Coherent Sampling of Sinewaves

F. Corrêa Alegria[1] and A. Cruz Serra[2]

Telecommunications Institute and Department of Electrical and Computer Engineering,

Instituto Superior Técnico, Technical University of Lisbon, Portugal

Abstract

This paper presents a study about the choice of input and sampling frequencies and the number of samples to acquire in order to achieve coherent sampling. This is important in the test of analog to digital converters (ADCs) by the Histogram Test Method, the Discrete Fourier Transform Method or the Sine-fitting Method.

It is shown that a trade off exists between closeness to the desired frequencies and test duration (through the number of acquired samples).

Several algorithms are presented and studied in terms of complexity, advantages and drawbacks.

I. INTRODUCTION

The most common ADC tests involve the sampling of a sinusoidal input signal at a constant rate (sampling frequency – f_s) and the use of the ADC output codes to estimate several characteristics of the converter. For instance, the Histogram Method [1, 2, 3] uses the output codes to compute an histogram of occurring codes and from that it estimates the transition voltages which permit the computation off code bin widths, gain, offset error, integral non-linearity (INL) and differential non-linearity (DNL) [1]. The sine-fitting method involves the least-squares fitting of a sinusoidal shape to the ADC output codes. The values of the fitted sinusoid are subtracted from the output codes resulting in an error signal from which the additive noise, phase noise, and aperture uncertainty (jitter) can be estimated. The DFT test [1] performs a discrete Fourier Transform on the output codes and estimates from the obtained spectrum the ADC additive noise (SNR), harmonic distortion (THD, SFDR) and spurious distortion (TSD, SFDR).

All these methods assume that the input signal is a perfect sinusoid sampled at specific time instants to guarantee that the amplitude distribution of the sampled voltages is known. To achieve this, the acquisition must be performed during exactly one period of the stimulus signal. The high number of samples (millions) that must be acquired to guarantee a good precision on the test results leads to a stimulus signal frequency much lower than the sampling frequency which is not desired since the performance of ADCs depends on the input frequency. To circumvent this problem time-equivalent sampling is generally used. This involves the acquisition of samples during more than one sinusoid period. Special care must be taken to guarantee that the sample phases (in relation to the sinusoid) are uniformly distributed in the interval of 2π. This is attained if the number of periods (J) during which the acquisition is carried out is an integer mutually prime with the number of acquired samples (M). The sinusoid and sampling frequency must therefore satisfy [4]

$$\rho = \frac{f}{f_s} = \frac{J}{M}\,. \qquad (1)$$

Fig. 1 depicts the case where 40 samples are acquired during 2 periods of the stimulus signal. Because the integer 2 and 40 are not mutually prime (they have at least one common factor greater than 1, in this case the number 2), the number of distinct phases is just 20 as can be seen in Fig. 2.

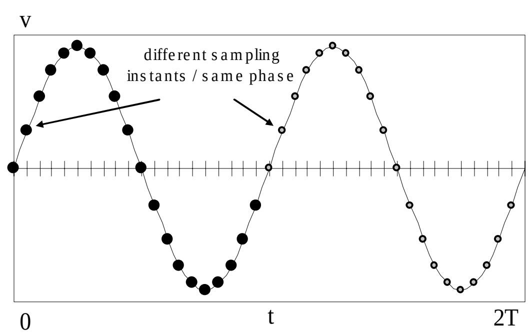

Fig. 1 – Representation of 2 periods of a sinusoid during which 40 samples were acquired.

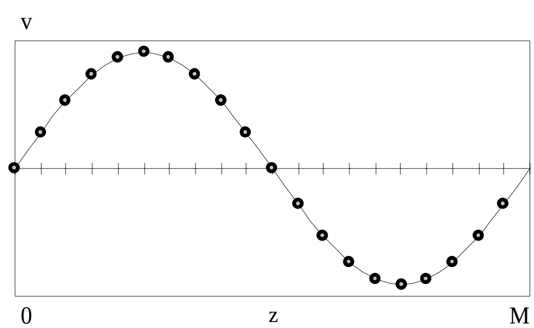

Fig. 2 – Representation of the samples phases (z), normalized from 0 to M, for the case of 40 samples acquired during 2 sinusoid periods.

The acquisition of 39 samples during the same 2 periods (Fig. 3) would however lead to 39 different sample phases since the integers 2 and 39 are mutually primes as can be seen in Fig. 4.

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[1] Phone: +351 218418485. E-mail: falegria@lx.it.pt.

[2] Phone: +351 218418490. E-mail: acserra@alfa.ist.utl.pt.

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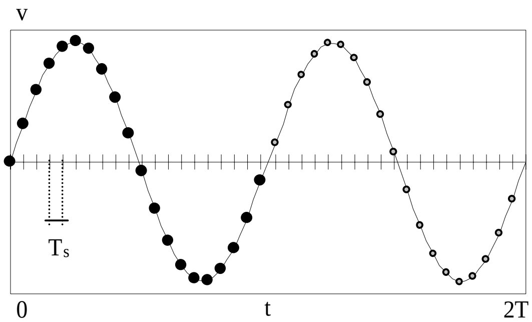

Fig. 3 – Representation of 2 periods of a sinusoid during which 39 samples were acquired.

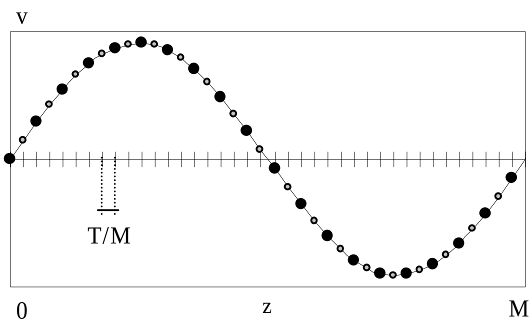

Fig. 4 – Representation of the samples phases (*z*), normalized from 0 to *M*, for the case of 39 samples acquired during 2 sinusoid periods.

II. Frequency Errors

The inevitable presence of frequency errors in the sinewave and in the sampling generators will cause the sample phases not to be uniformly distributed. To guarantee that an error $\Delta\rho$ in the frequencies ratio ρ leads to an error in the phases distribution of less than 50%, of the ideal phase spacing, the number of samples has to verify

$$\frac{\Delta\rho}{\rho} \leq \frac{1}{2 \cdot J \cdot M} . \tag{2}$$

This limit also guarantees that the variance of the number of counts of the cumulative histogram is lower than 1/4 [5, 6, 7].

Inserting (1) in (2) leads to

$$\Delta\rho \leq \frac{1}{2M^2} , \tag{3}$$

where

$$\Delta\rho = \left|\rho_{ideal} - \rho\right| . \tag{4}$$

Again using (1) leads to

$$\Delta\rho = \left| \frac{f_{ideal}}{f_{s\ ideal}} - \frac{f}{f_s} \right| . \tag{5}$$

Considering the real values of the frequency as being affected by maximum relative errors of $\pm\varepsilon_f$ and $\pm\varepsilon_{f_s}$, equation (5) can be written as

$$\begin{aligned} \Delta\rho &= \left| \frac{f_{ideal}}{f_{s\ ideal}} - \frac{f_{ideal} \cdot \left(1 \pm \varepsilon_f\right)}{f_{s\ ideal} \cdot \left(1 \pm \varepsilon_{f_s}\right)} \right| = \\ &= \frac{f_{ideal}}{f_{s\ ideal}} \left| 1 - \frac{1 \pm \varepsilon_f}{1 \pm \varepsilon_{f_s}} \right| = \\ &= \frac{f_{ideal}}{f_{s\ ideal}} \left| \frac{\pm\varepsilon_{f_s} - \left(\pm\varepsilon_f\right)}{1 \pm \varepsilon_{f_s}} \right| . \end{aligned} \tag{6}$$

Expression (6) implies that

$$\frac{f_{ideal}}{f_{s\ ideal}} \left| \frac{\varepsilon_{f_s} - \varepsilon_f}{1 + \varepsilon_{f_s}} \right| \leq \Delta\rho \leq \frac{f_{ideal}}{f_{s\ ideal}} \left| \frac{\varepsilon_{f_s} + \varepsilon_f}{1 - \varepsilon_{f_s}} \right| . \tag{7}$$

If the rightmost term is lower than $1/2M^2$, then $\Delta\rho$ will also be lower than that. So the number of samples must satisfy

$$\frac{f_{ideal}}{f_{s\ ideal}} \left| \frac{\varepsilon_{f_s} + \varepsilon_f}{1 - \varepsilon_{f_s}} \right| \leq \frac{1}{2M^2} . \tag{8}$$

This leads finally to an expression for the maximum number of samples that should be acquired consecutively:

$$M \leq \sqrt{\frac{1}{2} \frac{f_{s\ ideal}}{f_{ideal}} \frac{\left|1 - \varepsilon_{f_s}\right|}{\varepsilon_{f_s} + \varepsilon_f}} = M_{\max} . \tag{9}$$

III. Algorithms

The choice of input frequency and number of samples that satisfy both (1) and (9) is not straight forward. In the following different algorithms are studied.

A. Standard

In the IEEE waveform digitizer standard [1] an algorithm is presented which is based on the observation that an integer is mutually prime with one of its multiples subtracted by 1 [4]. The algorithm is stated the following way:

- Find an integer, *n*, such that the desired frequency (f_d) is approximately f_s / *n*.
- Let *J* = int(*M* / *n*) = the number of full cycles that can be recorder at this frequency.
- Let $f = J \times f_s / (nJ-1)$.
- This guarantees *nJ*–1 distinct sample phases.

This algorithm as some ambiguity in choosing the integer *n* (first step), which represents the number of samples acquired during one period of the sinusoid, because it does not state if one should round the ratio f_s/f_d up or down,

$$n = \left\lceil \frac{f_s}{f_d} \right\rceil \text{ or } n = \left\lfloor \frac{f_s}{f_d} \right\rfloor \text{ respectively} . \tag{10}$$

In Fig. 5 the actual frequency ratio f/f_s is represented as a function of the desired frequency ratio f_d/f_s, for large values of *M* (>100) where $nJ - 1 \approx nJ$ and $\rho \approx 1/n$. The thin and thick lines represent the cases where the ratio f_s/f_d is rounded up or down respectively. For instance, when the desired frequency is half the sampling frequency (ρ_d=1/2) the ratio f_s/f_d would be exactly 2 and no rounding would be required ($n= f_s/f_d =2$) which corresponds to exactly the center of the figure and means that 2 samples are acquired in each sinusoid period. Now, if the desired frequency is higher than half the sampling frequency and lower than the sampling frequency, $1/2<\rho_d<1$ (right side of the figure), the ratio f_s/f_d would be a number from 1 to 2. Rounding it up would lead to 2 samples per period (n=2) which would mean always ρ=1/2, leading to a considerable difference between the desired sinusoid frequency and the actual frequency used. This is an artifact of limiting the number of samples per period to an integer number (first step of the algorithm). Note that what was stated previously was that the samples acquisition has to be carried out during an integer number of sinusoid periods which is not the same thing.

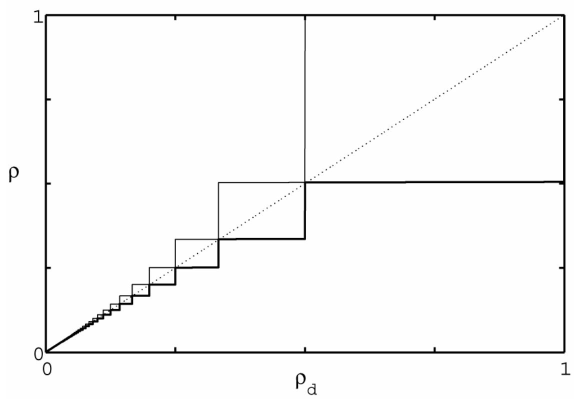

Fig. 5 – Representation of the actual frequency ratio as a function of the desired frequency ratio. The thin line represents the situation where f_s/f_d is rounded up and the thick line when it is rounded down.

The algorithm suggested in [1] also does not state which is the value one should use for M. Attending to what was said in the previous section, namely that the number of samples should be lower than M_{max} given by (9) one would presumably use this value in place of M to determine J. There is however a problem. The algorithm described guarantees that the number of samples to acquire (nJ–1) will be smaller that the value used for M but this could lead to an higher frequency than the desired one which would lower the bound M_{max}, given by (9), causing the number of samples, nJ–1, to be higher that M_{max} , which can be seen in Fig. 6 (thin line) where the relative difference between the actual number of samples to acquire and the maximum limit given by (9) was computed using:

$$\varepsilon_M = \frac{M - M_{max}}{M_{max}} . \tag{11}$$

This situation happens when the ratio f_s/f_d is rounded down to obtain n. The correct approach would be then to always round up f_s/f_d. As seen in Fig. 6 (thick line), this would lead to (9) being always satisfied. This would also imply a frequency lower that the desired one as can be seen in the thick line in Fig. 5.

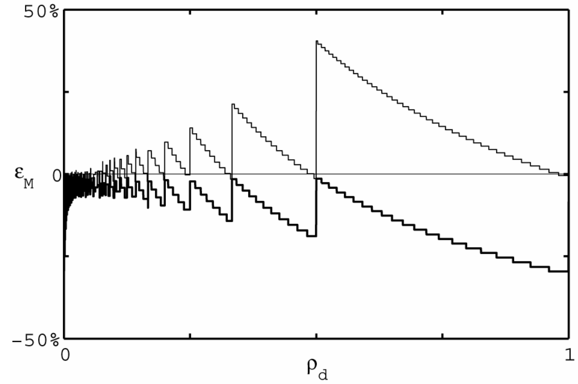

Fig. 6 – Representation of the relative difference of the number of samples defined by (11) as a function of the desired frequency ratio. The thin line represents the situation where f_s/f_d is rounded up and the thick line when it is rounded down. Frequency errors of 25ppm were used.

The actual frequency ratio and number of samples to acquire would be

$$\rho = \frac{J}{M} \text{ and } M = nJ - 1 \text{ with } J = \left\lfloor \frac{M_{max}}{n} \right\rfloor \text{ and } n = \left\lceil \frac{f_s}{f_d} \right\rceil . \tag{12}$$

B. Analytical

Note that the frequency ratio determined using (12) depends on the maximum number of samples (M_{max}) which in turn depends on the frequency ratio (equation (9)). These two expressions may be combined the following way. Substituting f_{sideal}/f_{ideal} by M/J in (9) leads to

$$M \le \sqrt{\frac{1}{2}\frac{M}{J}\frac{\left|1-\varepsilon_{f_s}\right|}{\varepsilon_{f_s}+\varepsilon_f}} . \tag{13}$$

Using $M=nJ–1$ expression (13) can be rewritten as

$$M \le \sqrt{\frac{1}{2}\frac{M}{\frac{M+1}{n}}\frac{\left|1-\varepsilon_{f_s}\right|}{\varepsilon_{f_s}+\varepsilon_f}} . \tag{14}$$

After some simplification,

$$M(M+1) \le n \cdot \frac{1}{2}\frac{\left|1-\varepsilon_{f_s}\right|}{\varepsilon_{f_s}+\varepsilon_f} . \tag{15}$$

Solving the second order equation leads to

$$M \le \frac{-1+\sqrt{1+2n\frac{\left|1-\varepsilon_{f_s}\right|}{\varepsilon_{f_s}+\varepsilon_f}}}{2} . \tag{16}$$

Inserting again $M=nJ–1$:

$$nJ - 1 \le \frac{1}{2}\left(-1+\sqrt{1+2n\frac{\left|1-\varepsilon_{f_s}\right|}{\varepsilon_{f_s}+\varepsilon_f}}\right) . \tag{17}$$

And finally solving for J and rounding down the right member of (17) leads to

$$J=\left\lfloor \frac{1}{2n}\cdot\left(1+\sqrt{1+2n\cdot\frac{\left|1-\varepsilon_{f_s}\right|}{\varepsilon_{f_s}+\varepsilon_f}}\right)\right\rfloor . \quad (18)$$

This is a more efficient way, in terms of the number of samples, of calculating J than the one given by (12) as can be seen in Fig. 7 (thick line).

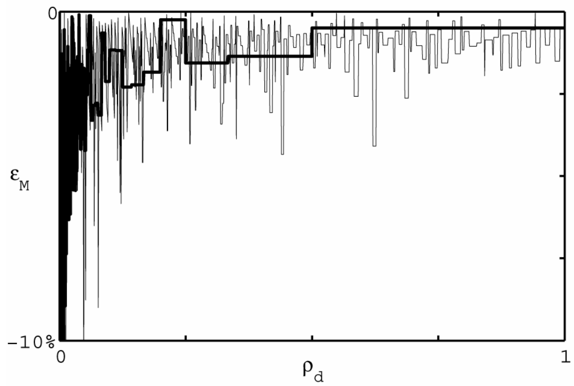

Fig. 7 – Representation of the relative difference of the number of samples defined by (11), using expression (18), as a function of the desired frequency ratio. The thin line represents the optimal solution described in III.C and the thick line the use of equation (18). Frequency errors of 25ppm were used.

The frequency however will still be in some cases far from the desired one (thick line in Fig. 8).

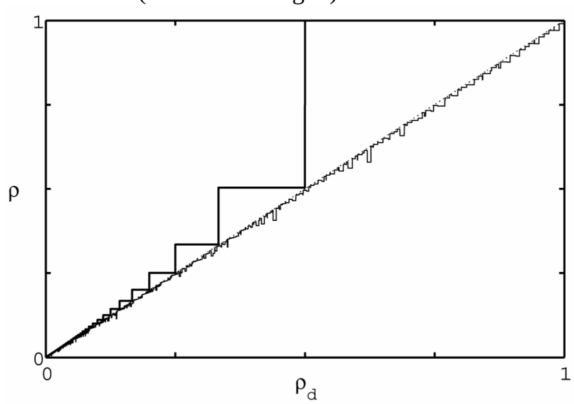

Fig. 8 – Representation of the actual frequency ratio as a function of the desired one. The thick line represents the situation when expression (18) is used and the thin line when the optimal solution is used (finding J that is mutually prime with M given by (9)). Frequency errors of 25ppm were used.

C. Optimal

An optimal solution is to use the number of samples given by (9):

$$M=\left\lfloor M_{\max}\right\rfloor \quad (19)$$

and look for an integer J lower than

$$J_{\max}=\left\lfloor \rho_d\cdot M\right\rfloor \quad (20)$$

so that J and M have no common dividers. This has to be done with the help of a computer which in most cases is not a problem since it is used anyway to process the results or even execute the ADC test. The efficiency of this method is seen in Fig. 7 (thin line) and especially in Fig. 8 (thin line) that shows the actual frequency ratio to be practically equal to the desired one.

IV. CONCLUSIONS

It was seen that the need to have, as much as possible, uniformly distributed phase samples, even in the presence of frequency errors, constrains the values of the number of consecutive samples that can be acquired. The algorithm suggested in [1] was clarified and a better solution was presented that more efficiently determines the number of samples (equation (18)).

Also the efficiency of the optimal solution based on a exhaustive search of two integers that are mutually prime was presented which highlights the advantages of using this alternative in terms of the closeness to the desired input frequency (thin line of Fig. 7) and the maximum number of samples (thin line of Fig. 8) that can be used, even though there is an added complexity and the need for a computer in the process.

V. REFERENCES

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