated profits of their parent petroleum companies. In order to overcome the above difficulties and explicitly take into account the different decision makers, existing and partially owned infrastructures, the PSC is modelled as a multi-entity network, thereby determining the contract tariffs per location/route, entity, and product, which are based on the model's decisions of committed volumes and entity capacities.

5.6 Formulation of the robust counterparts

To implement the RO model, we first take the PSC strategic and tactical planning MILP published in Fernandes *et al.* (2014), and make some improvements such as: incorporating petroleum products' importation at depot level by all petroleum entities; restricting bridge crossings to a maximum number of journeys per route per period; and simplifying the MILP notation and formulation, in order to improve the performance and facilitate the implementation of RO. This improved MILP, which is presented in Appendix C, maximizes the PSC total profits objective function that comprises of thirteen terms, as given below in eq. (5.1). Each objective function term is denoted as an objective sub-function and is derived using an equation or a set of equations: the refining profits $MR_{i,e,t}$ in eq.(5.2), the exportations costs $CE_{i,e,p,t}$ in eq.(5.3), the depot importation costs $CI_{j,e,p,t}$ in eq.(5.4), the crude oil inventory costs $CNC_{i,e,t}$ in eq.(5.5), the refined product inventory costs $CNR_{i,e,p,t}$ in eq.(5.6), the depot inventory costs $CND_{j,e,p,t}$ in eq.(5.7), the retail profits $ML_{k,e,p,t}$ in eq.(5.8), the retail inventory costs $CNL_{k,e,p,t}$ in eq.(5.9), the storage depot margins $MDE_{h,e,t}$ in eqs.(5.24-5.29), the storage depot tariffs $TDE_{h,e,t}$ in eqs.(5.14-5.19), the transportation margins $MTE_{h,l,m,e,t}$ in eqs.(5.20-5.23), and the transportation tariffs $TTE_{h,l,m,e,t}$ in eqs.(5.10-5.13).

At each infrastructure, the margins are incorporated by the shareholder companies of each installation, whereas the tariffs that are based on the non-amortized costs are incurred by the product owner companies. The improved MILP is constituted by four groups of constraints, specifically the volume balance constraints eqs. (5.33-5.43), the transportation network installation constraints eqs. (5.44-5.49), the storage depot installation and capacity constraints eqs. (5.50-5.59), and the minimum and maximum inventory constraints eqs. (5.60-5.67). In Appendix C, are provided the improved deterministic MILP, and the detailed RO formulation that comprises of four additional terms in the objective function and four additional groups of constraints, which respectively model the uncertainties in the crude costs, the product prices, the customer demand, and all three simultaneously.

5.6.1 Uncertainty in objective function parameters

In order to implement the robust optimization, we consider the deterministic PSC strategic and tactical planning MILP's objective function eq. (5.1), and begin by identifying and modelling the uncertain parameters iteratively in the objective function terms.

$$\begin{aligned}\psi \ =& Max\sum_{i\in I}\sum_{e\in E}\sum_{t\in T}\left(MR_{i,e,t}-\sum_{p\in p}CE_{i,e,p,t}-CNC_{i,e,t}-\sum_{p\in P}CNR_{i,e,p,t}\right)\\ &-\sum_{j\in J}\sum_{e\in E}\sum_{p\in P}\sum_{t\in T}\left(CI_{j,e,p,t}+CND_{j,e,p,t}\right)+\sum_{k\in K}\sum_{e\in E}\sum_{P\in P}\sum_{t\in T}\left(ML_{k,e,p,t}-CNL_{k,e,p,t}-CS_{k,e,p,t}\right)\\ &+\sum_{h\in H}\sum_{e\in E}\sum_{t\in T}\left(MDE_{h,e,t}-TDE_{h,e,t}\right)+\sum_{(h,l,m)|\exists\mu_{h.l.m}}\sum_{e\in E}\sum_{t\in T}\left(MTE_{h,l,m,e,t}-TTE_{h,l,m,e,t}\right)\end{aligned} \tag{5.1}$$

The robust optimization is introduced by considering two uncertain parameters $\tilde{\varphi}^{R}_{p,t}$ and $\tilde{\varphi}^{C}_{t}$, which are the ex-refinery product prices and the crude oil costs respectively. These parameters are used in eq.(5.2), which is the deterministic MILP's objective sub-function that maximizes the refinery margin $MR_{i,e,t}$. Here, $QR_{i,p,t}$ and $QCI_{i,t}$ are the refined product and imported crude volumes, whereas ξt^{R}_{p} and $\eta^{N}_{i,e}$ are the refinery tariffs and the entity participations respectively.

$$\begin{aligned}&\max_{\mathbf{QR},\mathbf{QCI}} MR_{i,e,t}=\sum_{i\in I}\sum_{e\in E}\sum_{t\in T}\left(\sum_{p\in P}QR_{i,p,t}\left(\tilde{\varphi}^{R}_{p,t}-\xi t^{R}_{p}\right)-QCI_{i,t}\times\tilde{\varphi}^{C}_{t}\right)\eta^{N}_{i,e}\\ &\text{subject to:}\qquad \text{constraints } (5.33-5.67)\end{aligned} \tag{5.2}$$

First of all, the worst-case perspective for the total network profit is reached when prices are lower than expected and costs are higher than expected. As a consequence, the uncertain parameters belong to the intervals $\tilde{\varphi}^{R}_{p,t}\in[\varphi^{R}_{p,t}-\hat{\varphi}^{R}_{p,t},\varphi^{R}_{p,t}]$ and $\tilde{\varphi}^{C}_{t}\in[\varphi^{C}_{t},\hat{\varphi}^{C}_{t}+\varphi^{C}_{t}]$, respectively.

By rewriting the independent random variables as a linear combination between their nominal values and their maximum deviations, we have $\tilde{\varphi}^{R}_{p,t} = \varphi^{R}_{p,t} - \hat{\varphi}^{R}_{p,t} Z^{R}_{p,t}$ and $\tilde{\varphi}^{C}_{p,t} = \varphi^{C}_{t} + \hat{\varphi}^{C}_{t} Z^{C}_{t}$, with $Z^{R}_{p,t}$ and Z^{C}_{t} being "normalized" random variables within the interval [0,1], since $Z^{R}_{p,t} = (\varphi^{R}_{p,t} - \tilde{\varphi}^{R}_{p,t}) / \hat{\varphi}^{R}_{p,t}$ and $Z^{C}_{t} = (\tilde{\varphi}^{C}_{t} - \varphi^{C}_{t}) / \hat{\varphi}^{C}_{t}$. Applying these concepts to our problem, we have the following development:

$$\max_{\mathbf{QR},\mathbf{QCI}} MR_{i,e,t} =$$

$$\left\{ \max_{\mathbf{QR},\mathbf{QCI}} \sum_{i\in I}\sum_{e\in E}\sum_{t\in T}\left(\sum_{p\in P} QR_{i,p,t}\left(\tilde{\varphi}^{R}_{p,t} - \xi t^{R}_{p}\right) - QCI_{i,t}\tilde{\varphi}^{C}_{t} \right)\eta^{N}_{i,e} \mid \text{constraints (5.33 - 5.67) hold} \right\} = \quad (5.2a)$$

$$\left\{ \max_{\mathbf{QR},\mathbf{QCI}} \max_{\substack{\mathbf{Z}^{R}\in\Omega^{R} \\ \mathbf{Z}^{C}\in\Omega^{C}}} \sum_{i\in I}\sum_{e\in E}\sum_{t\in T}\left(\sum_{p\in P} QR_{i,p,t}\left(\varphi^{R}_{p,t} - \hat{\varphi}^{R}_{p,t} Z^{R}_{p,t} - \xi t^{R}_{p}\right) - QCI_{i,t}\left(\varphi^{C}_{t} + \hat{\varphi}^{C}_{t} Z^{C}_{t}\right) \right)\eta^{N}_{i,e} \mid \text{constraints (5.33 - 5.67) hold} \right\} = \quad (5.2b)$$

$$\left\{ \max_{\mathbf{QR},\mathbf{QCI}} \max_{\substack{\mathbf{Z}^{R}\in\Omega^{R} \\ \mathbf{Z}^{C}\in\Omega^{C}}} \left[\sum_{i\in I}\sum_{e\in E}\sum_{t\in T}\left(MR_{i,e,t} - \sum_{p\in P}\left(QR_{i,p,t}\hat{\varphi}^{R}_{p,t}\eta^{N}_{i,e}\right)Z^{R}_{p,t} - \left(QCI_{i,t}\hat{\varphi}^{C}_{t}\eta^{N}_{i,e}\right)Z^{C}_{t} \right) \right] \mid \text{constraints (5.33 - 5.67) hold} \right\} \quad (5.2c)$$

in which $\Omega^{R} = \left\{ 0 \le Z^{R}_{p,t} \le 1, \forall p \in P, t \in T, \sum_{p\in P}\sum_{t\in T} Z^{R}_{p,t} \le \Gamma^{R} \right\}$ and

$\Omega^{C} = \left\{ 0 \le Z^{C}_{t} \le 1, \forall t \in T, \sum_{t\in T} Z^{C}_{t} \le \Gamma^{C} \right\}$ are the so-called uncertainty sets.

We use the following expressions $\tilde{\varphi}^{R}_{p,t} = \varphi^{R}_{p,t} - \hat{\varphi}^{R}_{p,t} Z^{R}_{p,t}$ and $\tilde{\varphi}^{C}_{p,t} = \varphi^{C}_{t} + \hat{\varphi}^{C}_{t} Z^{C}_{t}$ in eq.(5.2a) and obtain eq.(5.2b). The problem (5.2c) is finally obtained after some rearrangements. This follows the robust optimization paradigm where our aim is to maximize the worst-case revenue, which although not intuitive, is obtained by subtracting the two inner maximization problems from the total profits $MR_{i,e,t}$. Based on the idea of the *budget of uncertainty* (Bertsimas and Sim, 2003), we may control the level of conservativeness of the solutions, by imposing upper bounds on the normalized random variables. This is modelled using constraints $\sum_{p,t} Z^{R}_{p,t} \le \Gamma^{R}$ and $\sum_{t} Z^{C}_{t} \le \Gamma^{C}$, where the user-defined parameters Γ^{R} and Γ^{c} represent the risk attitude of the decision-maker. For instance, a whole range of solutions may be obtained by varying Γ^{R} from 0, where uncertainty is not incorporated at all, and that means that the decision-maker is risk-neutral, up to $\Gamma^{R} = |P||T|$, wherein is obtained the most conservative solution that is particularly recommended for the risk-averse decision-makers.

In order to derive a tractable (linear) robust counterpart for the problem (5.2c), notice that the first inner maximization problem in (5.2c) is equivalent to the following auxiliary problem:

$$\max_{\mathbf{z}^R} \sum_{p\in P}\sum_{t\in T}\hat{\varphi}^R_{p,t}\sum_{i\in I}\sum_{e\in E}QR_{i,p,t}\eta^N_{i,e}Z^R_{p,t}$$

$$\text{subject to:} \quad \sum_{p\in P}\sum_{t\in T}Z^R_{p,t} \le \Gamma^R \qquad (5.i)$$

$$0 \le Z^R_{p,t} \le 1, \forall p \in P \wedge t \in T$$

Then by considering the dual variables λ^R and $\mu^R_{p,t}$ respectively, for the first and second constraints in problems (5.i), we derive the dual problem of (5.i) which is as follows:

$$\min_{\lambda^R,\boldsymbol{\mu}^R} \Gamma^R\lambda^R + \sum_{p\in P}\sum_{t\in T}\mu^R_{p,t}$$

$$\text{subject to:} \quad \lambda^R + \mu^R_{p,t} \ge \hat{\varphi}^R_{p,t}\sum_{i\in I}\sum_{e\in E}QR_{i,p,t}\eta^N_{i,e} \qquad (5.ii)$$

$$\lambda^R \ge 0, \mu^R_{p,t} \ge 0, \forall p \in P \wedge t \in T$$

Notice that the primal problem (5.i) is feasible and bounded. In fact, it is straightforward that the trivial solution $\mathbf{Z}^R = 0$ is feasible. Thus, by strong duality we know that its corresponding dual is also feasible and bounded for all $\Gamma^R \in [0,|P||T|]$ and that their objective functions coincide. Based on this property, a tractable formulation for the nonlinear robust counterpart (5.2c) is derived by replacing the inner maximization problem (5.i) by its equivalent dual problem (5.ii). Using the same rationale, it is possible to linearize the second inner maximization problem in (5.2c). By injecting these dual problems into the main problem (5.1), we obtain the MILP robust optimization problem (5.1a), which is as follows:

$$\max\left(\psi^{CP}\right) = \psi - \left(\Gamma^R\lambda^R + \sum_{p\in P}\sum_{t\in T}\mu^R_{p,t}\right) - \left(\Gamma^C\lambda^C + \sum_{t\in T}\mu^C_t\right) \qquad (5.1a)$$

$$\text{subject to:} \quad \text{constraints } (5.33-5.67)$$

$$\lambda^R + \mu^R_{p,t} \ge \hat{\varphi}^R_{p,t}\sum_{i\in I, e\in E}QR_{i,p,t}\eta^N_{i,e}, \forall p \in P \wedge t \in T \qquad (5.68)$$

$$\lambda^C + \mu^C_t \ge \hat{\varphi}^C_t\sum_{i\in I, e\in E}QCI_{i,t}\eta^N_{i,e}, \forall t \in T \qquad (5.69)$$

$$\mu^R_{p,t} \ge 0, \ \forall p \in P \wedge t \in T \qquad (5.70)$$

$$\mu^C_t \ge 0, \ \forall t \in T \qquad (5.71)$$

$$\lambda^R \ge 0, \ \lambda^C \ge 0 \qquad (5.72)$$

The linear combination of the dual variables in (5.1a) penalizes, the objective function, and hence, provides the necessary protection against the variation of both, the ex-refinery product prices and the crude oil costs, when their corresponding budgets of uncertainty are increased.

As a consequence, the solution becomes more "robust" in detriment of some deterioration in the PSC profits. This is the essence of the RO methodology, and thus, it is essential to assign adequate values to the respective budgets of uncertainty.

A similar procedure is used to model the uncertainty in the excess inventory capacity costs by defining the uncertain parameters $\tilde{h}^{+}_{k,e,p,t} \in [h^{+}_{k,e,p,t}, h^{+}_{k,e,p,t} + \hat{h}^{+}_{k,e,p,t} Z^{+}_{k,e,p,t}]$. Note that the worst-case perspective leads to high excess inventory capacity costs, which results into the following auxiliary problem:

$$\begin{aligned} &\max_{\mathbf{Z}^{+}} \sum_{k\in K}\sum_{e\in E}\sum_{p\in P}\sum_{t\in T} h^{+}_{k,e,p,t} SL_{k,e,p,t} Z^{+}_{k,e,p,t} \\ &\text{subject to:} \quad \sum_{k\in K}\sum_{e\in E}\sum_{p\in P}\sum_{t\in T} Z^{+}_{k,e,p,t} \le \Gamma^{+} \\ &\qquad 0 \le Z^{+}_{k,e,p,t} \le 1, \forall k \in K \wedge e \in E \wedge p \in P \wedge t \in T \end{aligned} \tag{5.iii}$$

By iteratively using the previously presented procedure, we need to use the dual information of problem (5.iii) and reinject it into the original objective function, where the excess inventory capacity costs are the uncertain parameters. Consequently, the RO formulation considering all uncertain parameters is depicted in eq. (5.1b). Notice that it is not necessary to model the retail prices' uncertainty due to its strong correlation with that of the ex-refinery product prices. As the objective function maximizes the total profits, the retail prices' uncertainty may not be simultaneously modelled with that of the ex-refinery prices as this would mean considering this uncertainty twice, thereby doubly penalizing the objective function. Also, uncertainties related to the investment costs are not directly modelled as the investment decisions are here and now decisions. However this uncertainty is implicitly considered in the excess inventory capacity costs, which are cumulative functions of the pipeline, depot, and resource investment costs that are considered in the construction of the uncertainty sets as will be discussed in the section of the PSC real case.

$$\max\left(\psi^{\Theta}\right) = \psi - \left(\Gamma^{R}\lambda^{R} + \sum_{p\in P}\sum_{t\in T}\mu^{R}_{p,t}\right) - \left(\Gamma^{C}\lambda^{C} + \sum_{t\in T}\mu^{C}_{t}\right) - \left(\Gamma^{+}\lambda^{+} + \sum_{k\in K}\sum_{e\in E}\sum_{p\in P}\sum_{t\in T}\mu^{+}_{k,e,p,t}\right) \tag{5.1b}$$

$$\text{subject to:} \quad \text{constraints } (5.33-5.67), (5.68-5.72)$$

$$\lambda^{+} + \mu^{+}_{k,e,p,t} \ge \hat{h}^{+}_{k,e,p,t} \times SL_{k,e,p,t}, \forall k \in K \wedge e \in E \wedge p \in P \wedge t \in T \tag{5.73}$$

$$\mu^{+}_{k,e,p,t} \ge 0, \forall k \in K \wedge e \in E \wedge p \in P \wedge t \in T \tag{5.74}$$

$$\lambda^{+} \ge 0 \tag{5.75}$$

Bertsimas and Sim (2004) derived eqs. (5.iv), which approximates the probability for the violation of constraint *i*, given any feasible solution X^{*}_{j}, where $\Phi(\theta)$ is the cumulative distribution function of a standard normal, for all $i \in I$. Here J_i is defined as the set of coefficients in row *i* of the matrix *A* that is subject to uncertainty. The probabilistic bounds directly depend on the values of $|J_i|$ and Γ_i. In this problem, Γ^C can take values between 0 and $|J^C| = |T|$, and Γ^R between 0 and $|J^R| = |P||T|$. These may be explicitly chosen by the decision maker, so that the calculated probability of constraint violation is acceptable to the decision-maker, based on his risk attitude.

$$\Pr\left(\sum_j \tilde{a}_{i,j} X_j^* > b_i\right) \leq 1 - \Phi\left(\frac{\Gamma_i - 1}{\sqrt{|J_i|}}\right) \quad where\ \Phi(\theta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\theta} \exp\left(-\frac{y^2}{2}\right) dy \qquad (5.iv)$$

5.6.2 Uncertainty in demand parameters

The RO is now formulated for the uncertainty in the demand balancing constraints for. First, notice that this parameter $\delta_{k,e,p,t}$ is included in the set of equality constraints (5.43) and (5.44). Equality constraints containing uncertain data should be avoided as much as possible, since not rarely such constraints restrict the feasibility region drastically or even lead to infeasibility (Gorissen *et al.,* 2015). Thus, we propose (i) to rewrite the demand balancing constraints without the excess inventory variables; and (ii) to aggregate over time the uncertain demand parameter. Constraints (5.42) and (5.43) are reformulated as inequalities, thus creating in each time period, a "gap" that allows for the variation of the demand. In contrast to the deterministic model which uses equalities to enforce the tactical decisions, RO uses a protection function to create flexibility in the constraints so that the variations in the uncertain parameters can be incorporated.

For this purpose, the inventory constraints are now determined via the aggregation process. Basically, we rewrite the balancing constraints by dropping the excess inventory variables $SL_{k,e,p,t}$, and substituting them within the formulation $\sum_{\tau=1}^{t}\left(\sum_{(j,m)|\mu_{j,k,m}} QS_{j,k,m,e,p,\tau} - \delta_{k,e,p,\tau}\right), \forall k,e,p,t$, which takes a positive value if inventory is held in the period, thereby implying the excess capacity costs for holding the product inventory. Without loss of generality, the first period inventory $SL0_{k,p,e}$ is considered to be 0. A single decision variable $H_{k,e,p,t}$ accounts for the excess inventory capacity costs per location, entity, product and time period, wherein the parameters $h^+_{k,e,p,t}$ are the respective unit costs. As a result, an equivalent nominal problem is obtained in eq. (5.76), but flexible enough to accommodate demand variations. This procedure was successfully proposed by Bertsimas and Thiele (2006), Alem and Morabito (2012), and some references therein. Different to these works, the present formulation does not include backorders due to the fact that it is a strategic model.

$$H_{k,e,p,t} \geq h^+_{k,e,p,t} \sum_{\tau \leq t}\left(\sum_{j,m} QS_{j,k,m,e,p,\tau} - \delta_{k,e,p,\tau}\right) \qquad \forall k \in K \wedge e \in E \wedge p \in P \wedge t \in T \qquad (5.76)$$

Here, the cumulative uncertain demands $\tilde{\delta}_{k,e,p,\tau}$ are assumed independent and bounded random variables with an unknown distribution in the symmetric interval $\left[\delta_{k,e,p,\tau}-\hat{\delta}_{k,e,p,\tau},\delta_{k,e,p,\tau}+\hat{\delta}_{k,e,p,\tau}\right]$ centered around the nominal value $\delta_{k,e,p,\tau}$, where the amplitude of variation can be at most twice $\hat{\delta}_{k,e,p,\tau}$. Similarly, as in the previous section, we also define a normalized random variable $Z^{\delta}_{k,e,p,t}=\left(\tilde{\delta}_{k,e,p,t}-\delta_{k,e,p,t}\right)/\hat{\delta}_{k,e,p,t}$ that takes values in the range $[-1,1]$, such that $\tilde{\delta}_{k,e,p,\tau}=\delta_{k,e,p,\tau}+\hat{\delta}_{k,e,p,\tau}Z^{\delta}_{k,e,p,\tau}$.

Following the methodology proposed in Bertsimas and Thiele (2006), the budgets of uncertainty $\Gamma^{\delta}_{k,e,p,t}\in[0,t]$ are defined per customer location, entity, and product, adhering to the following rules:

i. $\Gamma^{\delta}_{k,e,p,1}\le\Gamma^{\delta}_{k,e,p,2}...\le\Gamma^{\delta}_{k,e,p,T}$ where $T=|T|$ is the last time period implying that $\Gamma^{\delta}_{k,e,p,t}$ either increases or maintains its value from one period to the next but does not decrease;

ii. $\Gamma^{\delta}_{k,e,p,t}-\Gamma^{\delta}_{k,e,p,t-1}\le 1,\forall k\in K,e\in E,p\in P,t\in T$ implying that the budget of uncertainty $\Gamma^{\delta}_{k,e,p,t}$ increases at most unitarily so that only one uncertain parameter can be added in each period.

The robust optimization framework adopted in this chapter, relies on finding the optimal solution for the worst-case instances of unknown and bounded parameters. The main goal is to guarantee near-optimality when these parameters vary within the uncertainty set. Differently from Bertsimas and Thiele (2006), where overall costs are minimized, our focus is to maximize the total revenue. As lower demands lead to lower revenues, the uncertain demand $\delta_{k,e,p,\tau}$ attains its worst-case realization at lower bound of the symmetric interval $\left[\delta_{k,e,p,\tau}-\hat{\delta}_{k,e,p,\tau},\delta_{k,e,p,\tau}+\hat{\delta}_{k,e,p,\tau}\right]$.

Based on the ideas already developed above, the eq. (5.76), whose cumulative demand presents uncertainty, is written as follows:

$$H_{k,e,p,t}\ge h^{+}_{k,e,p,t}\sum_{\tau\le t}\left(\sum_{j,m}QS_{j,k,m,e,p,\tau}-\tilde{\delta}_{k,e,p,\tau}\right)\qquad \forall k\in K\wedge e\in E\wedge p\in P\wedge t\in T \qquad (5.77)$$

which is equivalent to solving the following optimization problem in variable $Z^{\delta}_{k,e,p,\tau}$:

$$\begin{aligned}&\max_{\mathbf{Z}^{\delta}}\left(\sum_{\tau=1}^{t}\hat{\delta}_{k,e,p,\tau}Z^{\delta}_{k,e,p,\tau}\right)\\ &\text{subject to:}\quad \sum_{\tau=1}^{t}Z^{\delta}_{k,e,p,\tau}\le\Gamma^{\delta}_{k,e,p,t} \qquad \forall k\in K\wedge e\in E\wedge p\in P\wedge t\in T\wedge\tau\le t\\ &\qquad\qquad 0\le Z^{\delta}_{k,e,p,\tau}\le 1, \qquad \forall k\in K\wedge e\in E\wedge p\in P\wedge t\in T\wedge\tau\le t\end{aligned}\qquad (5.v)$$

Again, it is necessary to use the dual information provided by the auxiliary problem, to maintain the tractability of the robust counterpart. Notice that it is straightforward to see that problem (5.v) is feasible and bounded, thus implying that its corresponding dual is also feasible and bounded. Supported by the strong duality theory, it is known that the objective value of both problems coincide, and for this reason, the dual information of problem (5.v) can be incorporated into the original reformulated constraint eq. (5.77). Finally, the robust counterpart for the problem (5.1-5.29),(5.33-5.67) with demand uncertainty is obtained as follows:

$$\max\left(\psi^{\delta}\right) = \psi - \sum_{k\in K}\sum_{e\in E}\sum_{p\in P}\sum_{t\in T} H_{k,e,p,t} \tag{5.1c}$$

$$\text{subject to:} \quad \text{constraints } (5.33-5.41), (5.44-5.63), (5.65-5.67)$$

$$H_{k,e,p,t} \geq h^{+}_{k,e,p,t}\left(\sum_{\tau\leq t}\left(\sum_{j,m}\mathrm{QS}_{j,k,m,e,p,\tau} - \delta_{k,e,p,\tau}\right) + \Gamma^{\delta}_{k,e,p,t}\lambda^{\delta}_{k,e,p,t} + \sum_{\tau=t}\mu^{\delta}_{k,e,p,\tau,t}\right), \tag{5.78}$$
$$\forall k \in K \wedge e \in E \wedge p \in P \wedge t \in T$$

$$\sum_{\tau\leq t}\left(\sum_{j,m}\mathrm{QS}_{j,k,m,e,p,\tau} - \delta_{k,e,p,\tau}\right) + \Gamma^{\delta}_{k,e,p,t}\lambda^{\delta}_{k,e,p,t} + \sum_{\tau=t}\mu^{\delta}_{k,e,p,\tau,t} \leq \delta_{k,e,p,t+1} \times \beta^{Max}_{a}, \tag{5.79}$$
$$\forall k \in K, e \in E, p \in P, t \in T, t > 1 \wedge t < T, a = RETAIL$$

$$\lambda^{\delta}_{k,e,p,t} + \mu^{\delta}_{k,e,p,\tau,t} \geq \hat{\delta}_{k,e,p,\tau}, \qquad \forall k \in K \wedge e \in E \wedge p \in P \wedge t \in T \wedge \tau \leq t \tag{5.80}$$

$$\lambda^{\delta}_{k,e,p,t}, \mu^{\delta}_{k,e,p,\tau,t} \geq 0, \qquad \forall k \in K \wedge e \in E \wedge p \in P \wedge t \in T \wedge \tau \leq t \tag{5.81}$$

Here, the variables $\lambda^{\delta}_{k,e,p,t}$ and $\mu^{\delta}_{k,e,p,\tau,t}$ are the dual variables associated respectively with the first and second constraints of problem (5.v). As a result, eq. (5.64) that implements the inventory bounds in the original problem, is also replaced with eq. (5.79) so as to account for the robust terms. The robust counterpart provides the necessary "gap" to absorb the demand variations, which may be evaluated as the term $\Gamma^{\delta}_{k,e,p,t}\lambda^{\delta}_{k,e,p,t} + \sum_{\tau=t}\mu^{\delta}_{k,e,p,\tau,t}$ for each customer, entity, product, and time period. Wei *et al.* (2011) derive the bounds for the probability of constraint violation similar to those in eq. (5.77) wherein for a given solution $QS^{*}_{j,k,m,e,p,t}$, the probability of violation is approximated by eq. (5.vi) where $\Phi(\theta)$ is the cumulative distribution function of a standard normal for all $k \in K, e \in E, p \in P, t \in T$. The probability referred here that of the excess inventory capacity costs being lower than the costs related to the difference between the supplied product and the uncertain demands.

$$\Pr\left[H_{k,e,p,t} < h^{+}_{k,e,p,t}\sum_{\tau\leq t}\left(\sum_{j,m} QS*_{j,k,m,e,p,\tau} - \tilde{\delta}_{k,e,p,\tau}\right)\right] \leq 1 - \Phi\left(\frac{\Gamma^{\delta}_{k,e,p,t} - 1}{\sqrt{t}}\right) \tag{5.vi}$$

As referred in Adida and Perakis (2010), all the uncertain parameters may not vary simultaneously, and hence taking into account the PSC planning problem, and in contrast to varying the parameters over time, the budget of uncertainty could be shared by various products $\sum_{p\in P}\left|z_{k,e,p,\tau}\right| \leq \Gamma^{\delta}_{k,e,p,t}$, locations or entities.

5.6.3 Uncertainty in objective function and demand parameters

The crude costs, product prices, excess inventory capacity costs, and the customer demand uncertainties are now simultaneously considered. Note that the excess inventory capacity considered in the objective function of eqs. (5.1b), directly affect the constraints modelled above. Now, take for example, the restriction (5.77) which includes the cost $h^{+}_{k,e,p,t}$. As this restriction incorporates the uncertain demand, then the question is, what would be its robust equivalent if the cost would be uncertain? Would the formulation presented above in (5.1c) not be too conservative? In this sense, considering that the uncertain excess inventory capacity costs $\tilde{h}^{+}_{k,e,p,t}$ can be written as the linear combinations of their nominal values and respective deviations, we have $\tilde{h}^{+}_{k,e,p,t} = h^{+}_{k,e,p,t} + \hat{h}^{+}_{k,e,p,t} Z^{h+}_{k,e,p,t}$ with $Z^{h+}_{k,e,p,t} \in [0,1]$. So the robust equivalent corresponds to the maximization of the RHS of constraint (5.77) for all realizations of $Z^{h+}_{k,e,p,t}$, which can be obtained by solving the auxiliary problem (5.vii) for every *k,e,p,t*.

$$
\begin{aligned}
&\max_{\mathbf{Z}^{+}} \left(\hat{h}^{+}_{k,e,p,t} H^{+}_{k,e,p,t} Z^{h+}_{k,e,p,t} \right) \\
&\text{subject to:} \quad Z^{h+}_{k,e,p,t} \le \Gamma^{h+}_{k,e,p,t}, \\
&0 \le Z^{h+}_{k,e,p,t} \le 1,
\end{aligned}
\tag{5.vii}
$$

By using the dual variables $\lambda^{h+}_{k,e,p,t}$, $\mu^{h+}_{k,e,p,\tau,t}$ for the first and second constraints of problem (vii), the linear dual problem obtained is as follows:

$$
\begin{aligned}
&\min \left(\Gamma^{h+}_{k,e,p,t} \lambda^{h+}_{k,e,p,t} + \mu^{h+}_{k,e,p,t} \right) \\
&\text{subject to:} \quad \lambda^{h+}_{k,e,p,t} + \mu^{h+}_{k,e,p,t} \ge \hat{h}^{+}_{k,e,p,t} H^{+}_{k,e,p,t} \\
&\lambda^{h+}_{k,e,p,t}, \mu^{h+}_{k,e,p,\tau,t} \ge 0, \\
&H^{+}_{k,e,p,t} = \sum_{\tau=1}^{t} \left(\sum_{j,m} QS_{j,k,m,e,p,\tau} - \delta_{k,e,p,\tau} \right) + \Gamma^{\delta}_{k,e,p,t} \lambda^{\delta}_{k,e,p,t} + \sum_{\tau=t} \mu^{\delta}_{k,e,p,\tau,t}
\end{aligned}
\tag{5.viii}
$$

Following the rationale that has been developed in the previous sections, it is derived eq. (5.1d) which is the robust counterpart when considered the crude costs, the product prices, the excess capacity costs, and the customer demand uncertainties simultaneously.

$$
\begin{aligned}
&\max\left(\psi^{\delta\Theta} \right) = \psi^{\delta} \\
&\text{subject to:} \quad \text{constraints } (5.33-5.41), (5.44-5.63), (5.65-5.67), (5.68-5.75), (5.78-5.81)
\end{aligned}
\tag{5.1d}
$$

$$
H_{k,e,p,t} \ge h^{+}_{k,e,p,t} \left(\sum_{\tau \le t} \left(\sum_{j,m} QS_{j,k,m,e,p,\tau} - \delta_{k,e,p,\tau} \right) + \Gamma^{\delta}_{k,e,p,t} \lambda^{\delta}_{k,e,p,t} + \sum_{\tau=t} \mu^{\delta}_{k,e,p,\tau,t} \right) + \Gamma^{h+}_{k,e,p,t} \lambda^{h+}_{k,e,p,t} + \mu^{h+}_{k,e,p,t}, \quad \forall k \in K \wedge e \in E \wedge p \in P \wedge t \in T
\tag{5.82}
$$

$$\lambda^{h+}_{k,e,p,t} + \mu^{h+}_{k,e,p,t} \geq \hat{h}^{+}_{k,e,p,t}\left(\sum_{\tau\leq t}\left(\sum_{j,m}QS_{j,k,m,e,p,\tau} - \delta_{k,e,p,\tau}\right) + \Gamma^{\delta}_{k,e,p,t}\lambda^{\delta}_{k,e,p,t} + \sum_{\tau=t}\mu^{\delta}_{k,e,p,\tau,t}\right), \forall k \in K \wedge e \in E \wedge p \in P \wedge t \in T \quad (5.83)$$

$$\lambda^{h+}_{k,e,p,t}, \mu^{h+}_{k,e,p,\tau,t} \geq 0, \qquad \forall k \in K \wedge e \in E \wedge p \in P \wedge t \in T \quad (5.84)$$

In the formulation above, eqs. (5.82)-(5.83) provide the solution robustness for the excess inventory capacity costs by using $\Gamma^{h+}_{k,e,p,t} \in [0,1]$. Assuming integer values for these parameters, and considering the excess inventory capacity costs, there exist only two possibilities: the nominal problem when $\Gamma^{h+}_{k,e,p,t} = 0$; and the Soyster problem when $\Gamma^{h+}_{k,e,p,t} = 1$. As in the optimal solution, eq. (5.83) will be an equality, the expression $\Gamma^{h+}_{k,e,p,t}\lambda^{h+}_{k,e,p,t} + \mu^{h+}_{k,e,p,t}$ in the LHS of eq. (5.82) can be replaced with the RHS of eq. (5.83), which following simplification, results in the substitution of eqs. (5.82-5.84) by eq. (5.85).

$$H_{k,e,p,t} \geq \left(h^{+}_{k,e,p,t} + \hat{h}^{+}_{k,e,p,t}\right)\left(\sum_{\tau\leq t}\left(\sum_{j,m}QS_{j,k,m,e,p,\tau} - \delta_{k,e,p,\tau}\right) + \Gamma^{\delta}_{k,e,p,t}\lambda^{\delta}_{k,e,p,t} + \sum_{\tau=t}\mu^{\delta}_{k,e,p,\tau,t}\right), \forall k \in K \wedge e \in E \wedge p \in P \wedge t \in T \quad (5.85)$$

Following this reduction, the resulting robust formulation considering simultaneous cost, price, and demand uncertainties is the one obtained in eq. (5.1e).

$$\max\left(\psi^{\delta\Theta}\right) = \psi - \sum_{k\in K}\sum_{e\in E}\sum_{p\in P}\sum_{t\in T} H_{k,e,p,t} - \left(\Gamma^{R}\lambda^{R} + \sum_{t\in T}\sum_{p\in P}\mu^{R}_{p,t}\right) - \left(\Gamma^{C}\lambda^{C} + \sum_{t\in T}\mu^{C}_{t}\right) - \left(\Gamma^{+}\lambda^{+} + \sum_{k\in K}\sum_{e\in E}\sum_{p\in P}\sum_{t\in T}\mu^{+}_{k,e,p,t}\right) \quad (5.1e)$$

$$\text{subject to:} \quad \text{constraints } (5.33-5.41), (5.44-5.63), (5.65-5.67), (5.68-5.75), (5.78-5.81), (5.85)$$

The RO model will now be tested with the real case Portuguese PSC in order to validate the model and evaluate its solutions and performance. In the following sections, are presented: the real case Portuguese PSC; the construction of the uncertainty sets for the real case; followed by the presentation and discussion of the computational results. Consequently, are compared and discussed the resulting designs, the strategic and tactical decisions, followed by the solution trade-offs, performance statistics, relevant insights, and conclusions.

5.7 Petroleum supply chain real case

To evaluate the effectiveness of the proposed approach, the Portuguese PSC is taken as a case study and applied the developed RO model. Three designs are considered: 1) studying the existent PSC – current problem; 2) designing a new PSC from scratch – grassroots problem; 3) re-designing the existent PSC due to changes occurring in the markets – retrofit problem. Installation of new infrastructures, associated resources, and capacities are considered, as well as the expansion of existing ones. The network is also studied by considering it as a single or multiple entities supply chain.

Figure 5.4 presents the existent multi-echelon Portuguese PSC network, which commercializes and distributes the petroleum products to the customers, which are divided into eighteen national districts. The supply chain involves two refineries located at Sines and Matosinhos, each having currently a production capacity of 220 and 110 thousand barrels per day, which produce six petroleum products propane (C3), butane (C4), gasoline 95 (U5), diesel (GO), fuel oil (FU), and jet fuel (A1), which aggregate other varieties such as gasoline 98 and biodiesel. The products are supplied by pipeline, maritime or railroad transportation to twelve storage depots located in continental Portugal where are installed storage vessels, with capacities per product as shown in figure 5.4. The relative size of the depots can be identified by the darkness of the small squares that represent the storage capacities installed, for instance Matosinhos has 7 vessels for the storage of diesel (GO), which have a larger capacity than those at Aveiro or Leixoes. Storage depots as also the transportation infrastructure are owned by individual companies, partially owned by several companies or by independent logistics operators.

Refineries Matosinhos	Sines	Depots	C3	C4	U5	GO	FU	A1
		→AVEIRO			2	2		
A1		→PORTO						
		→MATOSINHOS	4	3	7	7	7	4
		→PERAFITA	5	2				
		→REAL	3	1	2	2		
		→LEIXOES			2	3	1	
	A1	→FARO	1	1				
		→AZAMBUJA	8	6	7	9		4
		→SINES	5	4	10	8	6	5
		→BARREIRO			2	1	1	
		→SETUBAL			2	2	1	
		→BANATICA	2	2		2		

Vessels per Product

Client Districts: AVEIRO, BEJA, BRAGA, BRAGANCA, CASTELO BRANCO, COIMBRA, EVORA, FARO, GUARDA, LEIRIA, LISBOA, PORTALEGRE, PORTO, SANTAREM, SETUBAL, VIANA CASTELO, VILA REAL

Figure 5.4. The downstream PSC in Portugal

Our aim is to provide an efficient design and planning tool for the current, grassroots, and retrofit networks considering single and multiple entities. The capacity expansions are considered individually per entity and product, and then aggregated per location. Crude oil purchased at the refineries is bounded by the refineries' capacities. Resources are considered to be static having rotations per period or dynamic having flowrates per hour and hence the flow of products in the transport and depot facilities is restricted to the capacity of the installed resources. Consequently the RO MILP may determine the installation of resources, thereby increasing capacity. Initial inventories are not considered, and the inventory costs include the added capacity costs and the product holding costs. Modularized fixed and variable costs are defined for depots and typical resources, whereas costs per kilometre and cubic meter of volume are defined for the transportation infrastructures. These costs were originally presented in Fernandes *et al.* (2013,2014), however, have now been improved from subsequent industry benchmarks.

In order to perform this study, it was necessary to obtain comprehensive data on the petroleum industry, which was a difficult task. A statistical database was constructed for a twelve year period between 2002 and 2013 using the crude oil and petroleum products demand, costs, and pricing data as well as the PSC structural information on existing facilities, taken from industry benchmarks and technical documentation. This data was collected from the Direcção Geral de Energia e Geologia (DGEG, 2015), Autoridade de Concorrência (AdC, 2008), the European Commission's Directorate-General for Energy (Eurostat, 2015), the U.S. Energy Information Administration (EIA, 2015), the Bureau of Transportation Statistics (BTS, 2015), the Platts Market Data – Oil (2015), the Organization for Economic Co-operation and Development (OECD, 2002), the Interstate Natural Gas Association of America (INGAA, 2009), and Miller *et al.* (2010) The database includes historical data relating to the crude oil costs, the ex-refinery prices, the retail prices, the customer demand, the infrastructure and vessel construction costs, the refining costs, the importation costs, the transportation and depot unitary costs, the GPS coordinates, and multi-modal distances. In order to test the RO model, uncertainties relating to the crude oil costs, the ex-refinery prices, the retail prices, construction costs, operating costs, and customer demand were considered in order to build the uncertainty sets, which are presented hereafter.

5.8 Construction of the uncertainty sets

For completeness, we now characterize in the necessary detail, the uncertain parameters modelled in the robust formulation, so as to test the RO MILP for the Portuguese PSC. Each uncertain parameter is analyzed per location, entity, product, and time period, based on the historical data and identified the corresponding nominal values as well as their amplitudes of variation. In robust optimization approach, it is sufficient to avail of the average nominal figures and the worst-case deviations. Therefore, we collected and correlated all the relevant data for the 12 year period between 2002 and 2013, which is directly used to calculate the model parameters. For a sector, which has very high uncertainty that repeats irregularly, it is difficult to predict the parameters for such a long planning horizon. Hence, the nominal values and the uncertainty band computed from the historical data are considered a good approximation.

5.8.1 Crude Oil Costs and Ex-refinery Product Prices

Typically, the refinery margin depends essentially on the ex-refinery product prices and crude oil free on board (FOB) prices. These prices are highly volatile as they compete in the international markets. One such important regional market is the North-West Europe (NWE) where the Antwerp-Rotterdam-Amsterdam (ARA) cluster plays an important role as a storage and supply hub for crude oil and petroleum products. Refineries purchase crude oil or petroleum products at the FOB price and arrange and pay the cost of insurance and freight (CIF) to import and transfer the crude oil to the refinery. Crude oil is then refined, producing petroleum products, which are bought by local companies at competitive CIF ARA prices. However, when production is deficit, the product is sourced externally, by paying a premium importation (CIF) cost to transport the product to the local destination in addition to the product CIF price at the ARA ports. The refineries export the surplus production at the FOB prices and support the exportation (CIF) costs to be competitive with the CIF ARA prices in the international markets.

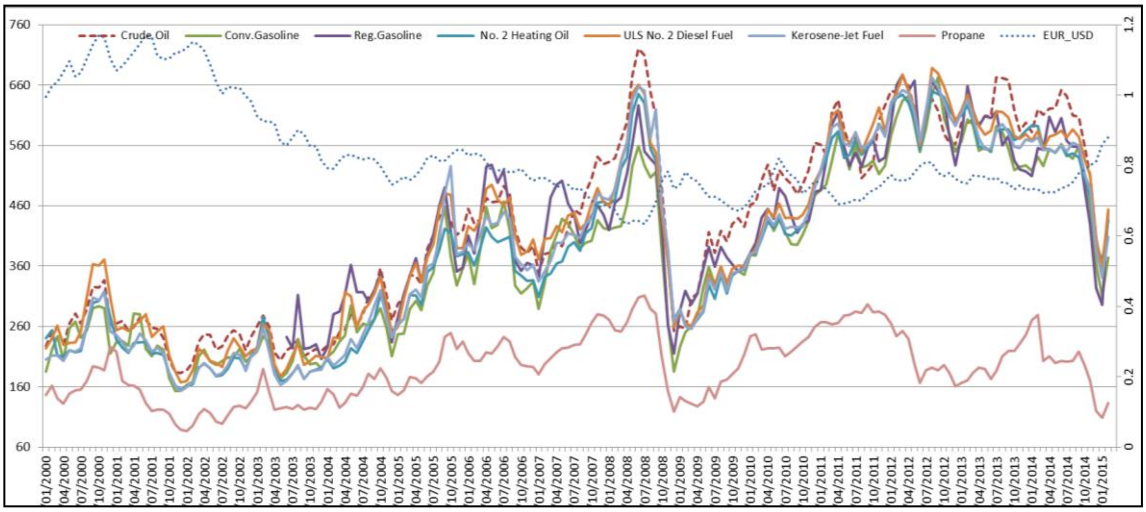


Figure 5.5. Time series for Brent Crude Oil and Petroleum products Ex-refinery FOB prices

Fig.5 presents the Brent Crude Oil and the petroleum products ex-refinery FOB prices' time series and uncertainty, collected from EIA (2015) and Platts (2015). It is observed a high volatility among product prices whose growth is inversely proportional to that of the U.S Dollar (plotted on the right axis). The prices increased from 200 up to 650 euros per cubic meter between years 2000 and 2008, crashed to 200 euros in 2009, increased to 580 euros by July 2014, and slumped to 300 euros by year end 2014.The time series in figure 5.5 was used to construct the uncertainty sets for the crude oil and product prices for the twelve year period between 2002 and 2013.

Table 5.1 presents the product prices and crude costs per year grouped into three sets: the first set shows the mean average or nominal crude costs and product prices per year, the second set shows the maximum worst-case deviation from the nominal prices per year, and the third set shows the standard deviation from the nominal prices per year. The worst-case scenario considered for the crude oil costs is the maximum positive deviation, which is when they are more expensive, whereas for the products' prices, it is considered the negative deviation where they fetch a lower price.

Table 5.1. Crude oil costs and Ex-refinery product prices uncertainty sets

	Nominal Ex-refinery Prices in €/m3							*Maximum Deviation from Nominal Prices*							*Standard Deviation from Nominal Prices*						
Period	*C4*	*C3*	*U5*	*A1*	*GO*	*FU*	*CRUDE*	*C4*	*C3*	*U5*	*A1*	*GO*	*FU*	*CRUDE*	*C4*	*C3*	*U5*	*A1*	*GO*	*FU*	*CRUDE*
2002	131.25	125.18	192.03	200.47	193.11	130.53	164.99	27.36	23.21	40.28	24.82	24.77	22.60	27.14	25.11	18.34	20.99	16.11	15.69	14.13	14.28
2003	130.81	132.84	197.13	207.53	206.42	124.14	159.79	46.21	34.70	21.54	31.37	29.90	24.67	21.02	28.83	31.74	18.26	27.30	30.66	16.21	16.22
2004	164.24	150.44	242.06	265.35	260.41	115.61	192.18	30.02	27.55	51.36	61.14	73.97	18.03	39.80	30.73	23.52	33.76	45.10	51.30	12.12	28.60
2005	191.83	185.24	322.06	377.40	366.54	174.81	274.26	50.79	36.00	89.59	101.88	90.73	62.64	63.55	48.04	38.63	59.66	57.96	53.46	33.63	39.32
2006	227.48	217.93	370.19	423.97	402.13	216.16	324.36	53.34	21.80	67.29	50.47	48.28	37.28	38.51	29.04	20.79	55.48	32.53	29.61	20.72	27.89
2007	254.93	231.76	380.55	423.97	398.70	232.43	329.19	45.17	40.58	96.52	66.79	70.73	69.89	71.09	41.49	37.00	43.91	48.42	45.46	41.80	42.41
2008	280.39	252.26	420.27	555.58	512.42	290.30	406.43	154.95	120.82	235.47	258.68	236.37	165.72	218.93	67.35	56.63	110.27	125.40	112.63	84.40	103.85
2009	199.76	174.47	309.08	329.82	305.43	231.35	273.94	60.47	51.40	90.09	59.38	51.45	74.19	68.15	42.84	39.68	50.23	39.19	35.12	47.18	44.01
2010	297.13	269.86	415.89	450.11	417.18	315.00	374.73	48.74	60.93	44.46	56.49	57.02	17.96	41.08	38.62	48.51	27.35	34.24	31.79	12.54	25.86
2011	352.02	303.43	530.36	604.86	553.51	413.14	499.52	27.87	28.17	46.24	53.49	56.17	54.67	47.88	17.93	23.48	26.82	25.58	26.81	23.52	21.79
2012	386.84	337.89	604.55	662.61	610.20	462.85	542.12	139.41	113.91	49.36	71.44	60.00	49.60	68.91	59.95	53.02	40.91	30.40	26.56	30.56	30.90
2013	345.33	287.83	573.10	622.43	574.63	427.93	516.69	60.65	40.66	24.65	37.16	31.11	13.67	29.00	39.24	22.43	21.41	32.28	24.03	9.29	22.14

5.8.2 Demand for Petroleum Products

The petroleum products' demand for the twelve years has been analysed and obtained the nominal customer demand for each customer district. The maximum and standard deviations are then obtained, with respect to the nominal demand, so that the model may be run to derive the RO solutions considering the worst-case demands. The monthly customer demand presented in table 5.2, is converted to the yearly basis prior to use by the model. This table has been constructed from the monthly and yearly statistics consulted in DGEG (2015). The deviations considered are the downside deviations, which in general would imply excess capacity.

Table 5.2. Customer demand uncertainty sets

2013	*Nominal Customer Demand in tons/month*						*Maximum deviation from Nominal Demand*						*Standard deviation from Nominal Demand*					
District	*C4*	*C3*	*U5*	*A1*	*GO*	*FU*	*C4*	*C3*	*U5*	*A1*	*GO*	*FU*	*C4*	*C3*	*U5*	*A1*	*GO*	*FU*
AVEIRO	1345	2521	5419		22718	1402	296	920	770	0	2635	347	212	577	461		1445	258
BEJA	369	465	796	51	5235	44	81	170	112	13	577	11	58	106	66	9	333	8
BRAGA	845	1973	6130		24859	775	186	720	870	0	2880	192	134	452	520		1581	142
BRAGANCA	164	288	911		5973	146	36	105	135	0	626	36	26	66	84		395	27
CASTELO BRANCO	266	448	1111		6425	353	59	164	156	0	718	87	42	103	92		407	65
COIMBRA	379	1306	3032		13536	959	83	477	432	0	1584	237	60	299	259		866	176
EVORA	494	563	1262	23	5493	48	109	206	176	6	596	12	78	129	104	4	352	9
FARO	863	2397	4872	14009	13821	123	190	875	679	3677	1634	31	136	549	399	2598	890	23
GUARDA	192	528	841		5222	124	42	193	120	0	559	31	30	121	72		338	23
LEIRIA	370	1233	3363		21919	515	81	450	483	0	2470	127	58	282	291		1386	95
LISBOA	3049	3314	18081	54188	63053	916	672	1210	2508	14221	7512	227	482	758	1466	10051	4089	168
PORTALEGRE	150	244	562		2848	22	33	89	78	0	317	5	24	56	46		180	4
PORTO	2640	4408	18917	15068	75472	2867	582	1609	2659	3955	8757	710	417	1009	1574	2795	4803	527
SANTAREM	958	2100	7902		32302	1725	211	767	1111	0	3774	427	151	481	657		2063	317
SETUBAL	2586	22122	8107		32717	1579	570	8076	1124	0	3653	391	408	5062	656		2071	290
VIANA DO CASTELO	242	592	1339		5486	547	53	216	190	0	605	135	38	136	113		349	100
VILA REAL	459	572	1130		6595	79	101	209	164	0	729	19	72	131	100		419	14
VISEU	572	1295	2405	10	11417	156	126	473	344	3	1290	39	90	296	207	2	722	29

5.8.3 Excess Inventory Capacity Costs

In practice, the excess inventory capacity is penalized using an excess capacity unit cost, which is previously obtained per destination node, entity, product, and time period as shown in eqs. (5.ix). Hence, the excess capacity costs $h^{+}_{k,e,p,t}$ are calculated per cubic meter, by aggregating the fixed and operating costs, based on the reference depot, resources, and transportation infrastructures, where *irr* is the internal rate of return (an arbitrary interest rate). The yearly statistics for the construction of these uncertainty sets is obtained from Eurostat (2015), BTS (2015) and INGAA (2009). The resulting table 5.3 presents the uncertainty sets for excess capacity comprising of the nominal costs and their maximum upside deviation for district Lisboa.

$$h^{+}_{k,e,p,t} = \sum_{r\in R}\left(\left(\xi r^{\mathrm{F}}_{\mathrm{r}} + \xi r^{\mathrm{FD}}_{\mathrm{m1}}\frac{\mu_{i1,k,m1}}{2}\right)(1+\mathrm{irr}\times\mathrm{t}) + \xi r^{O}_{r} + \xi r^{\mathrm{OD}}_{\mathrm{m1}}\frac{\mu_{i1,k,m1}}{2}\right)\frac{\tau c_{i1,NCP-1}}{\tau t_{i1,NCP-1}} + \sum_{r\in R}\left(\frac{\xi p^{F}_{r2,p}(1+\mathrm{irr}\times\mathrm{t}) + \xi p^{O}_{r2,p}}{\omega_{\mathrm{m1,c1',p}}\,\theta r_{\mathrm{ANY,r2,p}}}\right) + \gamma\,\varphi^{L}_{p,t}, \quad (5.ix)$$

$$\forall k\in K, e\in E, p\in P, t\in T, c1=\mathrm{ROTATION}, i1=\mathit{REFERENCE}, m1=\mathit{PIPELINE}$$

Table 5.3. Excess capacity cost uncertainty sets

	Nominal Excess Capacity Cost in €/m3						Maximum deviation from Nominal Cost in €/m3					
LISBOA	*C4*	*C3*	*U5*	*A1*	*GO*	*FU*	*C4*	*C3*	*U5*	*A1*	*GO*	*FU*
2002	67.40	94.42	35.58	44.86	31.66	24.66	2.70	2.33	2.85	5.00	0.82	1.20
2003	71.02	101.08	37.45	47.72	33.17	26.01	5.31	5.14	1.38	5.61	1.81	1.18
2004	78.76	108.71	39.76	51.47	35.87	26.00	9.33	11.92	4.59	14.58	3.11	0.93
2005	90.20	118.02	45.21	59.37	42.63	30.37	8.35	5.89	6.06	7.05	4.51	3.71
2006	88.60	117.63	50.39	65.21	46.65	33.71	24.43	30.99	9.71	24.23	2.42	2.10
2007	97.04	128.72	52.16	66.75	47.91	35.40	19.39	27.02	10.80	23.55	4.67	4.16
2008	100.28	135.38	59.50	82.35	60.81	40.88	44.76	61.24	24.04	40.58	12.00	7.47
2009	103.80	138.67	54.69	69.26	51.35	38.00	28.70	38.28	14.82	26.95	2.97	4.64
2010	109.51	146.42	58.28	73.67	54.11	44.55	16.38	18.05	6.00	18.46	3.79	2.76
2011	117.19	155.25	65.42	84.15	63.08	51.98	6.60	7.30	2.68	3.73	3.87	4.49
2012	122.16	161.11	70.48	88.49	67.18	57.15	17.44	15.61	3.49	8.22	3.06	2.79
2013	127.25	167.71	69.18	87.93	65.61	55.94	9.43	6.24	2.67	6.40	1.53	1.44

Thus far, we have developed the RO MILP formulation, collected and standardized the PSC data, and constructed the uncertainty sets. The remainder of the chapter presents the implementation results of the RO model, which include the real case test settings, followed by the discussion on the strategic and tactical decisions, the PSC profits maximization, the model solution performance statistics, the RO application main insights and modelling perspectives, and finally the conclusions.

5.9 Implementation Results

The RO model described in the previous sections is now implemented considering three network designs: the current design where are determined the tactical decisions of allocations, inventories, and transfer volumes for an existing network; the grassroots design where a new PSC network is designed from scratch, involving decisions of installation and operation of petroleum products' depots, transportation infrastructures and resource capacities besides the tactical decisions referred above; and the retrofit design, which involves redesign of an existing PSC network based on the market evolution for a twelve year horizon.

The model is developed in GAMS 24.5.6 and solved using the MILP commercial solver CPLEX 12.6 on an Intel Xeon 2 GHz CPU with 16 GB RAM and 16 parallel threads. The stopping criterion is 1% optimality gap or 24 hours of CPU time. As the model and the real case instances are large scale, we first obtain the GPS coordinates and travelling distances using the Google API interfaces and build concentration sets, using the methodology proposed by Rosing and ReVelle (1997), in order to reduce the model size. The concentration sets comprise of at most 12 potential depots per customer district, which are the gravitational centres of the demand within a 360 kilometre distance.

To facilitate the decision process, two different approaches may be used for the execution of the RO models: 1) select the appropriate values for the budgets of uncertainty Γ^C, Γ^R, Γ^δ prior to the model's executions, based on the probabilistic bound of constraint violation and the decision-makers risk attitude, or 2) execute the model exhaustively for all values of Γ^C, Γ^R, Γ^δ and choose the most appropriate solution that better fits the decision-makers interest. In the first case, the appropriate budgets of uncertainty could be defined based on the probability of constraint violation, determined by eq.(iv). The second alternative is not viable for large-scale problems as it requires to solve the model for each and every level of uncertainty, however, from a practical point of view, it is an attractive method because it can balance the budget of uncertainty with other consideration that may be relevant to the decision-maker. In the current study, we run the RO MILP for various budgets of uncertainty, and also compute the probabilities of constraint violation, hence, the decision-maker may choose one of many solutions.

Table 5.4 presents the RO model settings for the tests realized, in view of validating the robust MILP. For all three network designs (current; grassroots; retrofit), the model is executed considering the following settings: (N) Deterministic nominal solution, (C) crude costs uncertainty, (P) products prices uncertainty, (CP) simultaneous crude costs and product prices uncertainty, (D) customer demand uncertainty, and (DCP) simultaneous customer demand, crude costs, product prices, and excess inventory capacity costs uncertainty.

Table 5.4. Summary of the MILP Computational tests

	Budgets of Uncertainty				Network Designs			Strategic Decisions			Tactical Decisions		
MILP Execution Settings and Implementation Results for the different Robust Formulations	Γ^{C}	Γ^{R}	$\Gamma^{\delta}_{k,e,p,t}$	Γ^{+}	Current	Grassroots	Retrofit	Transportation Links	Storage Depots	Resource Capacities	Transfer Volumes	Inventory Volumes	Total PSC Profits
(N) Deterministic Nominal Solution	0	0	0	0	1	1	1	•	•	•	•	•	•
(C) Crude Cost Uncertainty	0..12	0	0	0	13	13	13	•	•	•	•	•	•
(P) Product Price Uncertainty	0	0..72	0	0	73	73	73	•	•	•	•	•	•
(CP) Costs + Price Uncertainty	0..12	0..72	0	0	91	91	91	•	•	•	•	•	•
(D) Customer Demand Uncertainty	0	0	0..12	0	13	13	13	•	•	•	•	•	•
(DCP) Demand +Cost +Price Uncertainty	0..12	0..72	0..12	0..72	13	13	13	•	•	•	•	•	•

For the uncertainty setting (CP), as the budget of uncertainty for costs is increased by 1, the budget of uncertainty for prices is incremented by 12, resulting in 91 model execution runs.

For the uncertainty setting (DCP), as the budgets of uncertainty for costs and for demand are increased by 1, the budgets of uncertainty for the prices and the excess capacity are increased by 6, resulting in 13 runs.

In order to study the most adverse cases, the maximum deviation uncertainty sets presented in tables 5.1,5.2,5.3 are used with the exception of demand uncertainty where are used the standard deviation uncertainty sets. As shown in table 5.4, the budgets of uncertainty are varied from 0-12 for the crude costs (so these costs may vary in all 12 periods under analysis), from 0-72 for the product prices (so the price may vary for all 6 products and in all 12 periods), from 0-12 for the demand uncertainty, and from 0-72 for the excess inventory capacity costs. Uncertainties are first tested individually by varying a single budget of uncertainty, while the remaining budgets of uncertainty are fixed to 0. Later, are performed the tests for combined uncertainty, by varying simultaneously several budgets of uncertainty.

The results are presented with a focus on the strategic decisions of the storage depots, the resource capacities, and the transportation links to install, operate, or to close; and the tactical decisions of the products' transfer and inventory volumes at the refineries, depots, transportation, and retail locations. The robust designs and planning solutions are analysed with respect to the impact on the total profits, detailed costs, and the service tariffs per storage depot, transportation route, entity, and product, for the three network designs, considering individual as well as simultaneous uncertainties, both with increasing levels of uncertainty.

As the model considers many settings including multiple entities, the MILP results for such characteristics were also verified in the exhaustive tests performed. However, the number of tests and combinations required to study these characteristics in adequate detail is considerably large, especially due to the dimension of the instance, and hence, the presentation of the multi-entity test results will not be covered in this chapter. Thus, the results discussed below, consider the maximization of the total profits for an uni-entity network, where the inventories are restricted to 1 month's demand, and enforced 100% demand satisfaction.

The RO MILP results are now presented and discussed, with a focus on the strategic and tactical decisions, the impact on the objective function values, and their probabilities of constraint violation.

5.9.1 Strategic Decisions

5.9.1.1 Infrastructure Storage Depots

In this section, are presented in figure 5.6, the depots that are installed and operated for the nominal as well as the 5 RO models C,P,CP,D,DCP, considering the current, grassroots, and retrofit designs. The nominal deterministic model installs 11, 5, and 6 storage depots for these designs. Although the models are run by varying the levels of uncertainty: for the crude costs up to $\Gamma^{C} = 12$, for product prices up to $\Gamma^{R} = 72$, for demand uncertainty up to $\Gamma^{\delta} = 12$, and for the combined costs and prices uncertainties, and for simultaneous demand, costs, and prices uncertainties, by varying all budgets simultaneously; it is not possible to present here all the depot installations decisions for all the budgets of uncertainty tested. Nevertheless, the total PSC profits are presented later for different levels of uncertainty.

In the figure, the optimal solutions are the green circles and the worst-case solutions are the yellow triangles and the green circles. In each case therefore, the robust solution determined by the decision maker's chosen probability of constraint violation lies between these two extreme solutions. It is observed that the increase in the budget of uncertainty results in an increase in the number of storage depots. The increase in the number of depots in the robust solutions, is related to the increase in depot inventories and the refined products' imported volumes.

Model	Design	Γ^C	Γ^R	Γ^δ	AVEIRO	AZAMBUJA	BANATICA	BARREIRO	FARO	LEIXOES	MATOSINHOS	PERAFITA	REAL	SETUBAL	SINES
(N)	Current				●	●	●	●	●	●	●	●	●	●	●
	Grassroots				●	●					●			●	●
	Retrofit				●	●			●		●			●	●
(C)	Grassroots	12			●	●		●			●				●
	Retrofit	12			●	●			●		●	●		●	●
(P)	Grassroots		72		●	●					●			●	●
	Retrofit		72		●	●		△	●		●			●	●
(CP)	Grassroots	12	72		●	●					●			●	●
	Retrofit	12	72		●	●		△	●		●		△	●	●
(D)	Grassroots			12	●	●		●			●				●
	Retrofit			12	●	●	●	●	●		●		△	●	●
(DCP)	Grassroots	12	72	12	●	●					●			●	●
	Retrofit	12	72	12	●	●		●	●		●	△		●	●

Figure 5.6. Depot selection for (N)ominal, (C)ost, (P)rice, (CP), (D)emand, (DCP) uncertainty. ● Optimal △ Worst-case

From the analysis of the results it can be stated that the number of depots decrease as the budgets of uncertainty are decreased, with consequent impacts in the profits. Here, the current network has the most depots, followed by the retrofit, and the grassroots the least. RO gains further relevance as it guarantees near-optimality within the pre-defined interval of uncertainties and it is also possible to determine the probability of constraint violation beyond these limits, which is not the case with the nominal solutions. From the worst-case solutions, it is observed that the product prices uncertainty, followed by the demand uncertainty are the ones who lead to an increase in depots.

5.9.1.2 Resource Capacities

To understand the model's decisions concerning the selection of resource capacities, we analysed the number of installed and operating vessels per storage depot and per product under study (GO,U5,FU,A1,C3 and C4) and shown in figure 5.7, for the nominal and robust C,P,CP,D,DCP models, considering the current, grassroots and retrofit designs. For each depot location, there are two columns, the left one provides the number of operating vessels in the optimal solution and the right one is their number in the worst-case solution. The model has also been solved for numerous settings of the budgets of uncertainty, determining for each design, a different number of vessels per product, which lies between the optimal and worst-case capacities shown in the figure. Clearly, this solution could be obtained by solving the model for a given budget of uncertainty, based on the decision-maker's acceptable probability of constraint violation. Nevertheless, it also makes sense to obtain the solutions for all budgets of uncertainty, and their respective probabilities of constraint violation, and thus the decision-maker can more appropriately choose a solution based on his risk attitude and the additional information available.

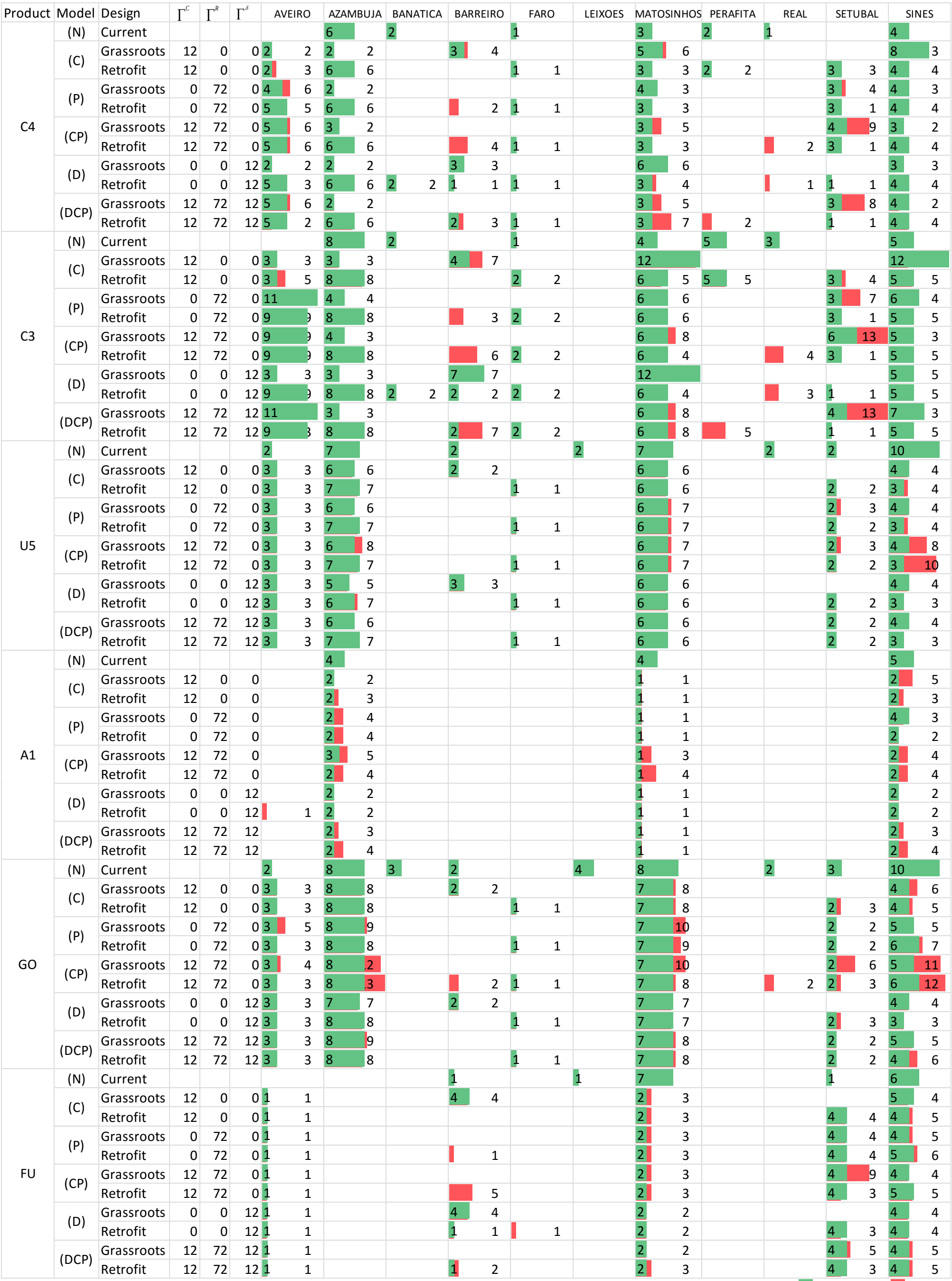


Figure 5.7. Product vessels for (N)ominal, (C)ost,(P)rice,(CP),(D)emand and (DCP) uncertainty. Optimal Worst-Case

As known, in the current networks, the resources defined per product and/or transport mode are fixed and do not change with uncertainty. However, in the retrofits their numbers are significantly reduced when compared to the current networks. The grassroots have the highest concentration of vessels, thereby resulting in very few storage depots.

Nevertheless, when considered the crude costs uncertainty, the number of vessels increases for both the grassroots and the retrofit designs. This number increases further when considered the products prices uncertainty and accentuates when considered both the costs and prices uncertainties. The numbers of vessels, when considered the demand uncertainty, are similar to those with crude costs uncertainty and the numbers for the simultaneous demand, costs, and prices uncertainties are similar to those when considered the product prices uncertainty. From the above, it becomes clear that moving towards the grassroots design, reduces the installed resources. This is also the case when reduced the budget of uncertainty. However, this comes at an expense of the increase in risk as these solutions may be sub-optimal in the presence of a higher level of uncertainty. Therefore RO is a powerful resource that can provide good trade-off solutions that stay feasible and near-optimal within the pre-established uncertainty bounds.

5.9.1.3 Transportation Infrastructure Links

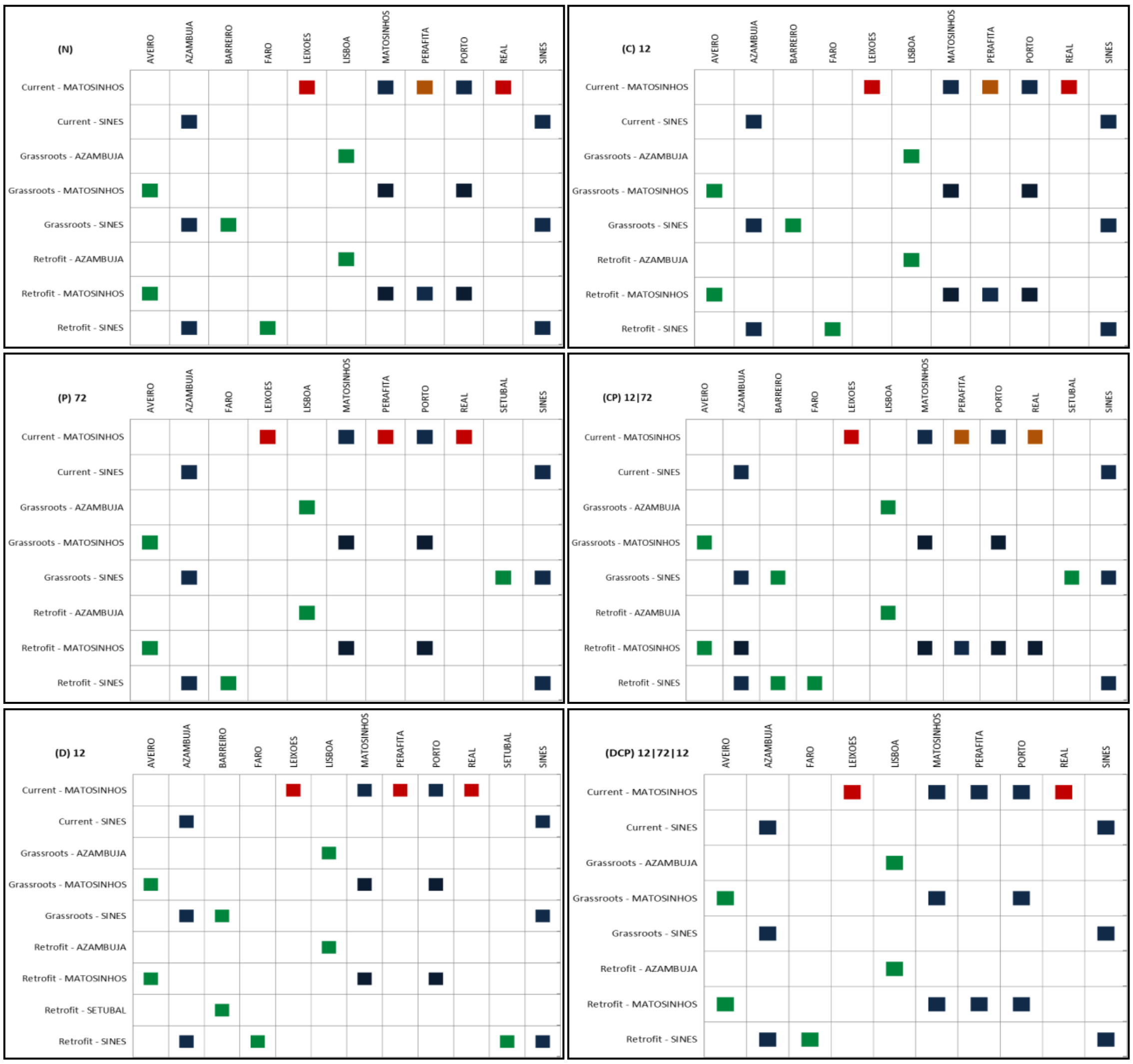


Figure 5.8. Pipeline links for (N)ominal and worst-case (C)rude costs, (P)roduct price, (CP), (D)emand, and (DCP)
■ operate current link ■ install new link ■ close current link in some designs ■ close current link

For completeness, we now present the decisions concerning the installation and operation of the transportation infrastructures for the optimal and the worst-case solutions, which result by solving the 5 uncertainty formulations (C,P,CP,D,DCP), see figure 5.8. As expected, the number of pipelines decreases from the current to the retrofit to the grassroots networks, associated with a rearrange of the existent transportation infrastructure, which means that the current PSC network design in Portugal is not optimal and could be improved.

Analogous to the earlier strategic decisions, there is a reduction, hence, concentration of pipelines in the grassroots networks. As in the previous decisions, as new depots are installed with the increase in uncertainty, so also are installed new pipeline links, specifically in the worst-case Grassroots and Retrofits. Clearly, the pipeline is preferred to the current maritime transport to many depots. A particularly interesting case arises with the increase in product prices and demand uncertainties, where the resulting increase in depots is accompanied by the appearance of a routed network, which includes the inter-depot transportation. This contrasts with the current and optimal networks, where only refinery-depot pipelines are observed. It is not possible to present here the solutions for all budgets of uncertainty, however, their objective values will be presented and analysed in the section on the total PSC profits maximization.

5.9.2 Tactical Decisions

5.9.2.1 Transfer Volumes and Inventories

The tactical decisions are now shown for the five robust settings (C,P,CP,D,DCP) for various levels of uncertainty as shown in figure 5.9. Here, the total product refined (QR) and exported (QE) are plotted on the left axis; and the product imported (QI), crude inventories (SC), and the product inventories at the refineries (SR), depots (SD) and retail locations (SL) are plotted on the right axis. For the sake of simplicity, only the retrofit designs are plotted for the execution runs of crude costs, product prices, simultaneous costs and prices, demand, and simultaneous all uncertainties. Due to 100% demand satisfaction, the primary and secondary transportation volumes do not change significantly, and hence are not presented. As shown in the figure The total refined product (QR) volumes begin at 253 million cubic meters (MM3) in the optimal designs, and decrease down to 230, 220, 160, 215, 180 MM3, respectively, for the crude costs, prices, both costs and prices, demand, and simultaneously all uncertainties when the respective budgets of uncertainty are increased up to maximum. The total exportation volumes (QE) are at 120 MM3 in the optimal design, and decrease to 105, 90, 40, 90, 50 MM3, respectively with the crude costs, products price, simultaneous costs and prices, demand, and simultaneous all uncertainties when budgets increased to maximum. The total imported product (QI) starts at 8 MM3 in the optimal design, and with the increase in the budgets, these increase up to 9,12, 11 respectively with crude costs, product prices, and demand uncertainties, and to 13 with both costs and prices as well as when considered all uncertainties. The total crude inventories (SC) are relatively stable between 12 and 14 MM3 irrespective of the type and level of uncertainty. The total refinery inventories (SR) are at 1 MM3 in the optimal designs, and increase up to 8 MM3 for price and both costs and prices uncertainties and up to 3 MM3 for the remaining uncertainties with the increase of budgets to maximum. The total depot inventories (SD) are 2 MM3 in the optimal designs, and increase to 7, 13, 19, 7, 10 MM3 for the costs, prices, both costs and prices, demand, and simultaneous all uncertainties at their maximum budgets. Finally, the total retail inventories (SL) are constant at around 4 MM3, irrespective of the levels of uncertainty. As observed, the major negative impact with uncertainties is the decrease in the exportation and the refined product volumes. The changes are higher when the budget of uncertainty for the crude costs is beyond 6 and that for the products prices beyond 12, which occurs when the probability of constraint violation decreases below 0.1 as shown in figures 5.10 and 5.11.

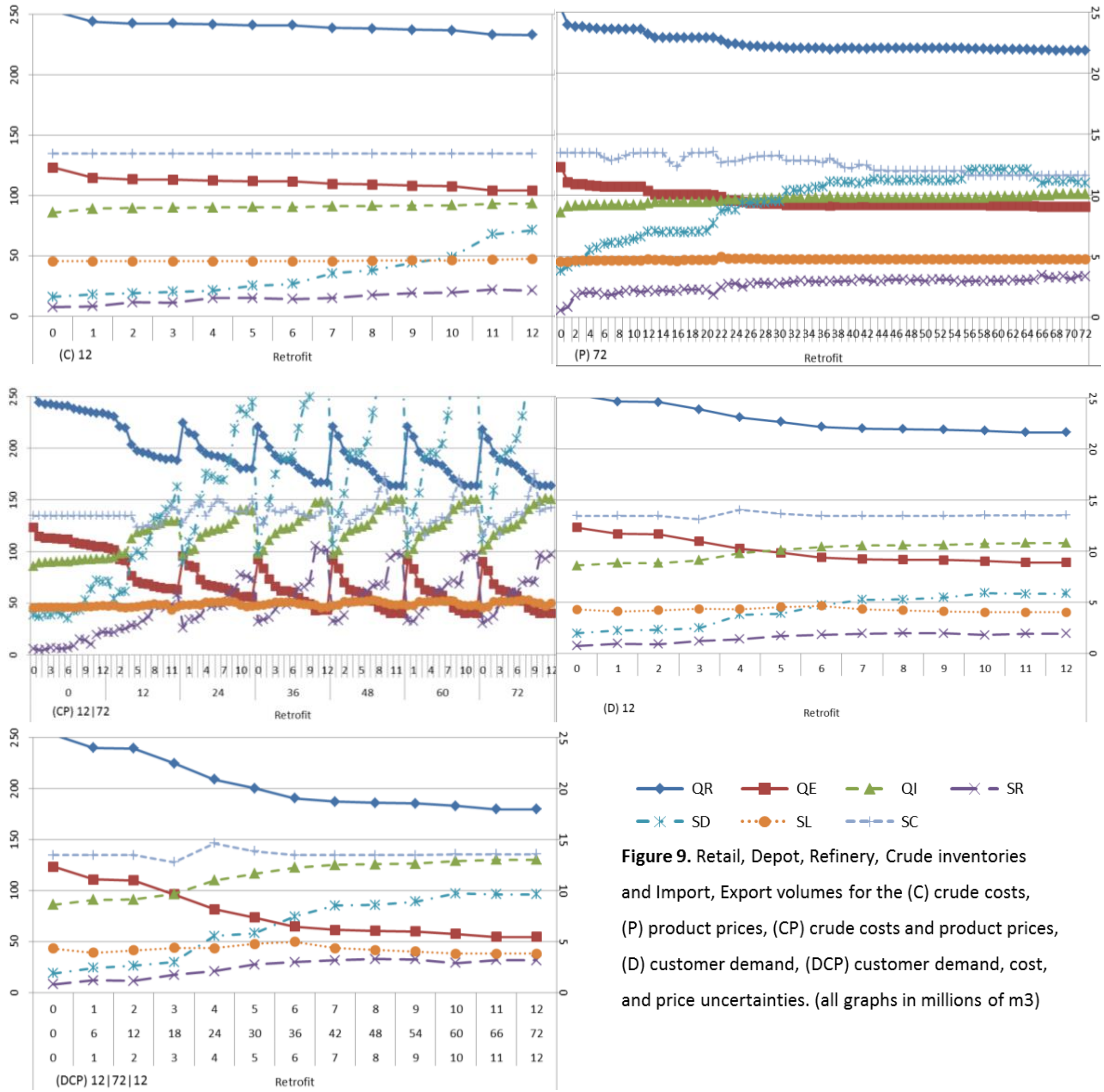


Figure 9. Retail, Depot, Refinery, Crude inventories and Import, Export volumes for the (C) crude costs, (P) product prices, (CP) crude costs and product prices, (D) customer demand, (DCP) customer demand, cost, and price uncertainties. (all graphs in millions of m3)

A conclusion to be drawn here is that the amplitude variation of the transfer volumes and inventories is not significant, as the 100% demand satisfaction and maximum one month inventory bounds, additionally restricts the flexibility for the uncertainty. The variation in the volumes is more significant when various uncertainties are simultaneously modelled as in the case of (CP) and (DCP). Note that the high level of fluctuation in (CP) is because the model is solved for some combinations and not for each and every level of cumulative uncertainty. It would be expected considerable changes if these parameters were to permit a higher tolerance.

5.9.3 PSC Total Profits Maximization

5.9.3.1 Crude Cost and Product Price Uncertainty.

To access more easily the trade-off solutions when individually modelled the crude costs uncertainty, the PSC total profits and the probabilities of constraint violation for varying levels of uncertainty are presented in figure 5.10, for the current, grassroots and retrofit network designs. When the budget of uncertainty for the crude costs is increased in the range $\Gamma^{C} \in [0,12]$, the total profits, given in billions of monetary units (BMU), reduce from 26.44 to 15.48 for the current network, from 27.13 to 16.24 for the grassroots and from 28.06 to 17.18 BMUs for the retrofit. The probability of constraint violation drops below 0.01 when the budget of uncertainty for the crude costs is at $\Gamma^c \geq 9$. This corresponds to the solution values of 16.86, 17.62, or 18.55 BMUs respectively for the current, grassroots, or retrofit network, implying a 34-36% decline in profits for a 99% confidence level.

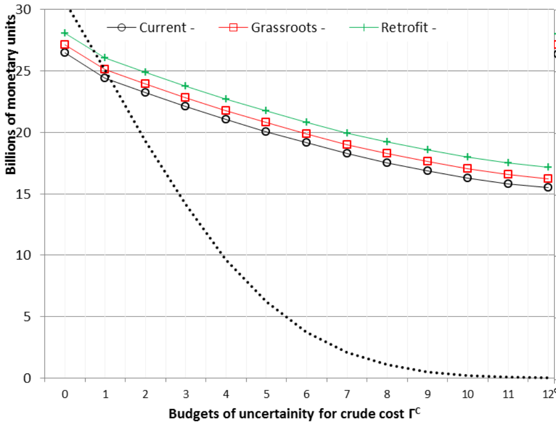


Figure 5.10. Total profits with crude costs uncertainty

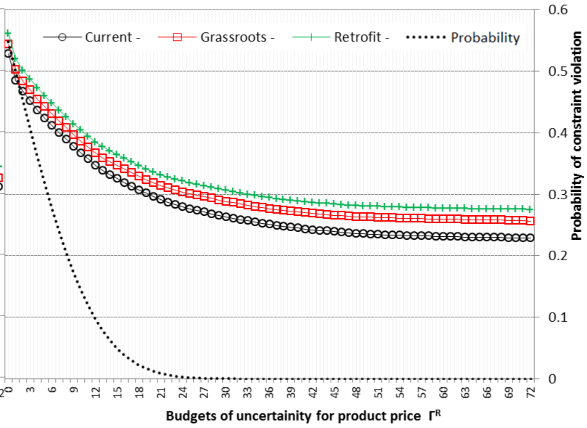


Figure 5.11 Total profits with product prices uncertainties

The total profits and the probability of constraint violation when individually considered the product prices uncertainties are shown in figure 5.11. It is observed that as the budget of uncertainty for products' prices is increased $\Gamma^R \in [0,72]$, the total profits' fall from 26.39 to 11.44 in the current network, from 27.09 to 12.84 in the grassroots and from 28.04 to 13.76 in the retrofit. The probability of constraint violation recedes to 0.01 with the budget $\Gamma^R = 21$, which corresponds to total profits of 14.54, 16.56 or 16.81 BMUs for the current, grassroots, or retrofit networks. This implies a decline of 39-45% in the profits, considering a confidence level of 99%. This means that for the retrofit design, a decision-maker willing to accept a probability of constraint violation of 0.01, would choose the solution corresponding to $\Gamma^R = 21$, thereby guaranteeing the total PSC profits of at least 16.81 BMUs. It should be noted that the model is executed for various budgets of uncertainty and the decision-maker will choose among the solutions based on his/her level of risk attitude. A particularly important insight here is that a lower probability of constraint violation, meaning a higher confidence level, is achieved with relatively lower budgets if the cardinality of the uncertainty parameters is high. This can be observed by comparing the plots of the probabilities in figures 5.10 and 5.11. Consequently, the degradation in performance is seen to be lower for uncertain parameters with higher cardinality.

5.9.3.2 Simultaneous Crude Cost and Product Price Uncertainty.

It is interesting to compare the impacts on the total profits when both crude costs and product prices uncertainties are increased simultaneously in the ranges $\Gamma^C \in [0,12]$ and $\Gamma^R \in [0,72]$. For every increase of Γ^C by 1, the model is executed 12 times by increasing Γ^R by 6 in every consecutive run. This corresponds to including 6 worst-case periods of products' prices, thereby executing the model 144 times. The robust solutions presented in figure 5.12, show that the decrease in total profits is more significant, reducing from 26.39 to 4.34 in the current network, from 27.09 to 6.04 in the grassroots, and from 28.04 to 7.05 in the retrofit in the simultaneous cost and price uncertainty worst-case solutions. To maintain the probability of constraint violation below 0.01, the worst-case crude costs may occur in at most 9 periods $\Gamma^{C}=9$ and for at most 36 combinations of product prices and periods $\Gamma^{R}=36$. At this level of uncertainty, the PSC profits fall to 6.25, 7.45 or 8.39 BMUs respectively, for the current, grassroots, or retrofit network, thereby implying a 70-76% reduction in the profits for a guarantee of 99%.

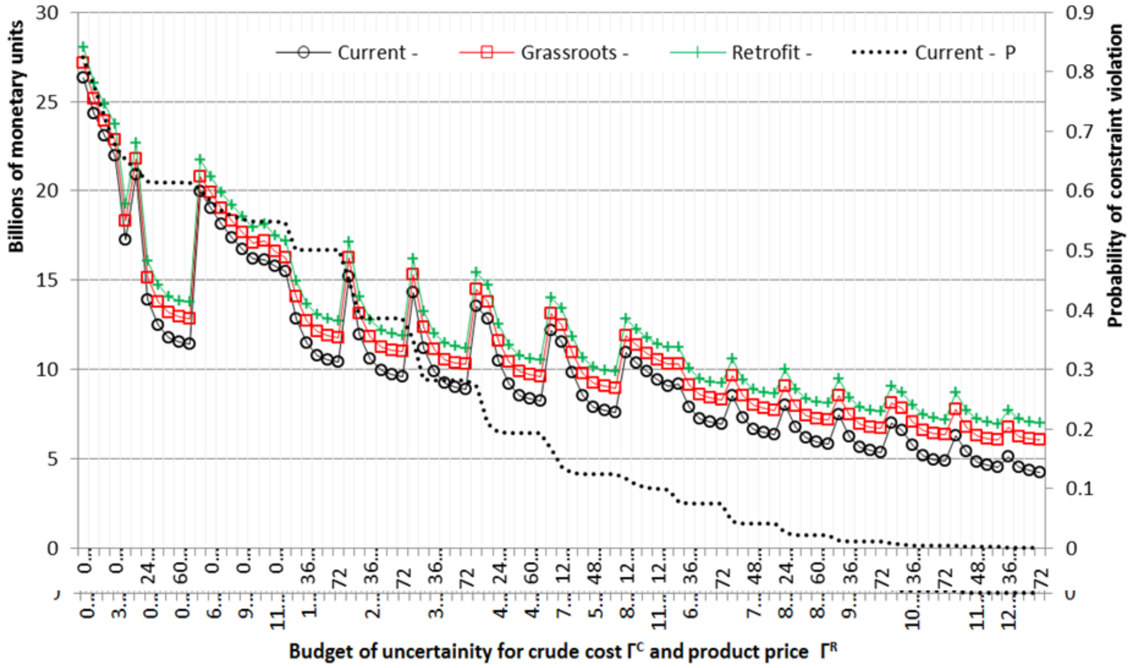


Figure 5.12. Total profits with simultaneous crude costs and product prices uncertainty

5.9.3.3 Cumulative Demand, Crude Cost, Inventory Cost and Product Price Uncertainties

We now turn our attention to the customer demand uncertainty $\Gamma^{D}_{k,p,e,t}\in[0,12]$ whose robust solution profits, for increasing levels of uncertainty, are presented in figure 5.13. The robust solutions for demand uncertainties are obtained by increasing $\Gamma^{D}_{k,p,e,t}$ by 1 in each consecutive execution of the MILP. This corresponds to adding 1 worst-case demand period per location, entity and product in every execution run. Considering only demand uncertainty, it is observed a reduction in the total profits, in the current network from 26.43 to 19.21, in the grassroots from 27.11 to 19.42, and in the retrofit from 27.99 to 20.04 BMUs, which implies a variation of 28% in the total profits, in order to protect against the highest level of demand uncertainty. A crucial aspect therefore is that considering the PSC has high uncertainty in many of its parameters, the profits deteriorate rapidly as the uncertainties are simultaneously increased. From a practical point of view, correlations, if existing, should be taken into account, and when simultaneously modelled, their budgets should be calibrated with caution so as to avoid excessive budgeting.

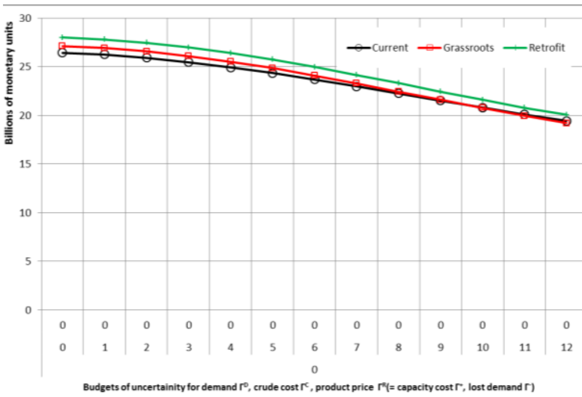


Figure 5.13. Total Profits for customer demand uncertainty

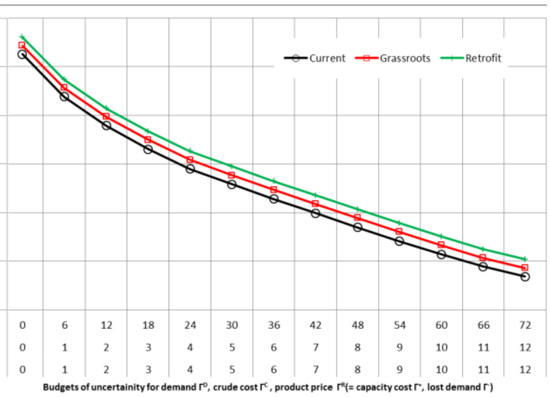


Figure 5.14. Total Profits with all uncertainties

To illustrate the impact of simultaneous uncertainty, figure 5.14 presents the total profits when increased simultaneously, the budgets of uncertainty in the following ranges: for demand $\Gamma^{D}_{k,p,e,t} \in [0,12]$, crude costs $\Gamma^{C} \in [0,12]$, products' prices $\Gamma^{R} \in [0,72]$, and excess inventory capacity costs $\Gamma^{+} \in [0,72]$. Here, the MILP is executed 12 times, by including in each iteration, one additional period of uncertainties. However, as Γ^{C} and $\Gamma^{D}_{k,p,e,t}$ are increased by 1, Γ^{R} and Γ^{+} are increased by 6, thereby including 6 period products' prices in every iteration. As expected, the changes between the optimal and the worst-case solution are highest when all uncertainties occur simultaneously, resulting in the decline of the total profits in the current network from 26.27 to 3.42, in the grassroots from 27.17 to 4.29, and in the retrofit from 28.03 to 5.19. The worst-case solution is derived with all parameters simultaneously at their worst-case and in all periods. As can be seen, this extreme case is overly pessimistic, however, such a situation is also most unlikely. This proves that the selection of the budget of uncertainty is crucial to the price of robustness. The study of the uncertainty parameters, correlations, and methodologies to better estimate probabilities, is consequently identified as a potential area for future research.

5.10 Model Solution Performance Statistics

The results presented above include 39 execution runs for crude costs, 219 for product prices, 270 for crude oil costs and product prices, 39 for customer demand, and 39 for simultaneous customer demand, crude costs and product prices uncertainties. Table 5.5 presents the model statistics for the execution of the different model formulations and network designs, providing the number of equations, the continuous and discrete variables, the model executions and CPU times, and the objective values and % GAPs from the best known upper bound. The CPU times increase between current to grassroots and retrofit designs, and between cost uncertainty to demand to the simultaneous uncertainties. The robust as well as optimal PSC profits increase from current to grassroots to retrofit design, however the increases are relatively small. Several factors contribute for these reduced differences: although the current, grassroots and retrofit designs are not the same, they have similarities; most of the current network infrastructures have already been amortized, whereas the grassroots and retrofit designs require new investments, and therefore have higher investment costs even though the distribution costs are reduced; the total distribution costs and profits are significantly lower than the refinery and retail margins and hence their relative impacts are smaller; the refinery, distribution and retails costs are not precisely known and could require further calibration; and the amortized installation costs are not discounted from the excess capacity costs, which if done would increase the differences in profits, this may be considered in future work. Nevertheless, the increase in profits from the current to the retrofit network, considering 100% demand satisfaction and restricted to maximum one month inventories, is about 1.69 BMU.

Table 5.5. Model Performance statistics

Model	Design	Equations	Variables	Discrete	Runs	Objective ($x10^9$)	GAP	CPU(s)
	Current	67 582	252 730	2 254	13	15.48 - 26.44	0.01 - 0.02	4 - 6
(C)	Grassroots	71 139	255 283	2 888	13	16.24 - 27.13	0.79 - 1	13 - 341
	Retrofit	79 772	303 993	3 267	13	17.18 - 28.06	0.44 - 0.98	8 - 330
	Current	67 639	252 790	2 254	73	11.44 - 26.39	0 - 0.18	4 - 5
(P)	Grassroots	71 199	255 343	2 888	73	12.82 - 27.17	0 - 1	10 - 267
	Retrofit	79 832	304 053	3 267	73	13.76 - 28.05	0.74 - 1	8 - 226
	Current	67 651	252 803	2 254	91	4.26 - 26.31	0 - 0.89	4 - 6
(CP)	Grassroots	71 212	255 357	2 888	91	6.04 - 27.18	0 - 1	9 - 2917
	Retrofit	79 845	304 067	3 267	91	6.97 - 28.04	0.31 – 1	8 - 18221
	Current	83 820	292 003	2 254	13	19.42 - 26.43	0 - 0.02	4 - 7
(D)	Grassroots	88 232	294 953	2 888	13	19.21 - 27.11	0.87 - 1	33 - 737
	Retrofit	96 865	343 663	3 267	13	20.04 - 27.99	0.85 - 1	15 - 92
	Current	83 902	293 801	2 254	13	3.42 - 26.27	0 - 0.95	5 - 14
(DCP)	Grassroots	88 318	296 769	2 888	13	4.29 - 27.17	0.94 - 1	24 - 15866
	Retrofit	96 951	345 479	3 267	13	5.19 - 28.03	0.89 - 1	13 - 9860

As mentioned earlier, the RO MILP was also executed for the multi-entity, not here presented, however each run still takes too much time, over 4 hours of runtime in some cases, as the combinatory problem increases significantly in size when we consider 5 entities simultaneously. The reason behind this is because each product is treated as a separate network as in reality, the entities may create capacity for some products at a location and for others at other locations. It is perceived, however, that the problem complexity could be decreased, if the network is converted to an entity level network, wherein, would be decided first the locations per entity and then the capacity to be installed per product at these locations. Reduction of the problem size for multi-entity networks is identified as an opportunity for future research, in which the authors are now investing.

5.11 RO Application Main Insights and Modelling Perspectives

To gain some understanding of the main insights of the developed research and of future implications, consider the performance gains verified in this research through the tractable RO model, which are substantial when compared to our stochastic MILP (2015) for the PSC strategic and tactical planning with demand uncertainty. Specifically, the stochastic MILP in Fernandes *et al.* (2015) for a similar setting and 9 demand uncertainty scenarios, took 59556 CPU seconds to solve for a 0.1% GAP from the best known upper bound. Simultaneous incorporation of cost and price uncertainty was therefore perceived to be very difficult to solve without the implementation of decomposition algorithms.

The RO model was executed for three different network designs, six types of uncertainty and for various levels of uncertainty from the optimal to the worst-case. Not all these strategic and tactical decisions, and respective installation and operational costs, tariffs and profits and probabilities could be shown. The decision-maker, however, can now make an informed choice among many available solutions. Specifically, if the uncertainty is low, the decision-maker may choose a lower budget of uncertainty, implying less infrastructures, less capacities, lower costs, lower flexibility and higher profits. On the contrary, if the uncertainty is high, the decision-maker would prefer to use a higher budget of uncertainty, implying more infrastructures, more capacities, higher costs, higher flexibility and lower profits. This task could however be helped by previously calculating the probabilities for each budget of uncertainty which have also been calculated and shown so the decision-maker can choose one of the solutions.

It was the objective of this research to verify if RO could be used to provide different strategic solutions for varying levels of uncertainty and that has been shown. Moreover, the RO formulation with the budget of uncertainty fixed at 0, determines the optimal solution. It should be noted that the many trade-off solutions between the optimal and the conservative ones, offer higher robustness and flexibility, at the cost of installing more resources, which imply lower profits. However a deterministic nominal solution would generate even lower profits if the costs, prices and demand uncertainties were to occur. The solutions are obtained without significantly deteriorating the computational performance in comparison to the deterministic model. To provide similar solutions with stochastic programming, it would require decomposition techniques or ε-constraint algorithms which would imply much more computational effort especially relevant for large scale models.

Thus highlighting the potential implications of this research, it can be stated that RO consolidates as a powerful methodology, supported by its fundamental strengths:

- Uncertainties may be incorporated without much information on their distributions;
- Wide variety of uncertainties including demands, costs, prices, time periods can be treated;

- Numerically tractable for large scale supply chain problems, where stochastic programming methods show serious complexity issues in computational terms;
- Advantageous to engaging managers and researchers on a common ground;
- Qualitatively similar optimal solutions as in stochastic programming;
- More and faster solutions when compared to scenario-driven and stochastic models.

In order to establish possible future model perspectives it should be considered that from the results presented it can be summarized that when only one uncertainty is modelled, the RO results show solution robustness with the profit remaining relatively stable, as well as model robustness wherein the solution stays feasible with a very high probability. However, when taken into account various uncertainties, RO can overly penalize the objective function, and hence the use of expert criteria to adequately define the concurrent budgets of uncertainty is identified as an improvement opportunity. Note, however, that this excessive penalization does not necessarily occur as it depends on the budget of uncertainty used. In this study the model was run with 100% demand satisfaction, which results in higher excess inventory capacity costs that further penalizes the objective function. However, it would be interesting to allow lower demand satisfaction rates, by incorporating the lost sales as tactical decisions. Although, these improvements have been implemented, the robust lost sales formulation requires further examination and a detailed presentation of the results, which are not covered in the current chapter. Consequently, these developments as well as the simultaneous modelling of the excess inventory capacity and the lost sales are considered for future work.

When simultaneously considering various uncertainties, the combined budgets of uncertainty, should be defined considering a practical and realistic point of view, in order to obtain more meaningful trade-off solutions between the optimal solution and the extremely pessimistic solution. Knowledge of correlations between uncertain parameters, for instance, between the crude oil costs and the ex-refinery product prices or the retail product prices, can be used to discard the uncertain parameters that do not increase the negative outcomes when occurring simultaneously; this could avoid excessive budgeting. Alternative formulations such as aggregated constraints could also be tested in order to overcome over conservatism of constraints when considering correlated uncertain parameters, or correlation among indices of an uncertain parameter. In order to combine the budgets of uncertainty, the scenario construction theory could be employed, however, without creating scenarios. Ben-Tal *et al.* (1998,2000) proposed the affine adjustable robust counterpart (AARC) RO methodology that could be used to model demand uncertainty. Here, the decision variables could be approached distinctly: first stage decisions as in the depot, transportation and resources installation and operation; and second-stage decisions for the transfer volumes and the product inventories. The tariffs could be determined as second stage decisions that is after the uncertainty is revealed. Moreover, the characterization of the price of robustness to build valuable test cases, as referred in Beyer and Sendhoff (2007) and Bertsimas *et al.* (2011) is seen as a potential area for future research.

Finalizing and as a conclusion that can be drawn from this research, and confirmed by the present findings, is that RO has high value for strategic planning, thereby sustaining the future potential for this line of research.

5.12 Conclusions

In this chapter, we presented the petroleum supply chain (PSC) design and planning problem, the PSC challenges and opportunities, reviews on PSC planning models and on robust optimization (RO), and a PSC strategic and tactical planning deterministic MILP. Three robust counterparts were formulated for uncertainties in the objective function and the demand parameters, using polyhedral uncertainty sets, in order to develop an uncertainty planning model as tractable as the deterministic MILP. The RO formulation implements protection against the crude costs, product prices, and customer demand uncertainties. Considering these uncertainties, the following decisions were determined: 1) the PSC infrastructures, capacities to install, operate, and close; 2) the strategic fair price costs and tariffs per location/route, entity and product; 3) the multi-stage product transfer and inventory volumes per period. Uncertainty sets were constructed for the real case Portuguese PSC and RO results were presented. Considering the PSC strategic and tactical planning problem, the RO methodology is observed to be tractable while obtaining robust and trade-off solutions, considerably faster than our earlier stochastic model.

Important insights and research opportunities are identified for using RO with PSC planning under uncertainty, and are the basis of our future research proposals: 1) better calibration of the excess inventory capacity costs in the current and retrofit designs by discounting the amortized investment costs; 2) modelling the excess inventory capacity simultaneously with the lost sales so the model may decide based on the respective costs, and allowing both situations to occur in the same period; 3) characterization of the price of robustness in order to improve the treatment of uncertainty; 4) study the effect of backorders on the PSC planning decisions; 5) reduction of the problem size when considering multi-entity networks; 6) extending the MILP to operational level decisions over the spot market.

From the present work further future lines can be devised. In particular, the authors are already working on the simultaneous modelling of strategic inventory capacities and lost sales decisions, and the correlations between uncertainties and costs as this was recognized as an important problem to address. Additionally it will be important to study strategic decisions over time, by considering depot and pipeline construction and labour costs, and interest rate uncertainties. Another direction is the study of the impacts on the collaborative decisions and the economy of scale entity tariffs. As new data becomes available, the stochastic and the robust models should be further compared using a common set of data. The design of uncertainty sets, the budgets of uncertainty and determination of their probabilities will consider the specificities characteristics of the PSC planning. The implementation of the RO AARC methodology for modelling strategic and tactical decisions, considering demand uncertainty should be also considered for future research.

CHAPTER 6. Conclusions and Future Research

6.1 Evaluation of the research realized

In this thesis we have developed four studies that were proposed as the research objectives for the Strategic and Tactical Planning problem of Downstream Petroleum Supply Chains (PSC) under crude costs, product prices and customer demand uncertainties.

A detailed cost, tariff, profit and price formation framework has been developed for the downstream PSC, which was used to model generalized network formulations that can also be applied to other supply chains with similar characteristics such as depot storage, distribution and resource equipment capacity. The price formation framework is well structured so as to design a network with little information on the potential locations.

The uncertainty in the downstream PSC has been studied and characterized, thereby resulting in costs, prices and demand scenarios with uncertainty sets that define the upper and lower bounds of their variations.

Four different models yet with complimentary use have been developed, the first being a deterministic MILP for individualistic strategic and tactical planning model that maximizes the total PSC profits and that is the base model of this thesis. The second is a deterministic MILP for the collaborative strategic and tactical planning model, which determines the multi-stage inventories and economy of scale costs, tariffs, profits and prices maximizing total profits including their participation in the logistic operator profits. The third is a two-stage stochastic MILP that maximizes the ENPV while considering customer demand uncertainties, which is then improved to a multi-objective stochastic MILP that simultaneously maximizes the ENPV and minimizes a risk measure such as variance, variability index, downside risk and CVaR. Finally a robust optimization model was developed that maximizes the total PSC profits by varying the level of uncertainty for crude costs, product prices and customer demand.

The above models were applied to the downstream PSC in Portugal, which was studied and structured with significant depth. The characterization of such PSC was performed and detailed historical data was gathered that outlines the Portuguese downstream PSC, which has been published with relevant detail so as to be available for other investigators.

Despite the specificities of the problem studied in this thesis, generalized models have been developed, and hence can be applied to other PSC case studies that share similar structural characteristics. Other industrial supply chains may also benefit from the presented models. Specifically, the thesis has achieved the objective of closing the distance between theoretical research and real-world applications, as we successfully applied the state of the art optimization theory to design models, and showed their capability to efficiently solve the real world problems of the PSC design and planning under uncertainty. The research contributions are summarized in further detail in the following section.

6.2 Research Contributions

As emphasized, the complexity and the uncertainties in the PSCs are constantly growing everyday due to their global operations and huge product volumes, which are subject to costs, prices and demand uncertainties. This justified the development in this thesis of the PSC design and planning models, and subsequent incorporation of the uncertainty and risk management features. The PSC faces many challenges, as presented in the introduction of this thesis, and along the work developed we have attempted to fill in the gaps by proposing generalized models to deal with the strategic and tactical planning problem as well as the crude oil costs, product prices and customer demand uncertainties.

In this section are presented the main contributions of this research, which are the mathematical models that were developed for the downstream PSC strategic and tactical planning problem. As these models have been published as independent papers in international peer reviewed journals, or are in the review process of submission, they constitute the main chapters of this thesis. Chapter 2 presented a novel deterministic MILP for the competitive strategic planning of the PSC network installations associated with tactical decisions of multi-stage transfer volumes. The model was applied to a detailed case study of the downstream PSC in Portugal. Chapter 3 extended the individualistic model, presented in chapter 2, to a dynamic collaborative MILP by implementing tactical decisions of inventories and introducing economy of scale tariffs along time. Chapter 4 generalized the collaborative model to a two-stage stochastic programming MILP, which considers customer demand uncertainties and maximizes ENPV. This model is further extended to a multi-objective MILP where four risk measures are modelled and implemented through an augmented ε-constraint algorithm (Mavrotas and Florios, 2010) so as to simultaneously maximize the ENPV and minimize a risk measure. Having faced difficulties in addressing the detailed uncertainty in Chapter 4, due to the computational intractability as a result of the model complexity, Chapter 5 presented a new approach to deal with uncertainty, the robust optimization method. An interesting development results in Chapter 5 where was proposed a robust optimization planning MILP, in order to model the crude costs, product prices and customer demand uncertainties. All four models have been tested with the real case downstream PSC in Portugal, thereby developing innovative ideas for the PSC strategic and tactical planning problem, while designing and optimizing the current, grassroots and retrofit designs for the Portuguese PSC. The models described in Chapters 2, 3, 4, and 5, represent novel approaches, with respect to the integrated decision making process of the PSC Downstream multi-entity network design and tactical planning with uncertainty.

The second chapter presented a novel MILP model that maximizes the total PSC profits for the strategic and tactical planning of downstream PSC networks. This has the novelty of modelling a bi-level transportation network flow formulation where the primary and distribution can be distinctly modelled to adhere to specific restrictions based on the product and level of transportation. The model integrates the strategic and tactical level decisions as well as the multiple PSC stages in a single deterministic mixed integer linear programing model. The strategic decisions involve the depot locations, transport modes, routes, resource capacities, and the network affectations, whereas the tactical decisions involve the crude oil and petroleum products' importation, exportation, production, storage and transportation volumes. The MILP generates the uni-entity and multi-entity PSC network designs, by solving the current network, the grassroots and the retrofit redesigns. It is observed that the MILP generates, for the Portuguese PSC better network designs with higher total PSC profits, when compared the current network to the grassroots and retrofit designs, for the uni-entity and multi-entity networks under individualistic operation. The results for the Portuguese PSC show higher logistic costs in the multi-entity when compared to the uni-entity networks. This suggests that the single entity network improves overall profits and could result in lower tariffs. The sensitivity analysis performed, by reducing the fixed and variable costs, verifies the installation of additional depots, a reduction of unmet demand, and an increase in the product transfer volumes and total profits.

The third chapter has the novelty of collaborative multi-entity strategic and tactical planning of downstream PSC networks, by maximizing the total PSC profits including the participations from partially owned logistic operator infrastructure profits, while considering a piecewise linearization function for company tariff differentiation and multi-stage dynamic inventories. Again, the current, grassroots and retrofit designs are solved for the uni-entity and multi-entity networks, determining depot locations, transport modes, routes, resource capacities and network affectations, crude oil and petroleum products' importation, exportation, production, storage and transportation volumes, and various stage dynamic inventories. Efficient grassroots and retrofit designs were generated for the uni-entity and multi-entity collaborative networks for the Portuguese PSC, where their total PSC profits supersede those of the current networks. In contrast to the individualistic model, the collaborative MILP generates multi-entity designs, whose overall profits are significantly improved and approximate those of the more efficient uni-entity designs. The multi-stage inventories optimize the procurement and exportation costs to benefit from the crude oil and petroleum product price variations. Product families and transportation mode families are used to standardize the modelling the products and/or transport mode dependent resources. Collaborative planning offsets the limitations of the current entity participation percentages in existing installations. The piecewise linearization function determines the economies of scale entity tariffs and prices, thereby permitting the collaborative and flexible utilization of competitor installation capacity. This results in considerable improvements in the costs, tariffs and profits for the PSC members. Inter-depot routing increases the flexibility in the network designs, inventories, and product flows.

The fourth chapter developed a stochastic MILP and a multi-objective stochastic MILP that consider customer demand uncertainty. The first-stage strategic decisions are scenario independent, and determine the installation and operation of depot locations, capacities, transportation modes, routes, and economies of scale entity tariffs, whereas the second-stage tactical decisions are scenario dependent, which determine the multistage crude oil and petroleum products inventories and transfer volumes. To manage the demand uncertainty, the stochastic MILP maximizes the expected net present value (ENPV) considering uncertain demand scenarios. By considering three 5 year duration periods optimistic, realistic and pessimist demand scenarios, are solved for the real-case Portuguese PSC single and multi-entity current, grassroots and retrofit networks designs. The stochastic RP solutions are compared to the EEV and WS solutions, and determined the EVPI and VSS measures. These measures show low values for the stochastic solutions, thereby identifying the need to improve the uncertain demand scenarios, and hence, is proposed a more detailed study of the market variations. The stochastic model was then improved to a multi-objective stochastic MILP by implementing the augmented ε-constraint method in order to simultaneously maximize the ENPV and minimize one of the risk measures: variance, variability index, downside risk, value-at risk and CVaR. As the multi-objective MILP maximizes the ENPV and minimizes a risk measure, better network designs are generated, which achieve higher NPVs in the presence of demand uncertainty, as the network is more flexible and adjustments are made at the tactical level for each demand scenario. The scenario NPVs cumulative probability distribution plots are obtained for the risk measures of variance, variability index, downside risk, and CVaR. Given two conflicting objective functions, the ε-constraint method optimizes first one objective function, divides its solution space into equidistant grid points, introduces a constraint to fix the first objective function at each grid point, and obtains the Pareto-optimal efficient solutions that optimize both the objective functions. The decision maker may choose an appropriate risk measure and subsequently one of many efficient solutions, based on his risk attitude. It is observed that the variability index and downside risk measures are more suitable for risk averse decision makers, whereas the CVaR is better for a risk-seeking decision maker.

The fifth chapter has the novelty of developing three robust counterpart formulations for modelling the crude costs, product prices, and customer demand uncertainties for the collaborative strategic and tactical downstream PSC planning MILP. Polyhedral uncertainty sets are used to develop the robust models as tractable as the deterministic MILP. Considering demand uncertainty for the Portuguese downstream PSC, the RO model in chapter 5 obtains robust trade-off solutions, considerably faster than the stochastic MILPs presented in chapter 4, and hence is observed to be adequate for modelling uncertainty for the PSC strategic and tactical planning problem. The RO approach circumvents the dimension problem by using a budget of uncertainty, defined by the decision maker, in order to obtain solution and risk trade-offs by means of an iterative strategy. Here, the range of possible uncertainty is defined by the uncertainty sets, whereas the range of protected uncertainty is defined by the budget of uncertainty parameter. The MILP may be executed specifically for a budget of uncertainty that better fits the decision-makers interest, or the model may provide solutions for various budgets of uncertainty. The former may use a level of uncertainty based on its previously calculated probability of constraint violation and the decision maker’s risk attitude. The second strategy is not viable for large-scale problems, however, it allows the decision maker to choose among various solutions. The RO approach obtains profits that are relatively stable when considered only one uncertainty, however, these decrease significantly when considered various uncertainties simultaneously. Here, budgets of uncertainty could be aggregated to take into account any existing correlations between uncertain parameters, in order to obtain realistic trade-off solutions. Finally, RO can model many types of uncertainties without knowing their precise distributions, besides it is numerically tractable for large supply chain problems compared to the stochastic programming methods.

As shown, the research presented here, led to important conclusions with respect to the planning of logistics infrastructures under uncertainty. Besides, many details of the reality were explored using the Downstream PSC in Portugal and its particular characteristics. To the best of our knowledge, this thesis is the first work concerning this area of study.

6.3 Future research perspectives

As has been shown, this thesis addressed very relevant topics within the PSC. However, the PSCs are complex supply chains and hence this work can be considered as a building block. There are adjacent areas where the proposed models could be applied so as to extend them to more levels of decisions and integrate a wider range of functions. Hence, many research directions are open for exploration to the scientific and industrial communities.

The developed multi-entity collaborative models are oriented to be used by single entities, a consortium of companies, the industry regulator or a logistic operator. Although the objective function maximizes the total network profits, independent terms are obtained per location or route, entity, product and time period. This objective function structure can be easily explored for multi-objective optimization wherein an objective function weight could be defined per entity, for instance their market share quotas. These objective function terms obtained per entity, could also be explored using a goal programming approach wherein entities may establish a minimum profit threshold. The strategic and tactical decisions could therefore be further enriched while determining the costs, tariffs and profits per location or route, entity and product under such conditions.

A different approach would be to apply perspectives that are not collaborative, and develop competitive models. For example, in Portugal, there is clearly a leader who dominates many parts of the PSC while the others are followers. Here, the model could develop a competitive or counter strategy based on known or projected strategic or tactical information of a competitor. Such a decision of an entity could directly affect the performance or future decision of the competitor who could launch a counter-strategy as in game theory. An entity could also make investments or hold inventories or sell products at higher prices so as to benefit from a stock out or in order to increase competition.

Smaller markets such as Portugal are seen to encourage competitive multi-entity individualistic networks; however, the regulations are taking a turn precisely in the opposite direction, which is shared infrastructures built and managed by a single entity, as predominant in the big countries.

As the collaborative model has achieved multi-entity network designs that are almost as efficient as the single entity designs, the contributions of this research are relevant for the network changes that may be implemented in the future.

In the case of the two-stage stochastic MILP developed, the study of the performance indicators VSS, EVPI, and EEV revealed low values for the stochastic solutions. Hence, fresh efforts are underway, with the Extended Pearson-Tukey Method from Clemen and Reilly (2003) and recently available data, to construct new uncertain scenarios and probabilities, based on the demand growth percentiles per localization, entity and product. The 15 year historical data is being used to obtain the future crude oil costs, product prices and customer demand forecasted scenarios. This could be corrected using the mean absolute deviation (MAD), mean square error (MSE) and/or mean absolute percent error (MAPE).

Even though the ε-constraint and robust optimization methodologies have been implemented to reduce the numerical complexity with uncertainty, other strategies could be used to manage the complexity of these problems. With respect to the two-stage stochastic programing MILP, the Sample Average Approximation (SAA) and Monte Carlo simulation based approaches proposed in Shapiro and Homem-de Mello (1998) could be used. These consider a controlled size of scenarios, in order to derive the optimal solution approximations, which consequently increase the model efficiency, the quality and the precision of the solutions.

Another important extension could be the application of the stochastic Benders and the Lagrangean decomposition procedures which were recently proposed in Oliveira *et al*. (2013, 2014), so as to evaluate the performance improvements. A sensitivity analysis could then be performed to investigate the differences in performance between the developed approaches and the decomposition approaches for such large scale problems. The computational performance could be evaluated with different algorithms or with different formulations for the nonanticipativity conditions of the first-stage decisions. Nevertheless, the incorporation of coherent risk measures such as the CVaR may result in a non-decomposable problem.

A direct implication of the strategic and tactical collaborative planning models is that they can be extended for operational planning decisions with respect to the shortage and excess of the supply, transformation, spot supply and spot selling of petroleum products. The strategic grassroots and retrofit designs, and the tactical entity tariffs, product flows and inventories could be used to define a contract agreement between the logistic operator and the petroleum companies. This setup can be thought of as minimum volume contracts where each operator is obliged to supply, unload and guarantee inventory volumes to comply within an established contract band, and pay a penalty when exceeding or defaulting from these volumes. After obtaining the strategic and tactical decisions, the models could be run again by fixing these strategic decisions and using the tactical decisions as the lower and upper bounds for the network volumes and initial and final inventories per location, entity, product and time period. The operational planning decisions could then be the mismatches (default or excess) between the earlier tactical decisions (contract volumes) and the realized volumes given the uncertain demand. The operational planning model could minimize the sum of the shortages and excesses in supply, transformation, spot supply and spot selling volumes which could be appropriately penalized. Some of these ideas have been proposed in Escudero *et al.* (1998), however, are not available in an integrated model.

At the operational level, it also becomes important to consider the multi-modal transportation times between nodes, in order to further optimize the available capacity usage in each time period. Although this information is made available in the model as parameters, it has not been explored in the model decisions, and hence is identified as a future research proposal.

The models were validated for the current, retrofit and grassroots designs, with real large-scale instances based on the Portuguese district-level PSC network. This has practical application as such work could be used to further study the Portuguese PSC by considering the municipality-level product consumption instead, or could be used to study the integration of the PSC networks in Portugal and Spain. The performance of the PSC could be studied with respect to the impact of differences in the Excise and VAT taxing systems between two neighbor countries. New insights could be arrived as to the effects and benefits of the fiscal harmonization of these taxes between the two countries, especially in the boundary areas, using other economic, sustainability and social indicators.

With respect to risk management, the more common risk measures from operational research have been studied. However, further research could be done in implementing other risk measures and comparing the results with the ones obtained. Likewise, other risk representation and management methodologies could be used, for instance, decision analysis to analyze different objectives. The risk measures could be optimized in the objective function or could be used in restrictions. Specifically, a supply chain risk management methodology and framework was proposed in Fernandes *et al*. (2011) that could be explored for the PSC.

The treatment of uncertainty could be further explored using other methodologies, such as extended stochastic programming or game theory. The PSCs are exposed to other sources of uncertainty, and hence their impacts should also be studied. Some examples are the availability and variety of the crude oils, the volumes and the specification of the petroleum products, the evolution of the investment and labor costs. Ben-Tal *et al.* (1998, 2000) proposed the affine adjustable robust counterpart (AARC) RO methodology that could be used to model demand uncertainty. Here too, the decision variables could be approached distinctly as first stage decisions and second-stage decisions which are determined after the uncertainty is revealed. Another potential research area is the characterization of the price of robustness to build valuable test cases, as referred in Beyer and Sendhoff (2007) and Bertsimas *et al.* (2011).

The RO models developed, could also be further improved so as to: better calibrate the excess inventory capacity costs in the current and retrofit designs where the amortized investment costs could be discounted; study the effect of backorders on the PSC planning decisions; and to reduce the problem size when considering multi-entity networks.

The authors are working to improve the RO MILP for simultaneous modelling of the excess inventory capacities and the lost sales tactical decisions, and studying the correlations between uncertainties and costs in order to avoid excessing budgeting. Additionally it is proposed to study strategic decisions over time, by considering depot and pipeline construction and labor costs, and interest rate uncertainties. Another line of interest is the study of the impacts on the collaborative decisions and the economy of scale entity tariffs. As new data becomes available, the stochastic and the robust models should be further compared using a common set of data. The design of uncertainty sets, the budgets of uncertainty and determination of their probabilities will consider the specificities of the PSC planning. It is also proposed to quantify the gains in using the robust optimization by comparing it to the perfect hindsight solution or by using Monte Carlo simulation to generate a big sample of values in the uncertainty range, and show the gain in feasibility over the original deterministic problem.

Another line of investigation relates to how the PSC network can readapt to high consequence events. Could the model address other relevant aspects such as contingency planning, regional changes, responsive planning, study of impacts of disruptions, new entity or product entrants, collapse of a relevant entity, re-dimension of transportation capacity, changes in product consumption trends, direct importation from Spain refineries or direct exportation to Spanish depots instead of using the regional hubs.

An area that has gained particular importance recently is resilience where multi-objective optimization could be developed for the PSC collaborative planning model. Cardoso *et al.* (2015) identify and implement for the SC, eleven resilience indicators from three categories found in the literature. The network design indicators include the node complexity that indicates the nodes in the network, the flow complexity that is the total number of forward and reverse flows in the network, the density that is the ratio between the number of total flows in the network and the number of potential flows, and the node criticality that is the total number of critical nodes in the network. The network centralization indicators include the out-degree and in-degree centrality based on the actual number of arcs, and the out-degree and in-degree centrality based on the actual amounts of flow circulating in these arcs. The operational indicators include the ENPV, the customer service level, and the investment amount.

Another important development involves the modelling of sustainability using economic, environmental and social dimensions. Barbosa-Póvoa (2012,2014) salient the importance of identifying appropriate forms to measure these dimensions while following a systemic and holistic approach for these problems, which implies a long-term engagement. The use of multi-objective approaches and establishment of trade-offs is suggested for building sustainable SCs, so that the companies may become agile, adaptive, and aligned in balancing the people and profits. Some issues that arise are energy consumption, natural resources scarcity, global warming, and climate changes, wherein society and legislation are forcing companies to act. PSCs if efficiently designed could help in reducing the negative impacts and the ecological footprint. Following these concepts, the PSC planning models could be improved to consider these three dimensions of sustainability. The first dimension has been modelled using the

ENPV and the environmental and social dimensions with respect to the PSC have yet to be studied so that a methodology could be developed to appropriately quantify these dimensions.

Finally, it would be very important to study the results with specialists from the PSC downstream, in order to compare the obtained results with company figures and obtain further managerial insights and proposals for model improvements, which could identify important design and model performance improvement implications. However, this involves confidential company information and although being developed, it may not be possible to share such results with the academic and industrial communities.

6.4 Closing remarks

In conclusion, this PhD thesis proposed many novel research ideas; developed four comprehensive models for the design and planning of the downstream PSC; implemented mixed integer linear programming, stochastic programming, uncertainty treatment and risk measures, augmented ε-constraint method, and robust optimization methodologies to the PSC problem; applied the models to the real case Portuguese PSC and presented a critical analysis of the results; identified several theoretical and practical implications; and finally explored future research directions. With this work we aim to have provided a strong contribution to the advance of the Petroleum Supply Chain Engineering and Management Science.

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Appendix A - Portuguese PSC Data of chapter 2

Table A.1 provides the product characteristics which include the product density used for mass to volume conversions, vessel rotations per month and typical vessel dimensions. Table A.2 gives the yearly product demand in cubic meters, based on historical data per per customer district, product and year. Table A.3 provides current market quotas per entity. Based on the product demand by customers given above in table A.2 and the entity market quota given in table A.3, are calculated the product demand volumes per entity, customer district, product and year. Table A.4 defines the installed refining capacities per refinery which is given in cubic meters and barrels of crude processing capacity per day. Table A.5 provides for each refinery, their typical yield of product mix per unit of crude oil. As stated earlier, each refinery, storage depots and transportation infrastructures are fully or jointly owned by the operating entities. In order to address the retrofit network problem, the model must consider the ownership participations by entities in the currently installed storage depots and existing transportation infrastructures. Table A.6 provides the ownership percentages per infrastructure with respect to the operating entities. Table A.7 provides the paid amortization periods in years per existing depot infrastructure. Table A.8 provides the paid amortization periods per existing transportation infrastructure and transportation mode in order to address the retrofit network problem. Table A.9 provides the transport mode dependent installed resource units; table A.10 the transport mode and product dependent installed resource units; and table A.11 the product dependent installed resource units. These are defined per location and per number of paid amortizations. The installed resource capacities are hence defined as the number of units which are directly proportional to the product volumes. In order to address the required number of resource units, their capacity in cubic meters is given in terms of vessel capacity for static equipment or flow rate per hour for the dynamic equipment. The typical capacities are defined in table A.12 for transport mode dependent resources, in table A.13 for transport mode and product dependent resources, and in table A.14 for the product dependent resources. These are given in cubic meters as vessel capacity for the static equipment or the flow rate per hour for the dynamic equipment. Table A.15 provides the product vessel rotations per product. Resource costs are typified as fixed, fixvar (fixed variable), operating, opervar (operating variable) and unitary costs. Table A.16 provides the resource costs when independent of product and table A.17 gives the resource costs which are product dependent. Table A.18 provides the price per product at the exit of each activity stage of the PSC. Table A.19 presents the maximum permitted distances defined per transportation mode, distribution stage and product. The transport distances are crucial to the design of the PSC network. The key data sets in the model are the refinery, depot and customer locations as well as the distances separating each of them by different transport modes. Table A.20 provides the rail distances, table A.21 gives the pipeline distances, while table A.22 and A.23 provide the maritime and road travelling distances respectively between refineries, depots and customer locations. The road, rail and maritime travelling distances have been previously obtained by recurring to the google maps application program interfaces.

Finally, table A.24 provides the GPS latitude and longitude coordinates for each customer district, depot and refinery location. The point-to-point pipeline distances between location pairs given in table A.23 have been calculated using the (Weisstein, 2014) big circle formula based on the GPS coordinates given in table A.24.

Table A.1. Product characteristics

Product	*C3*	*C4*	*U5*	*U8*	*GO*	*FU*	*A1*	*B5*	*CRUDE*
DENSITY	0.582	0.573	0.748	0.750	0.837	0.944	0.832	0.837	0.932
TYPICAL_M3	2200	2200	10000	10000	25000	10000	10000	5000	100000

Table A.2. Demand per customer district in m^3 per year

District	*Prod*	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*	*2005*	*2006*	*2007*	*2008*	*2009*	*2010*
AVEIRO	C3	47687	46971	45158	45297	44905	42730	40280	40520	37639	34488	80819	76778
AVEIRO	C4	109183	96647	89630	84991	79850	72917	73658	68452	63946	62313	251803	239213
AVEIRO	U5	59925	74290	79916	94722	104913	107452	113005	110098	103861	98251	250343	237826
AVEIRO	U8	39444	44224	43123	41803	43299	36103	32434	25218	20535	13080	30277	28763
AVEIRO	GO	272949	290585	302609	318122	356897	350849	340566	344454	346956	330578	906652	861319
AVEIRO	FU	159026	155929	136456	100378	92126	95889	87827	75867	72796	70129	154664	146930
AVEIRO	A1			54	198			190	244				
AVEIRO	B5	13863	15330	19152	21587	14155	12025	12136	17819	12307	25113	95977	91178
BEJA	C3	18158	18060	16857	16893	15875	14505	13894	12971	12298	11355	9550	9073
BEJA	C4	7791	8527	10731	11801	11820	11638	12262	11924	13023	12985	8386	7967
BEJA	U5	14213	16176	17719	20055	20104	19857	19948	19639	19115	19505	16641	15809
BEJA	U8	8272	8799	8355	7797	7897	7198	5961	4450	3510	2678	2026	1925
BEJA	GO	61282	71320	82419	79352	75370	72944	72948	76425	83933	91728	77095	73241
BEJA	FU	4855	4898	6087	5454	1970	1698	1678	1397	2334	1917	1537	1460
BEJA	B5	15845	15048	15682	19669	14994	15273	16005	16490	17667	18171	15598	14818
BRAGA	C3	60785	57679	51589	49182	45747	39515	39281	35702	33084	30003	24123	22917
BRAGA	C4	99871	97756	88588	84394	82560	75069	81428	77917	73294	66326	41849	39757
BRAGA	U5	59307	74481	75501	101134	113651	116934	116604	118595	119941	115919	107350	101983
BRAGA	U8	42688	48301	44535	48683	50344	42075	35336	26868	23127	14372	14165	13456
BRAGA	GO	246366	295203	328842	364536	372368	361178	351068	369837	387503	352774	332201	315591
BRAGA	FU	313857	264220	219266	186710	168427	189435	172079	151999	132773	86125	78335	74418
BRAGA	B5	13072	18538	25987	32522	17041	14727	14286	16178	17309	40714	44700	42465
BRAGANCA	C3	14095	13890	13198	12741	12582	10949	10610	9524	9054	7850	4662	4429
BRAGANCA	C4	8033	8640	9377	9473	9660	8869	10061	8654	8447	8213	4667	4433
BRAGANCA	U5	5529	7238	7459	7888	7856	7761	9262	9125	9325	7667	6825	6483
BRAGANCA	U8	4827	6139	5977	5620	6259	5963	5978	4520	4080	2714	2255	2143
BRAGANCA	GO	45809	50797	54256	53615	58863	48379	54740	57435	60360	45910	42092	39987
BRAGANCA	FU	2690	2015	2804	3063	2132	1400	2794	3197	2258	1231	1392	1323
BRAGANCA	B5	11391	14619	13125	15558	10925	10153	10295	10552	10164	14246	17574	16695
CASTELO BRANCO	C3	18813	18244	17007	16696	15667	9795	9936	8863	8614	7905	6441	6119
CASTELO BRANCO	C4	16649	18279	21414	21457	21047	17097	17593	14475	12433	11633	6964	6615
CASTELO BRANCO	U5	15598	20610	21598	23491	23563	22467	25165	25118	25709	20415	16831	15989
CASTELO BRANCO	U8	8473	10661	9891	9109	9469	8080	8230	6183	4749	3128	2370	2251
CASTELO BRANCO	GO	91718	109116	103440	108735	106116	94470	104976	101562	106155	91136	76527	72701
CASTELO BRANCO	FU	37610	35518	30070	23863	19697	17563	19002	13207	12764	23540	21764	20675
CASTELO BRANCO	B5	9706	13496	15129	18679	13019	13311	13743	13144	14399	17317	14561	13832
COIMBRA	C3	34187	32679	25771	20622	19277	16853	16389	14582	13841	12921	9697	9213
COIMBRA	C4	50410	52911	47127	43028	43665	39062	40316	38409	37245	38131	21110	20055
COIMBRA	U5	47439	56144	59364	68703	71572	75014	77690	80297	82344	70508	58032	55131
COIMBRA	U8	28752	29964	27796	26599	24916	22234	20849	16935	14743	10016	8007	7607
COIMBRA	GO	201943	210117	242767	235423	236977	224994	236162	225576	260659	229664	204973	194724
COIMBRA	FU	126355	121916	104948	96938	72179	50443	42645	36022	45079	53172	25054	23801
COIMBRA	B5	16341	18892	23060	23762	18339	18848	19038	18373	18223	19909	20458	19435
EVORA	C3	23383	22998	21856	21615	20646	18720	18391	16896	16558	15539	13161	12503
EVORA	C4	15363	17054	18792	17860	15700	14128	15491	14566	14938	14647	8831	8389
EVORA	U5	17437	23512	24648	27456	27325	27651	28139	25735	24200	21716	17795	16905
EVORA	U8	12755	12224	11759	10695	10500	8625	7684	5110	3965	2854	2097	1992
EVORA	GO	87294	106063	110442	104069	100118	99115	102855	95227	89582	85017	70159	66651
EVORA	FU	24301	21485	10843	5548	4117	4867	3924	2872	4226	5089	4546	4319
EVORA	B5	18226	19290	18502	19399	17327	17127	11897	11664	12761	14223	10751	10213
FARO	C3	34304	34316	35385	35883	34893	31661	31688	29047	28380	26845	22983	21833
FARO	C4	51017	54567	55220	56365	58042	54076	60535	59981	62218	60826	45889	43595
FARO	U5	71778	88261	95584	105742	107833	103734	105342	106264	107755	109669	100636	95605

FARO	U8	35245	40000	40040	38452	39709	29719	24983	19015	15716	10024	9073	8620
FARO	GO	193249	215357	243314	257909	262013	239370	252263	255334	273587	298554	254140	241433
FARO	FU	17387	17732	15273	17758	12115	8420	8553	6292	5961	4847	3264	3101
FARO	A1			202052	215916	219681	239639	231780	244463	239942	239942	239942	227945
FARO	B5	14521	14637	37385	12532	17061	12558	16254	11602	13051	15251	10719	10183
GUARDA	C3	13973	13672	13040	12533	11751	13509	13199	12044	11788	10816	9317	8852
GUARDA	C4	15005	16733	17579	17709	17211	15928	17596	16449	16395	15444	10905	10360
GUARDA	U5	10832	13686	15008	16039	17773	16873	17250	16475	15328	14645	11463	10890
GUARDA	U8	8092	7419	7460	6836	7573	6411	6356	4710	3489	2638	1855	1762
GUARDA	GO	98955	130761	153151	148636	138308	130165	123040	89532	73864	64294	54106	51400
GUARDA	FU	7599	6702	6129	5492	5334	2738	17423	8764	2918	2345	1401	1331
GUARDA	B5	8691	12675	15139	18423	10309	10753	10554	8954	8447	19501	15195	14435
LEIRIA	C3	22651	21502	20588	19328	18936	16482	16420	15127	14231	12358	9544	9066
LEIRIA	C4	107134	87059	73264	61068	53998	46102	44929	41627	42541	37894	18473	17550
LEIRIA	U5	53618	64329	67727	76963	76952	77452	82095	76400	71104	67628	56678	53844
LEIRIA	U8	33853	35697	33549	31683	32247	27887	26084	19730	15585	13247	10104	9599
LEIRIA	GO	359335	411349	428970	417625	408067	411576	425770	395582	376025	334499	284735	270499
LEIRIA	FU	119142	56762	43466	55128	45832	41292	43096	39405	22858	7983	5890	5595
LEIRIA	B5	23311	30736	36597	43697	21626	22683	22257	20400	21297	47277	35865	34072
LISBOA	C3	155251	140935	138241	139974	123586	154786	110401	98259	92460	89114	71877	68283
LISBOA	C4	172206	155382	140710	155738	146888	207073	132491	122885	117397	105506	67774	64385
LISBOA	U5	339075	384889	414011	464195	466934	511382	461499	443403	427900	443600	355280	337516
LISBOA	U8	182651	168276	156676	150692	140392	134471	91373	63475	48347	39269	28191	26781
LISBOA	GO	1036897	1077121	1139919	1162159	1124283	1314920	1152208	1136534	1154289	1268883	1022436	971314
LISBOA	FU	877500	765972	800199	960952	563832	186195	366592	117602	110570	95905	41687	39603
LISBOA	A1			507591	534490	589941	594567	634582	695828	723612	723612	723612	687431
LISBOA	B5	41694	47223	52389	46022	33278	41672	27816	28387	28817	61013	48301	45886
PORTALEGRE	C3	9454	9270	9033	8833	8134	6365	6307	5620	5649	5169	3951	3754
PORTALEGRE	C4	8668	10159	12209	12103	11831	9506	8875	7174	7625	7543	4197	3987
PORTALEGRE	U5	10801	12671	13191	14452	13003	12343	12601	11716	11186	8888	7363	6995
PORTALEGRE	U8	5608	6109	5677	5195	4445	3599	3644	2462	1671	925	713	678
PORTALEGRE	GO	52143	85112	179951	63039	53878	48229	49583	48239	47197	44180	33177	31518
PORTALEGRE	FU	12946	5671	4804	5123	3746	3118	2832	1126	216	1306	517	491
PORTALEGRE	B5	9216	10128	12263	11945	10994	10122	9720	8773	9542	9922	6212	5902
PORTO	C3	100643	97744	102084	101117	87249	87738	87514	78441	74969	70099	62040	58938
PORTO	C4	136319	138136	145206	142288	137326	125963	143490	122291	121323	115436	79526	75550
PORTO	U5	216964	267052	279317	343547	343334	343747	369092	363070	349339	349994	286491	272166
PORTO	U8	126432	139507	129141	128417	117401	98738	90634	70675	58871	52196	34319	32603
PORTO	GO	833215	931780	946969	1084694	1058971	1040619	1108020	1046967	1072490	1070699	887533	843156
PORTO	FU	447838	350579	283635	263050	209691	183428	198463	192121	184813	110147	78520	74594
PORTO	A1			134688	135731	149007	155589	170960	189424	216061	216061	216061	205258
PORTO	B5	74424	69424	81971	96346	62337	66158	67075	61143	61645	140616	108826	103385
SANTAREM	C3	40912	38479	37942	37081	34754	31499	31427	29539	28771	26905	23132	21975
SANTAREM	C4	67565	71276	75799	62782	57023	51194	51003	48457	47304	51158	30441	28919
SANTAREM	U5	47353	57180	56745	70830	81369	105541	114560	113711	110205	91504	99117	94161
SANTAREM	U8	26091	28629	25771	26700	28800	28269	28106	21739	18529	11654	12787	12148
SANTAREM	GO	281431	342487	326483	329441	353492	417073	426770	395177	409193	366393	381376	362307
SANTAREM	FU	158978	132124	132464	111422	110407	106173	70256	75528	83974	64286	69056	65603
SANTAREM	B5	46376	50072	46360	47075	41315	38229	34686	27100	30469	37220	29759	28271
SETUBAL	C3	69275	60725	66811	98670	80617	70188	69948	86222	197211	193844	44729	42492
SETUBAL	C4	67047	64044	59227	56539	55267	91718	58183	123481	57448	97798	70565	67037
SETUBAL	U5	107512	132789	143719	161705	165916	174385	176369	177386	168895	159467	188489	179064
SETUBAL	U8	54397	57823	53867	47753	45599	37315	31871	23808	17909	12302	15143	14386
SETUBAL	GO	323219	360323	395153	417338	406042	429729	433965	442447	457336	459218	583099	553944
SETUBAL	FU	1546477	1142158	1321865	1657627	824359	1066859	1464594	765820	713723	439390	268731	255294
SETUBAL	B5	48188	49670	45842	52608	50257	50143	50187	56231	55255	67040	63117	59962
VIANA DO	C3	21320	21220	17093	14588	13976	12015	11764	10364	9786	8401	6135	5829
VIANA DO	C4	21419	22803	22583	20300	20944	20113	23315	21152	20316	20024	11548	10971
VIANA DO	U5	20664	23595	27239	29098	28940	25475	28016	26341	26055	25662	22255	21142
VIANA DO	U8	10121	12563	13367	12601	12032	8434	7830	5429	4685	3206	2862	2719
VIANA DO	GO	78820	96477	96314	91013	93093	82363	90899	78770	80227	81275	73567	69889
VIANA DO	FU	42989	48287	44160	39145	39251	36726	36943	25834	14170	12678	11978	11379
VIANA DO	B5	7368	9839	10507	11522	6121	4841	5240	6147	6765	15029	14706	13970
VILA REAL	C3	24905	24481	22244	21311	19747	17124	16473	14372	14085	12181	12414	11793
VILA REAL	C4	13895	15230	17365	17536	17792	17200	18085	15666	15042	13315	9379	8910
VILA REAL	U5	13209	16584	16362	18428	19277	19510	21566	21149	19685	17821	33525	31849
VILA REAL	U8	8344	10536	10107	9964	10639	9273	9046	6642	5434	3716	3218	3057
VILA REAL	GO	63896	80038	110534	84177	90473	92049	98441	87868	80911	78840	75827	72035
VILA REAL	FU	3456	5088	4151	2918	2854	3850	3307	3430	1645	1188	948	900
VILA REAL	B5	10061	13104	13602	16039	12416	11440	11308	10244	9701	13526	17276	16412

VISEU	C3	27734	26947	26162	25225	23905	20975	19893	17837	16973	15118	12567	11938
VISEU	C4	41035	41742	44277	40763	38588	31298	34433	31422	31606	30282	18936	17989
VISEU	U5	27735	32921	33922	38743	42242	41822	46118	48465	51649	41715	35977	34178
VISEU	U8	15087	18068	17144	16577	17467	15048	14719	11989	9884	6865	5842	5549
VISEU	GO	156327	165971	154076	182010	196499	187651	197012	189885	189663	156016	144366	137148
VISEU	FU	63033	57602	47361	66129	73239	43139	36271	34992	31017	30333	30380	28861
VISEU	B5	13456	19290	22634	27378	15903	14602	16083	16201	15745	28228	23657	22474

Table A.3. Market quota percentages per entity

Entity	*A*	*B*	*C*	*D*
QUOTA	0.46	0.22	0.21	0.11

Table A.4. Refinery crude processing capacity

Refinery	*m^3*	*bb/d*
SINES	35200	220000
MATOSINHOS	14400	90000

Table A.5. Product yield at refinery as percentage of a unit m^3 of crude

Refinery	*C3*	*C4*	*U5*	*U8*	*GO*	*FU*	*A1*	*B5*
MATOSINHOS	0.0096	0.0144	0.201	0.058	0.39	0.137	0.079	0.09
SINES	0.0096	0.0144	0.201	0.058	0.39	0.137	0.079	0.09

Table A.6. Entity participation percentages in refineries, depots and transport infrastructures

Source	*Destination*	*Resource*	*A*	*B*	*C*	*D*	*Single*
SINES	*AZAMBUJA*	*PIPELINE*	65	20	15	0	100
SINES	*SINES*	*PIPELINE*	100	0	0	0	100
MATOSINHOS	*PORTO*	*PIPELINE*	100	0	0	0	100
MATOSINHOS	*MATOSINHOS*	*PIPELINE*	100	0	0	0	100
MATOSINHOS	*REAL*	*PIPELINE*	0	50	50	0	100
MATOSINHOS	*LEIXOES*	*PIPELINE*	0	0	0	100	100
MATOSINHOS	*PERAFITA*	*PIPELINE*	100	0	0	0	100
MATOSINHOS	*MATOSINHOS*	*REFINERY*	100	0	0	0	100
SINES	*SINES*	*REFINERY*	100	0	0	0	100
AZAMBUJA	*AZAMBUJA*	*DEPOT*	65	20	15	0	100
SINES	*SINES*	*DEPOT*	100	0	0	0	100
PORTO	*PORTO*	*DEPOT*	100	0	0	0	100
SETUBAL	*SETUBAL*	*DEPOT*	100	0	0	0	100
MATOSINHOS	*MATOSINHOS*	*DEPOT*	100	0	0	0	100
REAL	*REAL*	*DEPOT*	0	50	50	0	100
LEIXOES	*LEIXOES*	*DEPOT*	0	0	0	100	100
FARO	*FARO*	*DEPOT*	0	100	0	0	100
BANATICA	*BANATICA*	*DEPOT*	0	0	100	0	100
PERAFITA	*PERAFITA*	*DEPOT*	100	0	0	0	100
BARREIRO	*BARREIRO*	*DEPOT*	0	100	0	0	100
AVEIRO	*AVEIRO*	*DEPOT*	0	100	0	0	100

Table A.7. Current depots and paid amortization periods

	MATOSINHOS	*REAL*	*LEIXOES*	*AZAMBUJA*	*SINES*	*PERAFITA*	*SETUBAL*	*FARO*	*BANATICA*	*BARREIRO*	*AVEIRO*
PERIODS	12	12	12	12	12	12	7	12	12	7	3

Table A.8. Current transportation routes and paid amortization periods

	AVEIRO	*AZAMBUJA*	*BANATICA*	*BARREIRO*	*LEIXOES*	*MATOSINHOS*	*PERAFITA*	*PORTO*	*REAL*	*SETUBAL*	*SINES*
MATOSINHOS											
PIPELINE					12	12	12	12	12		
MARITIME	3		12	12						12	
SINES											
PIPELINE		12									12
MARITIME	12		3	3						2	

Table A.9. Current transport mode dependent resource units, per location and paid amortization periods

		PIPELINE			MARITIME	RAIL
Resources	*Periods*	*INTERFACE*	*PIPE*	*PUMP16*	*DOCK*	*STATION*
MATOSINHOS	12	2	1	2	1	
SINES	2					1
SINES	7				1	
SINES	12	3	1	3		

Table A.10. Current transport mode and product dependent resource units, per location and paid amortization periods

Location	*Transport*	*Resource*	*Periods*	*C3*	*C4*	*U5*	*U8*	*GO*	*FU*	*A1*	*B5*
AVEIRO	*ROAD*	*ARM*	*3*	0	0	2	2	4	0	0	1
AZAMBUJA	*ROAD*	*ARM*	*12*	5	4	10	9	28	0	11	4
BANATICA	*ROAD*	*ARM*	*12*	1	1	0	0	9	0	0	1
BARREIRO	*ROAD*	*ARM*	*7*	0	0	2	2	4	1	0	1
FARO	*ROAD*	*ARM*	*12*	1	1	0	0	0	0	0	0
MATOSINHOS	*ROAD*	*ARM*	*12*	0	0	7	1	12	10	2	2
REAL	*ROAD*	*ARM*	*12*	2	1	6	1	13	0	2	0
LEIXOES	*ROAD*	*ARM*	*12*	0	0	7	1	11	2	2	3
PERAFITA	*ROAD*	*ARM*	*12*	9	5	0	0	0	0	0	0
SETUBAL	*ROAD*	*ARM*	*7*	0	0	3	3	17	3	0	1
SINES	*ROAD*	*ARM*	*12*	1	0	4	3	9	0	0	1

Table A.11. Current product dependent resource units, per location and paid amortization periods

Location	*Resource*	*Periods*	*C3*	*C4*	*U5*	*U8*	*GO*	*FU*	*A1*	*B5*
AVEIRO	*VESSEL*	*3*	0	0	1	1	1	0	0	1
AZAMBUJA	*VESSEL*	*12*	8	6	4	3	4	0	4	3
AZAMBUJA	*VESSEL*	*4*	0	0	0	0	4	0	0	0
BANATICA	*VESSEL*	*12*	2	2	0	0	2	0	0	1
BARREIRO	*VESSEL*	*7*	0	0	1	1	1	1	0	1
FARO	*VESSEL*	*12*	1	1	0	0	0	0	0	0
MATOSINHOS	*VESSEL*	*12*	4	3	5	2	5	7	4	3
REAL	*VESSEL*	*12*	3	1	2	0	2	0	0	0
LEIXOES	*VESSEL*	*12*	0	0	2	0	2	1	0	2
PERAFITA	*VESSEL*	*12*	5	2	0	0	0	0	0	0
SETUBAL	*VESSEL*	*7*	0	0	1	1	2	1	0	1
SINES	*VESSEL*	*12*	5	4	6	4	6	6	5	4

Table A.12. Typical capacity of transport mode dependent resources, in m^3 equivalent per hour

	PIPELINE			MARITIME	RAIL
	PIPE	*PUMP16*	*INTERFACE*	*DOCK*	*STATION*
m^3 per hour	100000	600	600	600	240

Table A.13. Typical capacity of transport mode and product dependent resources, in m^3 equivalent per hour

		C3	*C4*	*U5*	*U8*	*GO*	*FU*	*A1*	*B5*
ROAD	*ARM*	20	20	20	20	20	20	20	20

Table A.14. Typical capacity of product dependent resources, in m^3

Product	*C3*	*C4*	*U5*	*U8*	*GO*	*FU*	*A1*	*B5*	*CRUDE*
VESSEL	2200	2200	10000	10000	25000	10000	10000	5000	100000

Table A.15. Product vessel rotations per transportation mode

Product	*C3*	*C4*	*U5*	*U8*	*GO*	*FU*	*A1*	*B5*	*CRUDE*
PIPELINE	4	4	4	4	4	4	4	4	4
MARITIME	2	2	2	2	2	2	2	2	2
RAIL	2	2	2	2	2	2	2	2	2
ROAD	1	1	1	1	1	1	1	1	1

Table A.16. Cost of resource independent of product

	PIPELINE	MARITIME	RAIL	ROAD	DEPOT	PUMP16	INTERFACE	DOCK	STATION	ARM
FIXED	0.083333	0.083333	0.083333	0.083333	6045833	83333.33	83333.33	83333.33	83333.33	11458.33
FIXVAR	28416.67									
OPERATING	0.016667	0.016667	0.016667	0.016667	1209167	16666.67	16666.67	16666.67	16666.67	2291.667
OPERVAR	5683.333									

Table A.17. Cost of resource depending on product

TYPE COST	RESOURC	C3	C4	U5	U8	GO	FU	A1	B5
UNITARY	DEPOT	1.630	1.604	2.094	2.100	2.344	2.643	2.330	2.344
FIXED	VESSEL	2,068,00	1,329,00	1,243,00	1,243,00	2,363,00	1,243,00	1,906,00	621,50
OPERATIN	VESSEL	413,600	265,800	248,600	248,600	472,600	248,600	381,200	124,30
UNITARY	PIPELINE	0.042	0.041	0.054	0.054	0.060	0.068	0.060	0.060
UNITARY	MARITIME	0.084	0.083	0.108	0.108	0.121	0.136	0.120	0.121
UNITARY	RAIL	0.116	0.115	0.150	0.150	0.167	0.189	0.166	0.167
UNITARY	ROAD	0.265	0.261	0.341	0.342	0.382	0.430	0.379	0.382
TARIFF	EXPORT	2.552	2.552	2.552	2.552	2.552	2.552	2.552	2.552
TARIFF	RETAIL	20.976	20.976	20.976	20.976	20.976	20.976	20.976	20.976
TARIFF	IMPORT	2.552	2.552	2.552	2.552	2.552	2.552	2.552	2.552
TARIFF	REFINERY	12.306	12.306	12.306	12.306	12.306	12.306	12.306	12.306

Table A.18. Price of product at end of activity

		1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
CRUDE	CRUDE	103.6714	192.8973	170.3213	164.418	160.7698	193.4144	262.7134	315.2445	324.2863	415.3588	274.659	374.9567
RETAIL	C3	247.7436	278.7116	294.1955	283.9988	277.6973	334.0843	453.7842	430.2373	486.2656	563.1292	456.7978	564.9639
RETAIL	C4	222.115	249.8793	263.7615	254.6196	248.97	299.5238	406.841	385.73	435.9623	504.8744	409.5429	506.5194
RETAIL	U5	255.2175	264.3324	276.4856	266.9026	260.9805	313.9731	426.4673	498.8604	507.3893	570.308	443.9805	556.2696
RETAIL	U8	243.0141	273.3908	288.5792	278.5771	272.396	327.7064	445.1212	520.6808	529.5828	595.2536	463.4005	580.6011
RETAIL	GO	249.9075	303.4592	303.4592	292.9413	286.4415	344.6039	468.0729	524.5244	527.7804	683.5657	470.7232	592.2314
RETAIL	FU	157.9673	191.8175	191.8175	185.1691	181.0606	217.8252	295.8704	356.5242	368.35	472.6258	383.817	505.9396
RETAIL	A1	252.2106	283.737	299.5001	289.1195	282.7045	340.1081	461.9663	540.3853	549.6242	617.7802	480.9373	602.5732
RETAIL	B5	220.3433	295.8896	295.8896	285.6341	279.2964	336.0079	456.3971	510.1973	517.122	661.2007	432.2325	546.4819
REFINING	C3	116.4908	131.0521	138.3328	133.5382	130.5752	157.0887	213.3725	250.186	266.3343	292.6881	256.6342	346.1498
REFINING	C4	103.4617	116.3944	122.8607	118.6024	115.9708	139.5189	189.5075	223.7842	251.0536	277.1868	224.8888	303.3314
REFINING	U5	193.051	199.9456	209.1385	201.8898	197.4103	237.4947	322.5874	360.6544	377.6982	421.0833	310.5833	418.9167
REFINING	U8	177.2733	199.4324	210.512	203.2157	198.7067	239.0544	324.7059	377.5801	387.4464	426.044	310.5833	418.9167
REFINING	GO	195.7966	237.753	237.753	229.5126	224.4201	269.9889	366.724	406.03	409.0792	532.0536	319.3333	435.4167
REFINING	FU	108.6176	131.8928	131.8928	127.3214	124.4964	149.7756	203.439	225.2439	226.9354	295.1551	177.1492	241.5461
REFINING	A1	190.7765	214.6236	226.5471	218.6951	213.8426	257.2637	349.4394	390.6751	409.1376	456.1341	336.4362	453.7871
REFINING	B5	164.7398	221.2221	221.2221	213.5545	208.8162	251.2166	341.2257	377.7987	380.6359	495.0599	297.1301	405.1421
PUBLIC	C3	410.3568	461.6515	487.2988	497.5577	523.205	563.2148	613.7915	661.4231	661.7112	752.9156	622.4171	755.3958
PUBLIC	C4	367.9061	413.8944	436.8885	446.0862	469.0803	504.9512	550.2958	593	593.2583	675.0278	558.0292	677.2514
PUBLIC	U5	840	870	910	920	970	1033	1146.454	1277.72	1319.938	1390.029	1232.317	1372.776
PUBLIC	U8	800	900	950	970	1020	1098	1196.6	1333.608	1377.673	1450.83	1286.219	1432.822
PUBLIC	GO	560	680	680	670	710	789	936.061	1044.1	1079.462	1263.081	1002.531	1152.831
PUBLIC	FU	186.0793	225.9535	225.9535	222.6306	235.922	262.1725	311.0386	372.4306	383.2686	487.9258	398.9124	521.2396
PUBLIC	A1	830.275	934.0593	985.9515	1006.708	1058.601	1139.552	1241.884	1384.077	1429.809	1505.735	1334.895	1487.046
PUBLIC	B5	350	470	470	380	420	489	612.6581	673.8333	725.3765	937.8657	680.9765	827.3639

Table A.19. Permitted distance in kilometers per distribution stage and transportation mode

		C3	C4	U5	U8	GO	FU	A1	B5
PRIMARY	PIPELINE	500	500	1000	1000	1000	30	1000	1000
PRIMARY	MARITIME	1000	1000	1000	1000	1000	1000	1000	1000
PRIMARY	RAIL	1000	1000	1000	1000	1000	1000	1000	1000
PRIMARY	ROAD	0	0	0	0	0	0	0	0
SECONDAR	PIPELINE	0	0	0	0	0	0	300	0
SECONDAR	MARITIME	0	0	0	0	0	0	0	0
SECONDAR	RAIL	0	0	0	0	0	0	300	0
SECONDAR	ROAD	1000	1000	1000	1000	1000	1000	1000	1000

Table A.20. Rail distance in kilometers between locations

Source	*Destination*	*Kilometers*
SINES	*FARO*	155

Table A.21. Pipeline distance in kilometers between locations

	AVEIRO	*AZAMBUJA*	*BANATICA*	*BARREIRO*	*BEJA*	*BRAGA*	*BRAGAN*	*CASTELO BRANCO*	*COIMBRA*	*EVORA*	*FARO*	*GUARDA*	*LEIRIA*
AVEIRO	0	193	246	245	330	113	225	148	57	263	449	129	111
AZAMBUJA	193		58	53	161	307	388	160	146	110	267	234	83
BANATICA	246	58		12	152	359	446	215	202	124	237	292	136
BARREIRO	245	53	12		141	358	441	207	199	112	230	286	135
BEJA	330	161	152	141		436	475	224	274	68	123	314	230
BRAGA	113	307	359	358	436		155	228	164	368	557	164	224
COIMBRA	57	146	202	199	274	164	249	100		207	394	115	67
EVORA	263	110	124	112	68	368	410	158	207		190	248	167
FARO	449	267	237	230	123	557	597	346	394	190		435	344
LEIRIA	111	83	136	135	230	224	316	124	67	167	344	174	
LEIXOES	67	260	311	311	396	50	191	200	122	328	515	153	177
LISBOA	241	51	120	100	148	354	439	207	196	118	237	285	131
MATOSINHOS	70	262	314	313	399	49	193	204	125	331	518	156	180
PERAFITA	70	263	314	314	399	48	192	204	125	332	519	156	180
PORTO	62	256	308	307	390	52	188	193	116	322	510	145	173
REAL	65	258	310	309	394	52	192	199	120	327	514	152	175
SANTAREM	172	27	84	79	169	284	362	134	122	110	281	208	63
SETUBAL	260	67	35	24	117	373	448	207	211	94	207	290	149
SINES	330	147	134	120	98	442	512	264	280	120	146	351	219
VIANA DO	130	321	371	371	459	41	189	260	185	392	579	202	239
VILA REAL	116	292	348	345	401	69	110	182	147	334	524	103	214
VISEU	69	214	271	267	323	119	176	109	73	255	446	62	139

	LEIXOES	*LISBOA*	*MATOSINHOS*	*PERAFITA*	*PORT-ALEGRE*	*PORTO*	*REAL*	*SANTAREM*	*SETUBAL*	*SINES*	*VIANA DO CASTELO*	*VILA REAL*	*VISEU*
AVEIRO	67	241	70	70	201	62	65	172	260	330	130	116	69
AZAMBUJA	260	51	262	263	139	256	258	27	67	147	321	292	214
BANATICA	311	120	314	314	185	308	310	84	35	134	371	348	271
BARREIRO	311	100	313	314	174	307	309	79	24	120	371	345	267
BEJA	396	148	399	399	161	390	394	169	117	98	459	401	323
BRAGA	50	354	49	48	291	52	52	284	373	442	41	69	119
COIMBRA	122	196	125	125	146	116	120	122	211	280	185	147	73
EVORA	328	118	331	332	99	322	327	110	94	120	392	334	255
FARO	515	237	518	519	283	510	514	281	207	146	579	524	446
LEIRIA	177	131	180	180	141	173	175	63	149	219	239	214	139
LEIXOES	5	307	5	4	260	8	5	239	327	397	63	87	96
LISBOA	307		309	310	177	303	305	77	100	170	367	342	265
MATOSINHOS	5	309	5	1	263	8	5	241	329	399	60	89	100
PERAFITA	4	310	1		263	12	5	242	330	400	60	88	100
PORTO	8	303	8	12	253		8	234	322	392	70	81	88
REAL	5	305	5	5	258	8	5	237	325	395	65	87	95
SANTAREM	239	77	241	242	119	234	237		89	158	301	267	188
SETUBAL	327	100	329	330	168	322	325	89		108	388	356	277
SINES	397	170	399	400	214	392	395	158	108	5	458	423	343
VIANA DO	63	367	60	60	321	70	65	301	388	458		111	152
VILA REAL	87	342	89	88	247	81	87	267	356	423	111		80
VISEU	96	265	100	100	173	88	95	188	277	343	152	80	

Table A.22. Maritime travelling distance in kilometers between locations

	AVEIRO	*BANATICA*	*BARREIRO*	*LEIXOES*	*MATOSINHOS*	*REAL*	*SETUBAL*	*SINES*
AVEIRO			295	71	71	71	334	354
BANATICA				319	319	319		98
BARREIRO	295			353	353	353		116
LEIXOES	71	319	353				383	403
MATOSINHOS	71	319	353				383	403
REAL	71	319	353				383	403
SETUBAL	334			383	383	383		68
SINES	354	98	116	403	403	403	68	

Table A.23. Road travelling distance in kilometers between district capitals

	AVEIRO	*BEJA*	*BRAGA*	*BRAGANCA*	*CASTELO*	*COIMBRA*	*EVORA*	*FARO*	*GUARDA*	*LEIRIA*	*LISBOA*	*PORT - ALEGR*	*PORTO*	*SANTAREM*	*SETUBAL*	*VIANA DO*	*VILA REAL*	*VISEU*
AVEIRO	40		126	285	252	63			159	124			78	188		150	179	86
AZAMBUJ		190			182		173	326		102	54	188		27	91			
BANATIC		171					127	273		216	86	221		146	44			
BARREIR		161			275		116	262		189	58	210		119	33			
BEJA		40					80	147			176	180			143			
BRAGA	126		40	225					252	235			56			62	119	179
COIMBRA	63		174		224	40			205	74		231	121	138		193		89
EVORA		80			190		40	287		240	132	102		160	99			
FARO		147						40										
LEIRIA	124				166	74	240			40	147	171		79	195			
LEIXOES	84		58	215		127			208	191			11			69	98	134
LISBOA		176			227		132	277		147	40			84	64			
MATOSIN	84		58	215		127			208	191			11			69	98	134
PERAFITA	90		62	244	260	160			199	229			15			76	112	127
PORTO	78		56	213		121			202	185			40			77	94	130
REAL	84		58	215		127			208	191			11			69	98	134
SANTARE		205			159	138	160	306		79	84	164		40	125			
SETUBAL		143					99	245		195	64	193		125	40			
SINES		100					130	165			162	218			114			
VIANA DO CASTELO			62	279									77			40	173	
VILA			119	120					182				94			173	40	98
VISEU	86			204	177	89			82	196							98	40

Table A.24. Location GPS Coordinates

	AVEIRO	*BEJA*	*BRAGA*	*BRAGANCA*	*CASTELO*	*COIMBRA*	*EVORA*	*FARO*
Longitud	-8.65362	-7.86523	-8.4229	-6.75919	-7.49189	-8.4292	-7.9096	-7.93511
Latitude	40.64118	38.01562	41.55176	41.80718	39.82389	40.21149	38.57098	37.01536

GUARDA	*LEIRIA*	*LISBOA*	*PORTALEGRE*	*PORTO*	*SANTAREM*	*SETUBAL*	*VIANA DO CASTELO*	*VILA REAL*
-7.26785	-8.80725	-9.13552	-7.43235	-8.61024	-8.68708	-8.89307	-8.83016	-7.74635
40.53713	39.74434	38.70716	39.2914	41.14997	39.23618	38.52451	41.69457	41.2959

VISEU	*MATOSINHO*	*AZAMBUJ*	*SINES*	*PERAFITA*	*BANATICA*	*BARREIRO*	*REAL*	*LEIXOES*
-7.91428	-8.705	-8.86912	-8.8759	-8.7005	-9.20453	-9.07666	-8.68	-8.679
40.65754	41.21	39.06862	37.95134	41.216	38.67568	38.66424	41.175	41.1898

Appendix B – Portuguese PSC Data of chapter 3

The real-case data used to test the current model may be consulted in Chapter 2, however some parameters have been restructured and new tables added, which are presented in this appendix. Tables A.9, A.10 and A.11 from Chapter 2 are now replaced by table B.9, providing the installed resource units per location, transport mode family, resource, product family and number of paid amortizations. Similarly, tables A.12, A.13 and A.14 from Fernandes *et al.* (2013) are replaced by table B.12, giving the typical capacities of resources in cubic meters for volume capacity or the flow rate per hour, defined per transport mode family, resource and product family. In order to generalize the resource constraints as these may be related only to transportation modes, or related only to product or to both, the concept of families is introduced in table B.25 which provides the composition of the product families and table B.26 provides the transport mode families. As the collaborative model implements cost, tariff and price linearization to provide economies of scale, the table B.27 provides the transfer volume thresholds and table B.28 the tariff linearization coefficients. Table B.29 which defines the transportation mode capacity characteristics is introduced in order to model resource capacities by taking into account some time related characteristics of the transportation mode.

Table B.9. Installed resource units per location, transport mode family, resource, product family

Depot	*Family*	*Resource*	*Periods*	*C3*	*C4*	*U5*	*U8*	*GO*	*FU*	*A1*	*B5*	*ANY*
AVEIRO	*ANY*	*VESSEL*	3			1	1	1			1	
AVEIRO	*ROAD*	*ARM*	3			2	2	4			1	
FARO	*ANY*	*VESSEL*	12	1	1							
FARO	*ROAD*	*ARM*	12	1	1							
SETUBAL	*ANY*	*VESSEL*	7			1	1	2	1		1	
SETUBAL	*ROAD*	*ARM*	7			3	3	17	3		1	
MATOSINHOS	*ANY*	*VESSEL*	12	4	3	5	2	5	7	4	3	
MATOSINHOS	*PIPELINE*	*PUMP16*	12									2
MATOSINHOS	*PIPELINE*	*INTERFACE*	12									2
MATOSINHOS	*MARITIME*	*DOCK*	12									1
MATOSINHOS	*ROAD*	*ARM*	12			7	1	12	10	2	2	
AZAMBUJA	*ANY*	*VESSEL*	4					4				
AZAMBUJA	*ANY*	*VESSEL*	12	8	6	4	3	4		4	3	
AZAMBUJA	*ROAD*	*ARM*	12	5	4	10	9	28		11	4	
SINES	*ANY*	*VESSEL*	12	5	4	6	4	6	6	5	4	
SINES	*PIPELINE*	*PUMP16*	12									3
SINES	*PIPELINE*	*INTERFACE*	12									3
SINES	*MARITIME*	*DOCK*	7									1
SINES	*RAIL*	*STATION*	2									1
SINES	*ROAD*	*ARM*	12	1		4	3	9			1	
PERAFITA	*ANY*	*VESSEL*	12	5	2							
PERAFITA	*ROAD*	*ARM*	12	9	5							
BANATICA	*ANY*	*VESSEL*	12	2	2			2			1	

BANATICA	*ROAD*	*ARM*	12	1	1			9			1	
BARREIRO	*ANY*	*VESSEL*	7			1	1	1	1		1	

Table B.9 (continued). Installed resource units per location, transport mode family, resource, product family

Depot	*Family*	*Resource*	Periods	C3	C4	U5	U8	GO	FU	A1	B5	ANY
BARREIRO	*ROAD*	*ARM*	7			2	2	4	1		1	
REAL	*ANY*	*VESSEL*	12	3	1	2		2				
REAL	*ROAD*	*ARM*	12	2	1	6	1	13		2		
LEIXOES	*ANY*	*VESSEL*	12			2		2	1		2	
LEIXOES	*ROAD*	*ARM*	12			7	1	11	2	2	3	

Table B.12. Resource typical capacities in cubic meters

Family	*Resource*	*C3*	*C4*	*U5*	*U8*	*GO*	*FU*	*A1*	*B5*	*ANY*
ANY	*VESSEL*	2200	2200	10000	10000	25000	10000	10000	5000	
PIPELINE	*PUMP16*									600
PIPELINE	*INTERFACE*									600
MARITIME	*DOCK*									600
RAIL	*STATION*									240
ROAD	*ARM*	20	20	20	20	20	20	20	20	

Table B.25. Product families

Family	*C3*	*C4*	*U5*	*U8*	*GO*	*FU*	*A1*	*B5*
C3	1							
C4		1						
U5			1					
U8				1				
GO					1			
FU						1		
A1							1	
B5								1
ANY	1	1	1	1	1	1	1	1

Table B.26. Transport mode families

Family	*Pipeline*	*Maritime*	*Rail*	*Road*
PIPELINE	1			
MARITIME		1		
RAIL			1	
ROAD				1
ANY	1	1	1	1

Table B.27. Transfer volume thresholds

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
0	124150	248300	496600	744900	993200	1241500	1489800	1738100	1986400	10000000

Table B.28. Tariff linearization coefficients

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
0,1	0,19	0,36	0,51	0,64	0,75	0,84	0,91	0,96	0,99

Table B.29. Transportation mode capacity characteristics

Transport	*Capacity*	*Ways*	*Before*	*After*	*Work*	*Days*
PIPELINE	800	1	4	8	24	30
MARITIME	3000	2	8	16	24	30
RAIL	512	2	4	4	14	28
ROAD	32	2	0.5	0.5	14	28

Appendix C - Deterministic and Robust MILP formulation of chapter 5

The Robust formulations developed in this chapter, are based on the strategic and tactical MILP presented in chapter 3, wherein may be consulted a detailed presentation of the original MILP formulation. The formulation presented below, first extends the published model by incorporating the importation of petroleum products, by all companies, at the storage depots; additional restrictions for bridge crossings on road routes, and several simplifications of the notation and formulation of the model, in view of the subsequent implementation of the robust optimization to deal with the crude costs, product prices and customer demand uncertainties. With a focus on these objectives, the model has been adequately adapted and presented below, beginning with the notation, the model decisions, the objective function and the network constraints and finally the RO formulation.

1 Notation

1.1 Sets / Indices

$a \in A$	set of activities, A={REFINERY,STORAGE,TRANSPORT,PRIMARY, SECONDARY}
$c \in C$	set of product characteristics, C={ DENSITY, TYPICAL_M3}
$d \in D$	set of route characteristics, D={ DISTANCE, DURATION, BRIDGE}
$e, e1 \in E$	set of petroleum entities/companies E={E1, E2, ..., EN }
$fc \in FC$	set of trip attributes, FC={CAPACITY,WORK,DAYS,BEFORE,AFTER, WAYS}
$g \in G$	set of GPS coordinates, G={ LATITUDE, LONGITUDE }
$n, o \in N$	set of all nodes
$p \in P$	set of products P={ P1, P2,..PN }
$r, r1 \in R$	set of resources including services and equipment
$t, \tau \in T$	set of time periods, T={1,2,..N}
$u \in U$	set of volume thresholds U={1,2,..N} for piecewise price linearization
$v \in V$	set of costs, V={TARIFF,FIXED,FIXVAR,OPERATING,OPERVAR, UNITARY}
$y \in Y$	set of amortization periods, Y={ 1,2,…,N }

1.2 Subsets / Indices

$i \in I \subseteq N$	set of refinery nodes
$IL \subseteq I$	set of local refineries
$j \in J \subseteq N$	set of depot nodes
$k \in K \subseteq N$	set of customer nodes
$m, m1 \in M \subseteq R$	set of transport modes, M={ PIPELINE, MARITIME, RAIL, ROAD }
$MC \subseteq M$	set of continuous flow transport modes MC={ PIPELINE }
$MP \subseteq M$	set of participated transport modes MP={ PIPELINE }

1.3 Subset unions / Indices

$fm \in FM = M \cup \{ANY\}$	set of transport mode families, FM={PIPELINE,MARITIME,RAIL,ROAD,ANY}
$fp \in FP = P \cup \{ANY\}$	set of products families FP={ P1, P2,..PN, ANY }
$h \in H = I \cup J$	set of refinery and depot nodes
$l \in L = J \cup K$	set of depot and customer nodes

1.4 Parameters

BN	big number for number of resources
HPP	hours per time period
MINTP	minimum time periods to stock
MAXTP	maximum time periods to stock
MAXBRIDGE	maximum trucks permitted per time period on routes with bridge
MAXUNSAT	maximum permitted unsatisfied demand
MD_m	distance multiplier per transportation mode to account for curved routes
MNA	maximum number of amortization periods
MV_m	medium velocity per transportation mode *m*
$FM_{fm,m}$	association of transportation mode family *fm* and transportation mode
$FP_{fp,p}$	association of product family *fp* with product *p*
$FR_{m,fc}$	trip parameters per transportation mode *m* and trip characteristic *fc*
IRR	internal rate of interest

NCD	number of depots in concentration set
NCP	number of capacity partitions per resource available to entity
NFF	fill factor in percentage
NT	number of time periods
NR	degree to radian coefficient for distance calculations
OCC	operating cost coefficient
RCD	radius for concentration set for demand at depot
$SC0_{i}$	initial period 0 inventory of crude at refinery *i*
$SD0_{j,e,p}$	initial period 0 inventory per depot *j*, entity *e*, and product *p*
$SL0_{k,e,p}$	initial period 0 inventory per customer *k*, entity *e*, and product *p*
$SR0_{i,e,p}$	initial period 0 inventory per refinery *i*, entity *e*, and product *p*
TPP	time aggregation in period, TPP=1 / 12 for monthly / yearly data
α_{a}	operator margin percentage per activity *a*
β_{a}^{Min}	minimum safety stock percentage per activity a
β_{a}^{Max}	maximum safety stock percentage per activity a
γ	working average cost of capital (WACC) coefficient for inventory costs
$h_{k,e,p,t}^{+}$	excess inventory capacity costs
$GPS_{n,g}$	global pointing system (GPS) coordinates *g* for each location node *n*
$\delta_{k,e,p,t}$	customer demand in m^3 at customer node *k*, entity *e*, product *p* and period *t*
$\varepsilon_{i,p}$	fraction yield of product *p* per unit measure of crude oil at refinery *i*
φ_{t}^{C}	price of crude oil at time *t*
$\varphi_{p,t}^{R}$	ex-refinery price of product *p* at time *t*
$\varphi_{p,t}^{L}$	retail price of product *p* at time *t*
$\eta_{n,e}^{N}$	capital participation percentage per entity *e* in refinery / depot node *n*
$\eta_{n,o,m,e}^{M}$	capital participation percentage per transportation infrastructure between nodes *n,o* and mode *m* by entity *e*
$\kappa d_{n,y}$	number of depreciated periods *y* for each existing depot at location *n*

$\kappa r_{n,fm,r,y,fp}$	current resource units per transport mode family *fm* and product family *fp*, per resource *r* at refinery or depot *n,* per number of depreciated periods *y*
$\kappa t_{n,o,m,y}$	number of depreciated periods *y* for current transport infrastructures between *n,o* for mode *m*; road routes are considered as existing and fully depreciated
$\mu_{n,o,m}$	transport distance between any two nodes *(n,o)* including refineries, depots or customer zones, using transport mode *m*
$\mu'_{n,o,m}$	transport duration between any two nodes *(n,o)* including refineries, depots or customer zones, using transport mode *m*
$\mu''_{n,o,m}$	bridge connection between any two nodes *(n,o)* including refineries, depots or customer zones, using transport mode *m=road*
$\theta r_{fm,r,fp}$	typical capacity of resource *r* for transport mode family *fm* and product family *fp*
ρ_i	crude processing capacity at refinery *i* in m^3 per day
τc_u	piecewise linearization function coefficients per step *u*
$\tau t_{h,u}$	piecewise linearization function thresholds per depot *h* and step *u*
$\upsilon^P_{m,p}$	maximum distance for primary distribution per transportation mode *m* for the petroleum product *p*; example: fuel-oil transportation by pipeline
$\upsilon^S_{m,p}$	maximum distance for secondary distribution per transportation mode *m* for the petroleum product *p*; example: JET transportation by pipeline
$\omega_{m,c,p}$	characteristic *c* per transport mode *m* per product *p*
ξr^F_r	fixed cost per resource *r*, independent of product
ξr^{FD}_r	fixed cost per unit distance of resource *r*, independent of product
ξr^O_r	operating cost per resource *r*, independent of product
ξr^{OD}_r	operating cost per unit distance of resource *r*, independent of product
ξr^V_r	volume based cost for using resource *r*, independent of product
$\xi p^F_{r,p}$	fixed cost of resource *r*, depending on product *p*
$\xi p^{FD}_{r,p}$	fixed cost per unit distance of resource *r*, depending on product *p*
$\xi p^O_{r,p}$	operating cost of resource *r*, depending on product *p*
$\xi p^{OD}_{r,p}$	operating cost per unit distance of resource *r*, depending on product *p*
$\xi p^V_{r,p}$	volume based cost for using resource *r*, dependant on product *p*
ξt^I_p	importation tariff per unit volume of product *p*
ξt^E_p	exportation tariff per unit volume of product *p*

ξt_p^R	refining tariff per unit volume of product *p*
ξt_p^L	retail tariff per unit volume of product *p*
ξt_p^U	unmet demand tariff per unit volume of product *p*
ζ_e	market quota per entity *e*

1.5 Parameters for modelling uncertainty

$\hat{\varphi}_t^C$	maximum deviation for price of crude oil at time *t*
$\hat{\varphi}_{p,t}^R$	maximum deviation for ex-refinery price of product *p* at time *t*
$\hat{h}_{k,e,p,t}^{+}$	maximum deviation for excess inventory capacity costs
$\hat{\delta}_{k,e,p,t}$	maximum deviation for customer demand in m³ at customer node *k*, entity *e*, product *p*, period *t*
Γ^C	budget of uncertainty for crude oil cost
Γ^R	budget of uncertainty for ex-refinery product price
Γ^+	budget of uncertainty for excess inventory capacity costs
$\Gamma_{k,p,e,t}^{\delta}$	budget of uncertainty for demand

2 Model Decisions

2.1 Binary Strategic Decision Variables

$$ID_j = \begin{cases} 1, & \text{if depot } j \text{ is to be installed} \\ 0, & \text{otherwise} \end{cases}$$

$$IT_{h,l,m} = \begin{cases} 1, & \text{if route using transport mode } m \text{ is to be installed between node } h \text{ and node } l \\ 0, & \text{otherwise} \end{cases}$$

$$OD_j = \begin{cases} 1, & \text{if depot } j \text{ is to be operated} \\ 0, & \text{otherwise} \end{cases}$$

$$OT_{h,l,m} = \begin{cases} 1, & \text{if route using transport mode } m \text{ is to be operated between node } h \text{ and node } l \\ 0, & \text{otherwise} \end{cases}$$

$$ZD_{j,e,u} = \begin{cases} 1, & \text{if } \pi t_{j,u-1} < QP_{j,l,m,p,e,t} + QS_{j,l,m,p,e,t} < \pi t_{j,u} \\ 0, & \text{otherwise} \end{cases}$$

$$ZT_{h,l,m,e,u} = \begin{cases} 1, & \text{if } \pi t_{h,u-1} < QP_{h,l,m,p,e,t} + QS_{h,l,m,p,e,t} < \pi t_{h,u} \\ 0, & \text{otherwise} \end{cases}$$

2.2 Integer Strategic Decision Variables

$IR_{j,fm,r,fp}$	number of resources to install per depot *j,* transport mode family *fm*, resource *r*, and product family *fp*
$OR_{j,fm,r,fp}$	number of resources to operate per depot *j*, per transport mode family *fm*, resource *r*, and product family *fp*
$ZR_{j,fm,r,e,fp}$	number of resource partitions to impute installation and operating costs per entity *e*, depot *j,* transport mode family *fm*, resource *r*, and product family f*p*

2.3 Non-negative Tactical Decision Variables

$QCI_{i,t}$	crude volume imported per refinery *i,* and period *t*
$QCP_{i,t}$	crude volume processed per refinery *i*, and period *t*
$QE_{i,p,t}$	exported volume per refinery *i*, product *p,* and period *t*
$QI_{j,e,p,t}$	imported volume per depot *j*, product *p,* entity *e,* and period *t*
$QP_{h,j,m,e,p,t}$	primary volumes transported from refinery/depot *h* to depot *j* by mode *m* per entity *e,* product *p,* and period *t*
$QR_{i,p,t}$	production volume per refinery *i*, product *p,* and period *t*
$QS_{j,k,m,e,p,t}$	secondary volumes transported from depot *j* to customer *k* by mode *m* per entity *e,* product *p*, and period *t*
$QU_{k,e,p,t}$	unsatisfied demand per customer *k*, entity *e*, and product *p* at end of period *t*
$SC_{i,t}$	crude inventory at refinery *i* at end of period *t*
$SD_{j,e,p,t}$	inventory at depot *j,* entity *e*, and product *p* at end of period *t*
$SL_{k,e,p,t}$	inventory at customer *k,* entity *e*, and product *p* at end of period *t*
$SR_{i,e,p,t}$	inventory at refinery *i,* entity *e*, and product *p* at end of period *t*

2.4 Non-negative Dual Variables for Robust Optimization

$\mu^{R}_{p,t}, \lambda^{R}$	Dual variables associated with the constraints of the inner maximization problem of the ex-refinery product price uncertainty
μ^{C}_{t}, λ^{C}	Dual variables associated with the constraints of the inner maximization problem of the crude oil cost uncertainty
$\mu^{+}_{n,e,p,t}, \lambda^{+}$	Dual variables associated with the constraints of the inner maximization problem of the excess inventory capacity cost uncertainty
$\mu^{\delta}_{n,p,e,\tau,t}$ $\lambda^{\delta}_{n,e,p,t}$	Dual variables associated with the constraints of the inner maximization problem of the demand uncertainty

2.5 Continuous Objective Sub-function Variables

ψ	PSC total profits

$MDE_{j,e,t}$ margin per depot *j*, entity *e*, and period *t*

$ML_{k,e,p,t}$ margin per retail customer *k*, product *p*, entity *e*, and period *t*

$MR_{i,e,t}$ margin per refinery *i*, entity *e*, and period *t*

$MTE_{h,l,m,e,t}$ transportation margin between nodes *h,l,* mode *m,* entity *e,* and period *t*

ψ^{CP} PSC total profits considering crude cost and product price uncertainties

ψ^{Θ} PSC total profits considering crude cost, product price and inventory capacity cost uncertainties

ψ^{δ} PSC total profits considering customer demand uncertainty

$\psi^{\delta\Theta}$ PSC total profits considering customer demand, crude cost, product price and inventory capacity cost uncertainties

2.6 Continuous Objective Sub-function Variables for Robust Optimization:

$\hat{R}$ trade-off value for robustness in ex-refinery prices uncertainty

$\hat{C}$ trade-off value for robustness in crude oil costs uncertainty

$\hat{D}$ trade-off value for robustness in customer demand uncertainty

$\hat{D}C$ trade-off value for robustness in customer demand and crude costs uncertainty

$\hat{D}H_{k,e,p,t}$ trade-off value for robustness in customer demand and excess inventory capacity costs uncertainties

$\hat{D}CH_{k,e,p,t}$ trade-off value for robustness in customer demand, crude costs, product prices and excess inventory capacity cost uncertainties

2.7 Non-negative Objective Sub-function Variables

$CDD_{h,t}$ depot costs per depot *h*, and period *t*

$CDFD_{h,t}$ depot fixed costs per depot *h*, and period *t*

$CDFE_{h,e,t}$ depot fixed costs per depot *h*, entity *e*, and period *t*

$CDOD_{h,t}$ depot operating costs per depot *h*, and period *t*

$CDOE_{h,e,t}$ depot operating costs per depot *h*, entity *e*, and period *t*

$CDU_{h,e,p,t}$ unitary costs per depot *h*, entity *e*, product *p*, and period *t*

$CE_{i,e,p,t}$ exportation costs per refinery *i*, entity *e*, product *p*, and period *t*

$CI_{j,e,p,t}$ importation costs per depot *j*, entity *e,* product *p*, and period *t*

$CNC_{i,e,t}$ inventory costs for crude at refinery *i,* entity *e*, period *t*

$CND_{j,e,p,t}$ inventory costs at depot *j*, entity *e,* product *p*, and period *t*

$CNL_{k,e,p,t}$ inventory costs at customer *k*, entity *e*, product *p*, and period *t*

$CNR_{i,e,p,t}$ inventory costs at refinery *i*, entity *e,* product *p*, and period *t*

$CRFD_{h,fp,t}$ resources fixed costs for depot *h,* resource *fp*, and period *t*

$CRFE_{h,e,fp,t}$ resources fixed costs for depot *h*, entity *e*, resource *fp*, and period *t*

$CROD_{h,fp,t}$ resources operating costs for depot *h,* resource *fp*, and period *t*

$CROE_{h,e,fp,t}$ resources operating costs for depot *h*, entity *e*, resource *fp*, and period *t*

$CTD_{h,l,m,t}$ transportation costs between nodes *h,l*, mode *m*, and period *t*

$CTFD_{h,l,m,t}$ transportation fixed costs between nodes *h,l*, mode *m*, and period *t*

$CTFE_{h,l,m,e,t}$ transportation fixed costs between nodes *h,l,* mode *m,* entity *e*, and period *t*

$CTOD_{h,l,m,t}$ transportation operating costs between nodes *h,l* for mode *m*, and period *t*

$CTOE_{h,l,m,e,t}$

transportation operating costs between nodes *h,l*, mode *m*, entity *e*, and period *t*

$CTU_{h,l,m,e,p,t}$ transportation unitary costs between nodes *h,l*, mode *m,* entity *e,* product *p*, and period *t*

$TDE_{h,e,t}$ depot tariffs at depot *h*, entity *e*, and period *t*

$TTE_{h,l,m,e,t}$ transportation tariff between nodes *h,l* for mode *m* for entity *e, and* period *t*

3 Objective Function

The MILP maximizes the PSC total profits, in eq. (5.1), which are the sum of the refining margin less the exportation, crude oil and refined product inventory costs; minus the product importation and the storage depot inventory costs; plus the retail stage revenue less the retail product inventory costs; the storage depot revenue less the storage depot tariffs; and the transportation stage revenue less the transportation tariffs. These objective function terms are obtained by their respective sub-functions eqs.(5.2-5.9) for the first nine terms, eqs.(5.10-5.13) for the transportation tariffs, eqs.(5.14-5.19) for the storage depot tariffs, eqs.(5.20-5.23) for the transportation margins, and eqs.(5.24-5.29) for the storage depot margins.

$$\psi = Max \sum_{i \in I} \sum_{e \in E} \sum_{t \in T} \left(MR_{i,e,t} - \sum_{p \in p} CE_{i,e,p,t} - CNC_{i,e,t} - \sum_{p \in P} CNR_{i,e,p,t} \right)$$
$$- \sum_{j \in J} \sum_{e \in E} \sum_{p \in P} \sum_{t \in T} \left(CI_{j,e,p,t} + CND_{j,e,p,t} \right) + \sum_{k \in K} \sum_{e \in E} \sum_{P \in P} \sum_{t \in T} \left(ML_{k,e,p,t} - CNL_{k,e,p,t} \right) \tag{5.1}$$
$$+ \sum_{h \in H} \sum_{e \in E} \sum_{t \in T} \left(MDE_{h,e,t} - TDE_{h,e,t} \right) + \sum_{(h,l,m) | \exists \mu_{h.l.m}} \sum_{e \in E} \sum_{t \in T} \left(MTE_{h,l,m,e,t} - TTE_{h,l,m,e,t} \right)$$

$$MR_{i,e,t} = \left(\sum_{p \in P} QR_{i,p,t} \left(\varphi^R_{p,t} - \xi t^R_p \right) - QCI_{i,t} \times \varphi^C_t \right) \eta^N_{i,i,e}, \; \forall i \in I, e \in E, t \in T \tag{5.2}$$

$$CE_{i,e,p,t} = QE_{i,p,t} \, \xi \, t^E_p \, \eta^N_{i,i,e}, \qquad \forall i \in I, e \in E, p \in P, t \in T, \tag{5.3}$$

$$CI_{h,e,p,t} = QI_{h,e,p,t} \times \left(\varphi^R_{p,t} + \xi \, t^I_p \right), \qquad \forall h \in H, e \in E, p \in P, t \in T \tag{5.4}$$

$$CNC_{i,e,t} = \gamma \times SC_{i,t} \times \varphi^C_t \times \eta^N_{i,e}, \qquad \forall i \in I, e \in E, t \in T \tag{5.5}$$

$$CNR_{i,e,p,t} = \gamma \times SR_{i,p,t} \times \varphi^R_{p,t} \times \eta^N_{i,e}, \qquad \forall i \in I, e \in E, p \in P, t \in T \tag{5.6}$$

$$CND_{j,e,p,t} = \gamma \times SD_{j,e,p,t} \times \varphi^R_{p,t}, \qquad \forall j \in J, e \in E, p \in P, t \in T \tag{5.7}$$

$$ML_{k,e,p,t} = \delta_{k,e,p,t} \left(\varphi^L_{p,t} - \xi t^L_p \right) - \sum_{(j,m) | \exists \mu_{j,k,m}} QS_{j,k,m,e,p,t} \times \varphi^R_{p,t}, \forall k \in K, e \in E, p \in P, t \in T \tag{5.8}$$

$$CNL_{k,e,p,t} = h^+_{k,e,p,t} \times SL_{k,e,p,t}, \qquad \forall k \in K, e \in E, p \in P, t \in T \tag{5.9}$$

3.1 Transportation tariffs

The transportation tariffs are determined per transportation mode, route, entity and time period in eqs.(5.10), by aggregating the fixed, operating and volume based costs, which are determined respectively in eqs. (5.11),(5.12) and (5.13). As the model considers retrofit design and the amortization of investment in MNA time periods, the fixed costs are generated only in the non-amortized periods.

$$TTE_{h,l,m,e,t} = \left(1 + \alpha^{a1}\right)\left(CTFE_{h,l,m,e,t} + CTOE_{h,l,m,e,t} + \sum_{p \in P} CTU_{h,l,m,e,p,t} \right) \forall h \in H, l \in L, m \in M \wedge (h,l,m) | \exists \mu_{h,l,m}, e \in E, t \in T, a1 = TRANSPORT \tag{5.10}$$

$$CTFE_{h,l,m,e,t} = \sum_{u \in U} \left(ZT_{h,l,m,e,u} \, \tau c_u \right) \left(\xi r^F_m + \xi r^{FD}_m \times \mu_{h,l,m} \right) \left(1 + irr \times t \right), \; \forall (h,l,m) | \exists \mu_{h,l,m}, e \in E, t \in T \wedge t \leq MNA \tag{5.11}$$

$$CTOE_{h,l,m,e,t} = \sum_{u \in U} ZT_{h,l,m,e,u} \, \tau c_u \left(\xi r^O_m + \xi r^{OD}_m \, \mu_{h,l,m} \right), \; \forall (h,l,m) | \exists \mu_{h,l,m}, e \in E, t \in T \tag{5.12}$$

$$CTU_{h,l,m,e,p,t} = \left(QP_{h,l,m,e,p,t} + QS_{h,l,m,e,p,t} \right) \xi \, p^V_{m,p} \, \mu_{h,l,m}, \qquad \forall (h,l,m) | \exists \mu_{h,l,m}, e \in E, p \in P, t \in T \tag{5.13}$$

3.2 Storage Depot tariffs

The storage depot tariffs are determined in eqs.(5.14) per storage depot, entity and time period, by aggregating the depot fixed and operating costs, the depot resources fixed and operating costs, and the depot volume based costs, which are determined respectively in eqs. (5.15),(5.16),(5.17),(5.18) and (5.19). It should be noted that this formulation implements modularized as well as volume dependent linearized costs per entity in order to incorporate economies of scale.

$$TDE_{h,e,t} = \left(1+\alpha^{a1}\right)\left(CDFE_{h,e,t} + CDOE_{h,e,t} + \sum_{fp\in FP}\left(CRFE_{h,e,fp,t} + CROE_{h,e,fp,t}\right) + \sum_{p\in P} CDU_{h,e,p,t}\right) \forall h\in H, e\in E, t\in T, a1 = DEPOT \quad (5.14)$$

$$CDFE_{h,e,t} = \sum_{u\in U} ZD_{h,e,u}\, \tau c_u\, \xi r_{r1}^{F}\left(1+irr\times t\right),\ \forall h\in H, e\in E, t\in T, t\le MNA, r1 = DEPOT \quad (5.15)$$

$$CDOE_{h,e,t} = \sum_{u\in U} ZD_{h,e}\, \tau c_u\, \xi r_{r1}^{O}, \qquad \forall h\in H, e\in E, t\in T, r1 = DEPOT \quad (5.16)$$

$$CRFE_{h,e,fp,t} = \sum_{fm,r} \frac{ZR_{m,h,r,e,p=fp}\left(\xi p_{r,p=fp}^{F} + \xi r_r^{F}\right)\left(1+irr\times t\right)}{NCP},\ \ \forall h\in H, e\in E, fp\in FP, t\in T, t\le MNA\ , \xi r_r^{F}\,\middle|\,\nexists \xi p_{r,p}^{F} \quad (5.17)$$

$$CROE_{h,e,fp,t} = \sum_{r\in R}\sum_{fm\in FM}\left(ZR_{h,fm,r,e,fp}/NCP\right)\left(\xi p_{r,p=fp}^{O} + \xi r_r^{O}\right), \forall h\in H, e\in E, fp\in FP, t\in T, \xi r_r^{O} \,|\, \nexists \xi p_{r,p}^{O} \quad (5.18)$$

$$CDU_{h,e,p,t} = \sum_{l\in L}\sum_{m\in M}\left(QP_{h,l,m,e,p,t} + QS_{h,l,m,e,p,t}\right)\xi p_{r,p}^{V}, \qquad \forall h\in H, e\in E, p\in P, t\in T, r = DEPOT \quad (5.19)$$

3.3 Transportation operator margins

The operator margin per transportation mode, route, entity and time period is given by eq.(5.20), which is the difference between the transportation route tariffs eq.(5.10) and costs eq.(5.21), where the latter is the sum of the fixed costs eq.(5.22), the operating costs eq.(5.23) and the volume based costs eq.(5.13).

$$MTE_{h,l,m,e,t} = \left(\sum_{e2\in E} TTE_{h,l,m,e2,t} - CTD_{h,l,m,t}\right)\left(\eta_{h,l,m,e}^{M} + \zeta_e\right),\ \ \forall (h,l,m) \,|\, \exists \mu_{h,l,m} \wedge m\in MP, e\in E, t\in T, \zeta_e \,|\, \nexists \eta_{h,l,m,e}^{M} \quad (5.20)$$

$$CTD_{h,l,m,t} = \left(CTFD_{h,l,m,t} + CTOD_{h,l,m,t} + \sum_{e\in E}\sum_{p\in P} CTU_{h,l,m,e,p,t}\right) \qquad \forall (h,l,m) \,|\, \exists \mu_{h,l,m}, t\in T \quad (5.21)$$

$$CTFD_{h,l,m,,t} = \left(IT_{h,l,m} - \sum_{y\in Y} \kappa t_{h,l,m,y}\right)\left(\xi r_m^{F} + \xi r_m^{FD}\times \mu_{h,l,m}\right)\left(1+irr\times t\right),$$
$$\forall (h,l,m) \,|\, \exists \mu_{h,l,m}, t\in T \wedge t\le MNA \wedge y+t > MNA \quad (5.22)$$

$$CTOD_{h,l,m,t} = OT_{h,l,m}\left(\xi r_m^{O} + \xi r_m^{OD}\, \mu_{h,l,m}\right), \qquad \forall (h,l,m) \,|\, \exists \mu_{h,l,m}, t\in T \quad (5.23)$$

3.4 Storage depot operator margin

The operator margin per storage depot, entity and time period is given by eq.(5.24), which is the difference between the storage depot tariffs eq.(5.14) and costs eq.(5.25), where the latter is the sum of the depot's fixed eq.(5.26) and operating costs eq.(5.27), the resources' fixed eq.(5.28) and operating costs eq.(5.29) and the volume based costs eq.(5.19).

$$MDE_{h,e,t} = \left(\sum_{e2\in E} TDE_{h,e2,t} - CDD_{h,t}\right)\left(\eta_{h,e}^{N} + \zeta_e\right) \quad \forall h\in H, e\in E, t\in T, \zeta_e \,|\, \nexists \eta_{h,e}^{N} \quad (5.24)$$

$$CDD_{h,t} = CDFD_{h,t} + CDOD_{h,t} + \sum_{fp\in FP}\left(CRFD_{h,fp,t} + CROD_{h,fp,t}\right) + \sum_{e\in E}\sum_{p\in P} CDU_{h,e,p,t} \qquad \forall h\in H, t\in T \quad (5.25)$$

$$CDFD_{h,t} = \left(ID_h - \sum_{(y+t)>MNA} \kappa d_{h,y}\right)\xi r_r^{F}\left(1+irr\times t\right), \qquad \forall h\in H, t\in T \wedge t\le MNA, r = DEPOT \quad (5.26)$$

$$CDOD_{h,t} = OD_h\, \xi r_r^{O}, \qquad \forall h\in H, t\in T, r = DEPOT \quad (5.27)$$

$$CRFD_{h,fp,t} = \sum_{fm,r}\left(IR_{h,fm,r,fp} - \sum_{(y+t)>MNA} \kappa r_{h,fm,r,y,fp}\right)\left(\xi p_{r,fp}^{F} + \xi r_r^{F}\right)\left(1+irr\times t\right), \forall h\in H, fp\in FP, t\in T \wedge t\le MNA, \xi r_r^{F} \,|\, \nexists \xi p_{r,fp}^{F} \quad (5.28)$$

$$CROD_{h,fp,t} = \sum_{fm\in FM, r\in R} OR_{h,fm,r,fp}\left(\xi p_{r,p}^{O} + \xi r_r^{O}\right), \qquad \forall h\in H, t\in T, fp\in FP, \xi r_r^{O} \,\middle|\, \nexists \xi p_{r,fp}^{O} \quad (5.29)$$

3.5 Entity tariffs based on non-amortized costs

Equations (5.11), (5.15) and (5.17), presented earlier, consider the reduction of the transportation route and storage depot entity tariffs, following the amortizations of the investment costs in each period. However, these tariffs may be fixed and based on the non-amortized costs, in which case eqs.(5.11), (5.15) and (5.17) are replaced respectively by eqs.(5.30), (5.31) and (5.32).

$$CTFE_{h,l,m,e,t} = \sum_{u\in U} ZT_{h,l,m,e,u}\,\tau c_u \left(1-\eta^{M}_{h,l,m,e}\sum_{y\in Y}\kappa t_{h,l,m,y}\right)\left(\xi r^{F}_{m} + \xi r^{FD}_{m}\,\mu_{h,l,m}\right)(1+irr\times t), \forall (h,l,m)\,|\,\exists \mu_{h,l,m}, e\in E, t\in T, t\le MNA \vee \left(\nexists \eta^{M}_{h,l,m,e} \wedge y+t > MNA\right) \quad (5.30)$$

$$CDFE_{h,e,t} = \sum_{u\in U}\left(ZD_{h,e,u}\,\tau c_u\left(1-\eta^{N}_{h,,e}\sum_{(y+t)>MNA}\kappa d_{h,y}\right)\right)\xi r^{F}_{r}\times(1+irr\times t), \forall h\in H, e\in E, t\in T \wedge \left(t\le MNA \vee \nexists \eta^{N}_{h,,e}\right), r = DEPOT \quad (5.31)$$

$$CRFE_{h,e,fp,t} = \sum\nolimits_{fm,r} k\times ZR_{h,fm,r,e,p}\left(\xi p^{F}_{r,p} + \xi r^{F}_{r}\right)(1+irr\times t)\big/ NCP,$$
$$\forall h\in H, e\in E, fp\in FP, t\in T, k=1\,|\,t\le MNA \vee \neg\exists \sum\nolimits_{fm,r,y1}\kappa r_{h,fm,r,y1,fp}\,\eta^{N}_{h,e}, \xi r^{F}_{r}\,|\,\nexists \xi p^{F}_{r,p} \quad (5.32)$$

4 Constraints

4.1 Volume balance equations

The volume balance constraints comprise of the first period and following periods processed crude constraints, eqs.(5.33-5.34), the restrictions related to the crude oil importation and processing volumes based on the refinery capacities, eqs.(5.35-5.36), the refined product volumes given by the crude oil's product mix yield, eq.(5.37), the volume balance constraints at refineries, eqs.(5.38-5.39), the volume balance constraints at the storage depots, eqs.(5.40-5.41), and the volume balance constraints at the retail stage, eqs.(5.42-5.43).

$$SC0_i + QCI_{i,t} = QCP_{i,t} + SC_{i,t}, \qquad \forall i\in I, t\in T \wedge t\le 1 \quad (5.33)$$

$$SC_{i,t-1} + QCI_{i,t} = QCP_{i,t} + SC_{i,t}, \qquad \forall i\in I, t\in T \wedge t > 1 \quad (5.34)$$

$$QCI_{i,t} \le \rho_i, \qquad \forall i\in I, t\in T \quad (5.35)$$

$$QCP_{i,t} \le \rho_i, \qquad \forall i\in I, t\in T \quad (5.36)$$

$$QR_{i,p,t}\,\omega_{m1,c1,p} = QCP_{i,t}\,\omega_{m1,c1,p2}\,\varepsilon_{i,p}, \quad \forall i\in I, p1\in P, t\in T, m1 = ANY, c1 = DENSITY, p2 = CRUDE \quad (5.37)$$

$$SR0_{i,p} + QR_{i,p,t} = \sum_{m\in MC, e\in E} QP_{i,i,m,e,p,t} + QE_{i\in IL,p,t} + SR_{i,p,t}, \forall i\in I, p\in P, t\in T \wedge t\le 1 \quad (5.38)$$

$$SR_{i,p,t-1} + QR_{i,p,t} = \sum_{m\in MC, e\in E} QP_{i,i,m,e,p,t} + QE_{i\in IL,p,t} + SR_{i,p,t}, \forall i\in I, p\in P, t\in T \wedge t > 1 \quad (5.39)$$

$$SD0_{j,p,e} + \sum_{(h\in H,m)|\exists \mu_{h,j,m}} QP_{h,j,m,e,p,t} + QI_{j1,e,p,t} = \sum_{(h\ne j,m)|\exists \mu_{j,h,m}} QP_{j,h,m,e,p,t} + \sum_{(k\in K,m)|\exists \mu_{j,k,m}} QS_{j,k,m,e,p,t} + SD_{j,e,p,t},$$
$$\forall j\in J, j1 = j \wedge m = MARITIME, e\in E, p\in P, t\in T, t\le 1 \quad (5.40)$$

$$SD_{j,e,p,t-1} + \sum_{(h\in H,m)|\exists\mu_{h,j,m}} QP_{h,j,m,e,p,t} + QI_{j1,e,p,t} = \sum_{(h\neq j,m)|\exists\mu_{j,h,m}} QP_{j,h,m,e,p,t} + \sum_{(k\in K,m)|\exists\mu_{j,k,m}} QS_{j,k,m,e,p,t} + SD_{j,e,p,t}, \quad \forall j\in J,\ j1 = j \wedge m = MARITIME, e\in E,\ p\in P, t\in T, t>1 \tag{5.41}$$

$$SL0_{k,e,p} + \sum_{(j,m)|\exists\mu_{j,k,m}} QS_{j,k,m,e,p,t} = \delta_{k,e,p,t} + SL_{k,e,p,t}, \forall k\in K, e\in E,\ p\in P, t\in T \wedge t\le 1 \tag{5.42}$$

$$SL_{k,e,p,t-1} + \sum_{(j,m)|\exists\mu_{j,k,m}} QS_{j,k,m,e,p,t} = \delta_{k,e,p,t} + SL_{k,e,p,t}, \forall k\in K, e\in E,\ p\in P, t\in T \wedge t> 1 \tag{5.43}$$

4.2 Transportation network

The transportation network operating capacities are determined per transportation mode, route and entity in eq.(5.44) using binary variables $ZT_{h,l,m,e,u}$. The route capacities determined per entity, enforce that the corresponding routes are operating with eq.(5.45), and are installed with eq.(5.46), consequently enforcing the installation and operation of resources for the installed and operating transportation modes. The installation of the existing transportation routes is enforced by eqs.(5.49).

$$\sum_{u\in U} \tau t_{h,u-1} ZT_{h,l,m,e,u} \le \sum_{t\in T}\sum_{p\in P} \frac{QP_{h,l,m,e,p,t} + QS_{h,l,m,e,p,t}}{NT} \le \sum_{u\in U} \tau t_{h,u} ZT_{h,l,m,e,u}, \forall (h,l,m)|\exists(\xi r^{F}_{m} \vee \xi r^{FD}_{m}) \wedge \mu_{h,l,m}, e\in E \tag{5.44}$$

$$\sum_{u\in U} ZT_{h,l,m,e,u} \le OT_{h,l,m} \times NB, \quad \forall (h,l,m)|\exists(\xi r^{F}_{m} \vee \xi r^{FD}_{m}) \wedge \mu_{h,l,m}, e\in E \tag{5.45}$$

$$OT_{h,l,m} \le IT_{h,l,m}, \quad \forall (h,l,m)\,|\,\exists\mu_{h,l,m} \tag{5.46}$$

$$OT_{h,l,m} \le \sum_{fm=m,r,fp} OR_{h,fm,r,fp}, \quad \forall (h,l,m)\,|\,\exists\mu_{h,l,m} \tag{5.47}$$

$$IT_{h,l,m} \le \sum_{fm=m,r,fp} IR_{h,fm,r,fp}, \quad \forall (h,l,m)\,|\,\exists\mu_{h,l,m} \tag{5.48}$$

$$\kappa t_{h,l,m,y} \le IT_{h,l,m}, \quad \forall (h,l,m)\,|\,\exists\mu_{h,l,m}, y\in Y \tag{5.49}$$

4.3 Storage Depot network

The operating resource capacities per storage depot, transport mode family and product family are determined for each entity by eqs.(5.50) and aggregated per depot in eqs.(5.52-5.53,5.56). These are determined as the operating resources that determine the operating costs and the installed resources that determine the fixed costs. Eq.(5.51) restrict the number of journeys on road routes with bridges, which restrict the transfer volumes, and consequently have an impact on the decision of the depots to install. The linearized cost parameters are used in eqs (5.57) to determine the depot operating and installation costs' affectations per entity. The installation and operation of resources per product and entity, also determine the operation or closure of existing depots or installation of new storage depots, as enforced in eqs.(5.55-5.56,5.58-5.59).

$$\left(ZR_{h,fm,r,e,fp}-1\right)\times\theta r_{fm,r,fp}\times\left((1-k)HPP+k\times TPP\right)/NCP$$
$$\leq\sum_{t\in T}\sum_{p}\frac{}{|FP_{fp,p}}\sum_{m}\frac{}{|FM_{fm,m}}\left(\left(\left(\sum_{j\neq h\wedge j\in\mu_{h,j,m}}QP_{h,j,m,e,p,t}+\sum_{k\in\mu_{h,k,m}}QS_{h,k,m,e,p,t}\right)\left((1-k)+\frac{k}{\omega m_{m,p}}\right)+SD_{h,e,p,t}\times k\right)\right)/NT \quad (5.50)$$
$$\leq ZR_{h,fm,r,e,fp}\theta r_{fm,r,fp}\left((1-k)HPP+k\times TPP\right)/NCP,\forall h\in H,e\in E,(fm,r,fp)\,|\exists\theta r_{fm,r,fp},k=1|\,fm=ANY$$

$$\frac{\sum_{l\in\mu''_{h,l,m},e\in E,p\in P}\left(QP_{h,l,m,e,p,t}+QS_{h,l,m,e,p,t}\right)}{\left(FR_{m,c1}\times FR_{m,c2}\times TPP\right)}\leq MAXBRIDGE,\forall h\in H,m\in M\wedge m=ROAD,t\in T \quad (5.51)$$

$$\sum_{y\in Y}\kappa r_{h,fm,r,y,fp}\leq IR_{h,fm,r,fp}, \quad \forall h\in H,\ fm\in FM,r\in R,\ fp\in FP,(fm,r,fp)\,|\exists\theta r_{fm,r,fp} \quad (5.52)$$

$$\sum_{e\in E}ZR_{h,fm,r,e,fp}\leq OR_{h,fm,r,fp}\times NCP, \quad \forall h\in H,\ fm\in FM,r\in R,\ fp\in FP,(fm,r,fp)\,|\exists\theta r_{fm,r,fp} \quad (5.53)$$

$$OR_{h,fm,r,fp}\leq OD_h\times NB, \quad \forall h\in H,\ fm\in FM,r\in R,\ fp\in FP,(fm,r,fp)\,|\exists\theta r_{fm,r,fp} \quad (5.54)$$

$$IR_{h,fm,r,fp}\leq ID_h\times NB, \quad \forall h\in H,\ fm\in FM,r\in R,\ fp\in FP,(fm,r,fp)\,|\exists\theta r_{fm,r,fp} \quad (5.55)$$

$$OR_{h,fm,r,fp}\leq IR_{h,fm,r,fp}, \quad \forall h\in H,\ fm\in FM,r\in R,\ fp\in FP,(fm,r,fp)\,|\exists\theta r_{fm,r,fp} \quad (5.56)$$

$$\sum_{u\in U}\pi t_{h,u-1}ZD_{h,e,u}\leq\sum_{fm,r,fp}ZR_{h,fm,r,e,fp}\theta r_{fm,r,fp}\times TPP/NCP\leq\sum_{u\in U}\pi t_{h,u}ZD_{h,e,u},\forall h\in H,e\in E,r=VESSEL \quad (5.57)$$

$$\sum_{u\in U}ZD_{h,e,u}\leq OD_h\times NB, \quad \forall h\in H,e\in E \quad (5.58)$$

$$OD_h\leq ID_h, \quad \forall h\in H \quad (5.59)$$

4.4 Minimum and Maximum inventory constraints

$$SC_{i,t}\geq\rho_i\times\beta_a^{Min}, \quad \forall i\in I,t\in T,a=CRUDE \quad (5.60)$$

$$SC_{i,t}\leq\rho_i\times\beta_a^{Max}, \quad \forall i\in I,t\in T,a=CRUDE \quad (5.61)$$

$$QCP_{i,t}\geq\rho_i\times\beta_a^{Min}, \quad \forall i\in I,t\in T,a=REFINING \quad (5.62)$$

$$SR_{i,p,t}\leq\rho_i\times\varepsilon_{i,p}\times\beta_a^{Max}, \quad \forall i\in I,p\in P,t\in T,t>1\wedge t\leq T-1,a=REFINING \quad (5.63)$$

$$SL_{k,e,p,t}\leq\delta_{k,e,p,t}\times\beta_a^{Max}, \quad \forall k\in K,e\in E,p\in P,t\in T,t>1\wedge t<T,a=RETAIL \quad (5.64)$$

$$OT_{n,o,m}=IT_{n,o,m}=0, \quad \forall n\in N,o\in O,m\in M,\wedge\mu_{n,o,m}=0 \quad (5.65)$$

$$QP_{h,j,m,e,p,t}=0 \quad \forall h\in H,j\in J,m\in M,e\in E,p\in P,t\in T,\wedge\left(\mu_{h,j,m}>\upsilon_{m,p}^{P}\right) \quad (5.66)$$

$$QS_{j,k,m,e,p,t}=0 \quad \forall j\in J,k\in K,m\in M,e\in E,p\in P,t\in T,\wedge\left(\mu_{j,k,m}>\upsilon_{m,p}^{S}\right) \quad (5.67)$$

The minimum and maximum bounds for inventories at crude oil, refinery, and retail stages as well as the primary and secondary transportation restrictions are enforced by eqs (5.60-5.67).

5 Crude Cost and Product Price Uncertainty

The robust counterpart formulation for the maximization of the PSC total profits, considering crude oil costs and product prices uncertainty is given by eqs.(5.1a), subject to the original problem constraints and additionally eqs.(5.68-5.72).

$$\max\left(\psi^{CP}\right)=\psi-\left(\Gamma^{R}\lambda^{R}+\sum_{t\in T}\sum_{p\in P}\mu_{p,t}^{R}\right)-\left(\Gamma^{C}\lambda^{C}+\sum_{t\in T}\mu_{t}^{C}\right) \quad (5.1a)$$

$$\text{subject to:} \quad \text{constraints } (5.33-5.67)$$

$$\lambda^{R}+\mu_{p,t}^{R}\geq\hat{\varphi}_{p,t}^{R}\sum_{i\in I,e\in E}QR_{i,p,t}\eta_{i,e}^{N}, \forall p\in P\wedge t\in T \quad (5.68)$$

$$\lambda^{C}+\mu_{t}^{C}\geq\hat{\varphi}_{t}^{C}\sum_{i\in I,e\in E}QCI_{i,t}\eta_{i,e}^{N}, \forall t\in T \quad (5.69)$$

$$\mu_{p,t}^{R}\geq 0, \forall p\in P\wedge t\in T \quad (5.70)$$

$$\mu_{t}^{C}\geq 0, \forall t\in T \quad (5.71)$$

$$\lambda^{R}\geq 0,\ \lambda^{C}\geq 0 \quad (5.72)$$

6 Crude Cost, Product Price and Excess Capacity Cost Uncertainty

The robust counterpart formulation for the maximization of the PSC total profits, considering the crude oil costs, product prices, and inventory capacity costs is provided by eqs.(5.1b), subject to the additional constraints, eqs.(5.73-5.75).

$$\max\left(\psi^{\Theta}\right)=\psi-\left(\Gamma^{R}\lambda^{R}+\sum_{t\in T}\sum_{p\in P}\mu_{p,t}^{R}\right)-\left(\Gamma^{C}\lambda^{C}+\sum_{t\in T}\mu_{t}^{C}\right)-\left(\Gamma^{+}\lambda^{+}+\sum_{k\in K}\sum_{e\in E}\sum_{p\in P}\sum_{t\in T}\mu_{k,e,p,t}^{+}\right) \quad (5.1b)$$

$$\text{subject to:} \quad \text{constraints } (5.33-5.67),(5.68-5.72)$$

$$\lambda^{+}+\mu_{k,e,p,t}^{+}\geq\hat{h}_{k,e,p,t}^{+}\times SL_{k,e,p,t}, \forall k\in K\wedge e\in E\wedge p\in P\wedge t\in T \quad (5.73)$$

$$\mu_{k,e,p,t}^{+}\geq 0, \forall k\in K\wedge e\in E\wedge p\in P\wedge t\in T \quad (5.74)$$

$$\lambda^{+}\geq 0 \quad (5.75)$$

7 Customer Demand Uncertainty

The robust counterpart formulation for the maximization of the PSC total profits, considering customer demand uncertainty are given by eqs.(5.1c), subject to the additional constraints, eqs.(5.78-5.81).

$$\max\left(\psi^{\delta}\right) = \psi - \sum_{k\in K}\sum_{e\in E}\sum_{p\in P}\sum_{t\in T} H_{k,e,p,t} \tag{5.1c}$$

$$\text{subject to:} \quad \text{constraints } (5.33-5.41), (5.44-5.63), (5.65-5.67)$$

$$H_{k,e,p,t} \geq h^{+}_{k,e,p,t}\left(\sum_{\tau\leq t}\left(\sum_{j,m}\mathrm{QS}_{j,k,m,e,p,\tau} - \delta_{k,e,p,\tau}\right) + \Gamma^{\delta}_{k,e,p,t}\lambda^{\delta}_{k,e,p,t} + \sum_{\tau=t}\mu^{\delta}_{k,e,p,\tau,t}\right), \quad \forall k\in K \wedge e\in E \wedge p\in P \wedge t\in T \tag{5.78}$$

$$\sum_{\tau\leq t}\left(\sum_{j,m}\mathrm{QS}_{j,k,m,e,p,\tau} - \delta_{k,e,p,\tau}\right) + \Gamma^{\delta}_{k,e,p,t}\lambda^{\delta}_{k,e,p,t} + \sum_{\tau=t}\mu^{\delta}_{k,e,p,\tau,t} \leq \delta_{k,e,p,t+1}\times\beta^{Max}_{a}, \quad \forall k\in K, e\in E, p\in P, t\in T, t>1 \wedge t<T, a = RETAIL \tag{5.79}$$

$$\lambda^{\delta}_{k,e,p,t} + \mu^{\delta}_{k,e,p,\tau,t} \geq \hat{\delta}_{k,e,p,\tau}, \qquad \forall k\in K \wedge e\in E \wedge p\in P \wedge t\in T \wedge \tau\leq t \tag{5.80}$$

$$\lambda^{\delta}_{k,e,p,t}, \mu^{\delta}_{k,e,p,\tau,t} \geq 0, \qquad \forall k\in K \wedge e\in E \wedge p\in P \wedge t\in T \wedge \tau\leq t \tag{5.81}$$

8 Crude Cost, Product Price, Excess Capacity costs, and Demand Uncertainties

The robust counterpart formulation for the maximization of the PSC total profits, considering the crude oil costs, product prices, inventory capacity costs, and demand uncertainties are given in eqs.(5.1e), subject to the associated constraints and additionally eqs.(5.84).

$$\max\left(\psi^{\delta\Theta}\right) = \psi - \sum_{k\in K}\sum_{e\in E}\sum_{p\in P}\sum_{t\in T} H_{k,e,p,t} - \left(\Gamma^{R}\lambda^{R} + \sum_{t\in T}\sum_{p\in P}\mu^{R}_{p,t}\right) - \left(\Gamma^{C}\lambda^{C} + \sum_{t\in T}\mu^{C}_{t}\right) - \left(\Gamma^{+}\lambda^{+} + \sum_{k\in K}\sum_{e\in E}\sum_{p\in P}\sum_{t\in T}\mu^{+}_{k,e,p,t}\right) \tag{5.1e}$$

$$\text{subject to:} \quad \text{constraints } (5.33-5.41), (5.44-5.63), (5.65-5.67), (5.68-5.75), (5.78-5.81), (5.85)$$

$$H_{k,e,p,t} \geq \left(h^{+}_{k,e,p,t} + \hat{h}^{+}_{k,e,p,t}\right)\left(\sum_{\tau\leq t}\left(\sum_{j,m}\mathrm{QS}_{j,k,m,e,p,\tau} - \delta_{k,e,p,\tau}\right) + \Gamma^{\delta}_{k,e,p,t}\lambda^{\delta}_{k,e,p,t} + \sum_{\tau=t}\mu^{\delta}_{k,e,p,\tau,t}\right), \forall k\in K \wedge e\in E \wedge p\in P \wedge t\in T \tag{5.85}$$