IEEE 1057 Jitter Test of Waveform Recorders

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Abstract—The jitter test of waveform recorders and analogto-digital converters (ADCs) is traditionally carried out using one of the methods recommended in the IEEE standard for digitizing waveform recorders, i.e., IEEE Standard 1057. Here, we study the uncertainty of one of those methods, point out the bias inherent to the estimator recommended for measuring the ADC jitter, and suggest an alternate estimator. Expressions are also presented for the determination of the precision of a given estimate from the number of samples used, the standard deviation of the additive noise present in the test setup, the jitter standard deviation, and the stimulus signal parameters. In addition, an expression for the computation of the minimum number of samples required to guarantee a given bound on the estimation uncertainty is presented, which is useful in optimizing the duration of the test.

Index Terms—Analog-to-digital converter (ADC), estimation, jitter, phase noise, test.

I. INTRODUCTION

THE JITTER, or aperture uncertainty, in analog-to-digital converters (ADCs), is a random variation in the instant of sampling. This ADC parameter is of special importance in ADCs used in digital communication receivers, where the decision between which symbols were transmitted is intimately related to the instant where the input signal is sampled [1], [2]. In radio receivers, the noise level and, therefore, the effective number of bits are not only dependent on the quantization noise. The jitter that is present in receiver ADC clocks is one of the main causes of loss of performance in wireless communications [3], [4]. The effects of ADC clock jitter on the system signal-tonoise ratio in waveform recorders are discussed in [5], whereas an improved jitter measurement method has been proposed in [6]. Many jitter estimators are proposed in prior work [7]–[10]. This paper, however, focuses on the jitter test methods proposed in IEEE Standard 1057 (both the 1994 version [11] and the 2007 version [12]). This IEEE standard suggests three different methods for jitter estimation. One of those is appropriate for use in systems where the clock signal is externally available [11, Sec. 4.9.2.3], [12, Sec. 12.2.3]. The other two can more

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generally be used [11, Sec. 4.9.2.1 and 4.9.2.2], [12, Sec. 12.2.1 and 12.2.2]. These three methods only permit the estimation of an upper bound on the amount of jitter present since the result obtained also includes other nonidealities, such as ADC pattern errors, amplitude noise, quantization noise, and harmonic distortion. The methods in [11, Sec. 4.9.2.1 and 4.9.2.2] ([12, Sec. 12.2.1 and 12.2.2]) were compared by using a lowbandwidth (100-Hz) seismic data recorder as the measurement system. In active marine seismology, the quality of data is directly related to the acquisition timing. The results of jitter estimation of this system are reported and discussed in [13]. These results have shown that the method suggested in [11, Sec. 4.9.2.2] ([12, Sec. 12.2.2]) is the only one appropriate when the amount of amplitude or quantization noise present is significant since it does not include their contribution when estimating jitter. This is the method we are going to study here. The analysis carried out will focus on the statistical properties of the estimated value of the jitter standard deviation. We will not consider, at present, the effect that harmonic distortion has on the estimator, which can be significant. Work is being carried out on this area and will be the subject of a future publication.

In Section II, we describe the test method. In Section III, we start analyzing the estimator statistics by computing its bias and concluding that the estimator suggested in [11] and [12] for this test method is biased, i.e., the expected value of the estimator is different from the true value [18, p. 455], even if the number of samples acquired is infinite (asymptotically biased). It is shown that the estimator is also inconsistent since the mean square limit [18, p. 446] of the estimator, as the number of samples tends to infinity, does not tend to the true value [18, p. 455].

In Section IV, we propose a new estimator that is asymptotically unbiased and consistent. We then proceed to the precision analysis of both estimators in Section V. In Section VI, we present the experimental results that validate the theoretical study presented. Finally, we derive an expression for the computation of the minimum number of samples required to guarantee a certain bound on the estimation uncertainty (see Section VII). In Section VIII, we sum up the results achieved and highlight future work that needs to be done to fully understand the uncertainty contributions of the jitter measurement method studied. This paper presents the first results obtained to achieve that goal.

II. JITTER TEST

Test 4.9.2.2 in [11] [12, test 12.2.2] is based on the fact that the presence of jitter in the sampling instant translates to an increase in the amplitude noise of the sampled voltage, which depends on the slope of the input signal. The jitter test consists of applying a low-frequency sine wave f_a to the ADC input, i.e.,

$$ya(t) = C + A\cos(2\pi f_a t + \varphi) \tag{1}$$

where C, A, and φ are the sine wave offset, amplitude, and initial phase, respectively. After that, a given number of samples M are acquired, whose voltage, after quantization, will be

$$za_i = Q \cdot round \left\{ \frac{C + A\cos\left[2\pi f_a(t_i + \delta_i) + \varphi\right] + n_i}{Q} \right\}$$
(2)

where Q is the ADC quantization step. Inevitably, those samples will be affected by amplitude noise n and jitter δ . Representing the effect of the quantizer by an additive term q, we can write (2) as

$$za_i = C + A\cos\left[2\pi f_a(t_i + \delta_i) + \varphi\right] + n_i + q_i.$$
 (3)

This assumes that the quantization error is independent of the stimulus signal. This assumption is valid only if the characteristic function of the stimulus signal is "band limited," i.e., if it is null outside an interval of length $2\pi/Q$ around 0 [16]. In the case of a sine wave, the characteristic function has an infinite bandwidth, i.e.,

$$\Phi_y(u) = J_0\left(\frac{2\pi A}{Q}u\right). \tag{4}$$

The higher the A in relation to Q, the higher the constant multiplying u, and the more concentrated the characteristic function around 0. As a consequence, if A is high enough, we can consider the characteristic function as "band limited" and consequently consider that the quantization error is uniform and independent of the signal.

The sine wave that best fits the acquired samples, in a least-square-error sense, is determined [11]. From the fitted sine wave parameters (\widehat{C}_a , \widehat{A}_a , and $\widehat{\varphi}_a$), the ideal value of the sampled voltage can be computed as

$$\widehat{ya}_i = \widehat{C_a} + \widehat{A_a} \cos(2\pi f_a t_i + \widehat{\varphi_a}).$$
(5)

From here, we compute the mean square difference between the ideal input voltage and the voltage of the actual sample, i.e.,

$$mse_a = \frac{1}{M} \sum_{i=1}^{M} (za_i - \widehat{ya}_i)^2.$$
 (6)

Then, another signal with a higher frequency f_b is applied to the ADC yb, the same number of samples zb is acquired, and the mean square error between the acquired samples and the fitted sine wave \hat{yb} is computed, i.e.,

$$mse_b = \frac{1}{M} \sum_{i=1}^{M} (zb_i - \hat{yb}_i)^2.$$
 (7)

Finally, the ADC jitter standard deviation is estimated using

$$\widehat{\sigma}_t = \frac{\sqrt{mse_b - mse_a}}{\sqrt{2\pi}f_b A}.$$
(8)

Note that, in the published version of IEEE Standard 1057 [11], there is a typo in (109). It should read f_2 , instead of f, in the denominator. In this paper, we use indexes "a" and "b," instead of "1" and "2," to represent the two different frequencies.

The two frequencies used should be as distinct as allowed by the system bandwidth to have as distinct values of mse_a and mse_b as possible.

III. BIAS OF THE IEEE JITTER TEST ESTIMATOR

In this section, we are going to compute the bias of estimator (8). To achieve this, we first determine the bias of the computed mean square errors mse_a and mse_b . From (6), we can write

$$E\{mse_a\} = \frac{1}{M} \sum_{i=1}^{M} E\{(za_i - \widehat{ya}_i)^2\}.$$
 (9)

In this paper, we will consider that the error in the estimation of the sine wave parameters is negligible; thus, we will substitute \widehat{ya} by ya, which is given by (1). Introducing (3) and (5) into (9) and making $\widehat{A} = A$, $\widehat{C} = C$, and $\widehat{\varphi} = \varphi$ lead to

$$E\{mse_{a}\} = \frac{1}{M} \sum_{i=1}^{M} E\left\{ \left[A\cos\left[2\pi f_{a}(t_{i}+\delta_{i})+\varphi\right] + n_{i}+q_{i}-A\cos(2\pi f_{a}t_{i}+\varphi)\right]^{2} \right\}.$$
 (10)

Using the trigonometric relation

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b) \tag{11}$$

we can write

$$E\{mse_{a}\} = \frac{1}{M} \sum_{i=1}^{M} E\{[n_{i}+q_{i}-A\sin(2\pi f_{a}\delta_{i})\sin(2\pi f_{a}t_{i}+\varphi) - A[1-\cos(2\pi f_{a}\delta_{i})]\cos(2\pi f_{a}t_{i}+\varphi)]^{2}\}.$$
(12)

This expression can be simplified in situations where the amount of jitter is small, compared with the sampling period. In those cases, we can use the fact that

$$\cos(a) \approx 1 \text{ and } \sin(a) \approx a, \quad \text{for } |a| \ll 1.$$
 (13)

This assumption is not valid in all situations as is the case, for instance, of high-frequency sampling oscilloscopes [14]. Here, however, we will consider only the situation where (13) is valid. From (12), we have

$$E\{mse_a\} = \frac{1}{M} \sum_{i=1}^{M} E\left\{ [n_i + q_i - 2\pi f_a \delta_i A \sin(2\pi f_a t_i + \varphi)]^2 \right\}$$
(14)

which can be written as

$$E\{mse_{a}\} = \frac{1}{M} \sum_{i=1}^{M} E\{n_{i}^{2} + q_{i}^{2} + (2\pi f_{a}\delta_{i}A)^{2}\sin^{2}(2\pi f_{a}t_{i} + \varphi) - 2(n_{i} + q_{i})(2\pi f_{a}\delta_{i}A)\sin(2\pi f_{a}t_{i} + \varphi)\}$$
(15)

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if we take into account that n, q, and δ are independent and have zero mean.

In this study, both additive noise and jitter are considered normally distributed random variables with standard deviations σ_n^2 and σ_t^2 , respectively. The quantization error can be considered a uniform random variable in an interval of length Q if the conditions in [16] are satisfied. In this case, its standard deviation is given by $Q/\sqrt{12}$. We can thus write (15) as

$$E\{mse_a\} = \sigma_n^2 + \frac{Q^2}{12} + (2\pi f_a \sigma_t A)^2 \frac{1}{M} \sum_{i=1}^M \sin^2(2\pi f_a t_i + \varphi).$$
(16)

Again, we will make another simplifying assumption. In this case, we will consider that the acquisition of the input signal is carried out during an integer number of periods J, i.e., the signal frequency, sampling frequency f_s , and the number of samples satisfy

$$\frac{f_a}{f_s} = \frac{J}{M}, \qquad J \in \mathbb{N} \text{ and } J \text{ is not a multiple of } \frac{M}{2}.$$
 (17)

In this case, the summation in (16) is

$$\sum_{i=1}^{M} \sin^2(2\pi f_a t_i + \varphi) = \frac{M}{2}.$$
 (18)

Note that the sampling instants are given by $t_i = i/f_s$. The assumption is reasonable, because we can choose whatever values we want for those frequencies and the number of samples. In practice, however, due to instrument inaccuracies, the actual value of those frequencies may not exactly be the values that were chosen and that satisfy (17), but they are close enough, considering typical frequency errors of smaller than 100 ppm. If a noninteger number of periods are acquired, a bias will affect the estimator. In this paper, however, we will not consider this scenario.

Using (18), we can write (16) as

$$E\{mse_a\} = \sigma_n^2 + \frac{Q^2}{12} + 2(\pi f_a \sigma_t A)^2.$$
(19)

The same reasoning can be applied to the samples acquired with the high-frequency sine wave, i.e.,

$$E\{mse_b\} = \sigma_n^2 + \frac{Q^2}{12} + 2(\pi f_b \sigma_t A)^2.$$
 (20)

The expected value of the square of estimator (8) is

$$E\left\{\widehat{\sigma}_{t}^{2}\right\} = \frac{E\{mse_{b}\} - E\{mse_{a}\}}{(\sqrt{2}\pi f_{b}A)^{2}}.$$
(21)

Inserting (19) and (20) into (21) leads to

$$E\left\{\widehat{\sigma}_{t}^{2}\right\} = \sigma_{t}^{2}\left(1 - \frac{f_{a}^{2}}{f_{b}^{2}}\right).$$
(22)

We are now ready to compute the expected value of estimator (8). For the first approximation, the expected value of the square

root of a variable is equal to the square root of its expected value [15, p. 113], i.e.,

$$E\{g(x)\} \approx g(E\{x\}). \tag{23}$$

We thus have

$$E\{\widehat{\sigma}_t\} \approx \sqrt{E\{\widehat{\sigma}_t^2\}}.$$
(24)

Inserting (22) leads to

$$E\{\widehat{\sigma}_t\} \approx \sigma_t \sqrt{1 - \frac{f_a^2}{f_b^2}}.$$
(25)

By observing (25), we conclude that estimator (8) is biased since $E\{\hat{\sigma}_t\} \neq \sigma_t$ [18, p. 455], even if the number of samples tends to ∞ (asymptotically biased [18, p. 455]). To minimize the estimation error, one should have a low value of f_a and a value of f_b that is as high as possible.

This estimator is also inconsistent since the mean square limit [18, p. 446] is different from the true value, i.e.,

$$\lim_{M \to \infty} \widehat{\sigma}_t \neq \sigma_t. \tag{26}$$

This can be shown by computing the mean square limit using [18, eqs. (14)-(51)], i.e.,

$$\lim_{M \to \infty} E\left\{ |\widehat{\sigma}_t - \sigma_t|^2 \right\} = \lim_{M \to \infty} E\left\{ \widehat{\sigma}_t^2 + \sigma_t^2 - 2\sigma_t \widehat{\sigma}_t \right\}$$
$$= \sigma_t^2 + \lim_{M \to \infty} E\left\{ \widehat{\sigma}_t^2 \right\} - 2\sigma_t \lim_{M \to \infty} E\left\{ \widehat{\sigma}_t \right\}.$$
(27)

Inserting (22) and (25) into (27) leads to

$$\lim_{M \to \infty} E\left\{ |\hat{\sigma}_t - \sigma_t|^2 \right\} = \sigma_t^2 \left[2 - \frac{f_a^2}{f_b^2} - 2\sqrt{1 - \frac{f_a^2}{f_b^2}} \right] \quad (28)$$

which is different from 0. Hence, the estimator is not consistent.

IV. NEW ESTIMATOR PROPOSED

As concluded in the previous section, estimator (8), which is recommended in method 4.9.2.2 of IEEE Standard 1057 [11] [12, method 12.2.2] to estimate jitter in waveform digitizers and ADCs in general, is biased and inconsistent. The expected value of this estimator is given by (25). Using this information, we suggest a new estimator, i.e.,

$$\widehat{\sigma}_t = \frac{\sqrt{mse_b - mse_a}}{\sqrt{2}\pi A \sqrt{f_b^2 - f_a^2}}.$$
(29)

If frequencies f_a and f_b can properly be chosen, i.e., if we can have $f_b \gg f_a$, then, in practice, the difference in using (29), instead of (8), is negligible. We maintain however that there is no reason to use a biased expression when an unbiased one is available, which is equally easy to use.

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 TABLE I

 Settings Used for the Monte Carlo Simulation

Setting	Value
Sine Wave Amplitude (A)	10 V
Sine Wave Offset (C)	0
Low Sine Wave Frequency (f _a)	100 kHz
High Sine Wave Frequency (f _b)	1 MHz
ADC Quantization Step (Q)	1 μV
Number of Acquired Samples (M)	1000 and 10000
Sampling Frequency (f _s)	100 MHz
Injected Additive Noise (σ_n)	50 mV
Injected Jitter (σ _t)	0 to 10 ns
Number of Repetitions (R)	1000
Confidence Level (v)	99.9 %

The expected value of the square of the estimator may be computed from (29), i.e.,

$$E\left\{\hat{\sigma}_{t}^{2}\right\} = \frac{E\{mse_{b}\} - E\{mse_{a}\}}{(\sqrt{2}\pi A)^{2} (f_{b}^{2} - f_{a}^{2})}.$$
(30)

Inserting (19) and (20) leads to

$$E\left\{\widehat{\sigma}_t^2\right\} = \sigma_t^2. \tag{31}$$

Using approximation (23) again, we can write

$$E\{\widehat{\sigma}_t\} = E\left\{\sqrt{(\widehat{\sigma}_t^2)}\right\} \approx \sqrt{E\{\widehat{\sigma}_t^2\}} = \sigma_t.$$
(32)

This proves that estimator (29) is approximately unbiased. Again, note that this is so, because we are considering neither the eventual presence of harmonic distortion in the stimulus signal or caused by the waveform recorder nor that the samples acquisition may have been carried out over a noninteger number of periods due to mismatch in the stimulus signal and sampling clock frequencies.

To validate the results obtained so far about the bias of the estimator, we used a Monte Carlo procedure. The test was repeated 1000 times on a simulated ADC having jitter and amplitude noise. The conditions of the test are presented in Table I.

The expected value of the jitter estimates obtained was computed, and its difference to the actual jitter standard deviation is shown in Fig. 1. The vertical bars represent the confidence intervals for a confidence level of 99.9% [15, p. 248].

It can clearly be seen that, for values of the jitter standard deviation greater than 0.5 ns, the result confirms that estimator (29) is unbiased.

The divergence observed for smaller values of the jitter standard deviation is due to the fact that, with such a low amount of jitter, the value obtained for mse_b is sometimes smaller than that obtained for mse_a . In the Monte Carlo simulations carried out, we estimated a value of 0 for the jitter standard deviation when that happened. Estimator (29) should actually be written as

$$\widehat{\sigma}_{t} = \begin{cases} \frac{\sqrt{mse_{b} - mse_{a}}}{\sqrt{2}\pi A \sqrt{f_{b}^{2} - f_{a}^{2}}}, & mse_{b} \ge mse_{a} \\ 0, & \text{otherwise.} \end{cases}$$
(33)



Fig. 1. Representation of the error of the expected value of the jitter estimation as a function of the actual standard deviation of jitter. The vertical bars represent the confidence interval for a confidence level of 99.9%.

The approximation used to compute the expected value of jitter standard deviation estimator (29) from the expected value of its square [see (23)] is not valid if the function is not continuous around the expected value of x. That is what happens when mse_a and mse_b are similar. Thus, the result obtained in (32) is not valid; hence, the conclusion that the estimator is unbiased is not true, as shown in Fig. 1, for low values of the jitter standard deviation.

Estimator (33) is, however, asymptotically unbiased, even for small values of the jitter standard deviation. To prove it, i.e., to prove that

$$\lim_{M \to \infty} E\{\widehat{\sigma}_t\} = \sigma_t \tag{34}$$

we can use [18, eqs. (14)–(65)] to exchange the limit and the expected value, i.e.,

$$\lim_{M \to \infty} E\{\widehat{\sigma}_t\} = E\left\{\lim_{M \to \infty} \widehat{\sigma}_t\right\}$$
(35)

where "l.i.m." is the mean square limit [18, p. 446]. Furthermore, the convergence in mean square limit implies convergence in probability (plim) [18, pp. 447], which can be interchanged with a continuous function [18, p. 450, eqs. (14)–(61)]. We thus have, inserting (29) into (35)

$$\lim_{M \to \infty} E\{\widehat{\sigma}_t\} = E\left\{\lim_{M \to \infty} \widehat{\sigma}_t\right\}$$
$$= E\left\{\frac{\sqrt{\lim_{M \to \infty} mse_b - \min_{M \to \infty} mse_a}}{\sqrt{2\pi}A\sqrt{f_b^2 - f_a^2}}\right\}.$$
(36)

The weak law of large numbers states that sums such as those in (6) and (7) converge in probability to their expected values [18, eqs. (14)–(68),], i.e.,

$$\lim_{M \to \infty} \left(\frac{mse - E\{mse\}}{M} \right) = 0.$$
 (37)

This allows us to write (36) as

$$\lim_{M \to \infty} E\{\widehat{\sigma}_t\} = E\left\{\frac{\sqrt{E\{mse_b\} - E\{mse_a\}}}{\sqrt{2\pi}A\sqrt{f_b^2 - f_a^2}}\right\}.$$
 (38)

Inserting (19) and (20) into (38) leads to

$$\lim_{M \to \infty} E\{\widehat{\sigma}_t\} = \sigma_t \tag{39}$$

which proves that the estimator is asymptotically unbiased [18, p. 455].

Note that, although sum mse_b may be lower than mse_a for small values of the jitter standard deviation, when the number of samples tends to infinity, sum mse_b is always higher than mse_a ; this is because they tend to their expected values, and the expected value of mse_b , which is given by (20), is higher than the expected value of mse_a , which is given by (19), considering that f_b is higher than f_a .

Estimator (29) is consistent (even for small amounts of jitter) since the mean square limit [18, p. 446] of $\hat{\sigma}_t$ is equal to σ_t [18, eqs. (14)–(78)]. This can be shown by computing the mean square limit using [18, eqs. (14)–(51)]:

$$\lim_{M \to \infty} E\left\{ |\widehat{\sigma}_t - E\{\widehat{\sigma}_t\}|^2 \right\}$$

$$= \lim_{M \to \infty} E\left\{ |\widehat{\sigma}_t - \sigma_t|^2 \right\}$$

$$= \lim_{M \to \infty} E\left\{ \widehat{\sigma}_t^2 + \sigma_t^2 - 2\sigma_t \widehat{\sigma}_t \right\}$$

$$= \sigma_t^2 + \lim_{M \to \infty} E\left\{ \widehat{\sigma}_t^2 \right\} - 2\sigma_t \lim_{M \to \infty} E\{\widehat{\sigma}_t\}$$

$$= 0.$$
(40)

Inserting (31) and (39) leads to

$$\lim_{M \to \infty} \widehat{\sigma}_t = 0 \tag{41}$$

which proves that the estimator is consistent.

To illustrate this, we repeated the Monte Carlo simulation previously done but now with ten times more samples. The difference between the expected value and the true value in this case, where $M = 10\,000$ (see Fig. 2), is smaller than that in the previous case for M = 1000. When the number of samples tends to infinity, the expected value of the estimator tends to the true value of the jitter standard deviation, even for small amounts of jitter.

V. PRECISION OF THE JITTER TESTS

In this section, we will focus on the uncertainty of the estimators (8) and (29). The precision of the estimates is related to the standard deviation of the random variable $\hat{\sigma}_t$. To compute it, we will start by computing the variance of mse_a and mse_b . From (6) and considering the different samples uncorrelated, we have

$$\operatorname{VAR}\{mse_a\} = \frac{1}{M^2} \sum_{i=1}^{M} \operatorname{VAR}\left\{(za_i - \widehat{ya}_i)^2\right\}.$$
(42)



Fig. 2. Representation of the error of the expected value of the jitter estimation as a function of the actual standard deviation of jitter. The vertical bars represent the confidence interval.

Using the same reasoning as in Section III, we can write

$$VAR\{mse_{a}\} = \frac{1}{M^{2}} \sum_{i=1}^{M} VAR\{n_{i}^{2} + q_{i}^{2} + (2\pi f_{a}\delta_{i}A)^{2} \sin^{2}(2\pi f_{a}t_{i} + \varphi) - 2(n_{i} + q_{i})(2\pi f_{a}\delta_{i}A) \sin(2\pi f_{a}t_{i} + \varphi)\}.$$
(43)

Taking into account that n and δ are normal random variables with standard deviations of σ_n and σ_t , respectively, and q is uniformly distributed between -Q/2 and Q/2, we have

$$\begin{aligned} &\operatorname{VAR}\left\{n_{i}^{2}\right\} = 2\sigma_{n}^{4} \\ &\operatorname{VAR}\left\{q_{i}^{2}\right\} = \frac{Q^{4}}{180} \\ &\operatorname{VAR}\left\{\delta_{i}^{2}\right\} = 2\sigma_{t}^{4}. \end{aligned} \tag{44}$$

Using (44), we can write (43) as

$$VAR\{mse_{a}\} = \frac{2\sigma_{n}^{4}}{M} + \frac{Q^{4}}{180 \cdot M} + 2\sigma_{t}^{4}(2\pi f_{a}A)^{4}\frac{1}{M^{2}}\sum_{i=1}^{M}\sin^{4}(2\pi f_{a}t_{i} + \varphi) + 4\left(\sigma_{n}^{2} + \frac{Q^{2}}{12}\right)\sigma_{t}^{2}(2\pi f_{a}A)^{2}\frac{1}{M^{2}} \times \sum_{i=1}^{M}\sin^{2}(2\pi f_{a}t_{i} + \varphi).$$
(45)

Again, we will consider that the acquisition of the input signal is carried out during an integer number of periods, as done in (16). The first summation in (45) is, under such conditions [see (17)], given by

$$\frac{1}{M}\sum_{i=1}^{M}\sin^4(2\pi f_a t_i + \varphi) = \frac{3}{8}.$$
 (46)

Introducing (18) and (46) into (45) leads to

$$VAR\{mse_a\} = \frac{2\sigma_n^4}{M} + \frac{Q^4}{180M} + 2\sigma_t^4 (2\pi f_a A)^4 \frac{3}{8M} + 4\left(\sigma_n^2 + \frac{Q^2}{12}\right)\sigma_t^2 (2\pi f_a A)^2 \frac{1}{2M}.$$
 (47)

The same reasoning can be applied to compute the variance of mse_b , which will lead to an expression similar to (47), with f_b in place of f_a , i.e.,

$$\operatorname{VAR}\{mse_b\} = \frac{2\sigma_n^4}{M} + \frac{Q^4}{180M} + 2\sigma_t^4 (2\pi f_b A)^4 \frac{3}{8M} + 4\left(\sigma_n^2 + \frac{Q^2}{12}\right)\sigma_t^2 (2\pi f_b A)^2 \frac{1}{2M}.$$
 (48)

The following step is to compute the variance of $\hat{\sigma}_t^2$ from the variances of mse_a and mse_b . Using (29), we can write

$$\widehat{\sigma}_t^2 = \frac{mse_b - mse_a}{(\sqrt{2}\pi A)^2 \, (f_b^2 - f_a^2)}.$$
(49)

Since random variables mse_a and mse_b are independent, we can write

$$\operatorname{VAR}\left\{\widehat{\sigma}_{t}^{2}\right\} = \frac{\operatorname{VAR}\left\{mse_{b}\right\} + \operatorname{VAR}\left\{mse_{a}\right\}}{(\sqrt{2}\pi A)^{4}\left(f_{b}^{2} - f_{a}^{2}\right)^{2}}.$$
 (50)

Inserting (47) and (48) leads to

$$\operatorname{VAR}\left\{\widehat{\sigma}_{t}^{2}\right\} = \frac{4\sigma_{n}^{4} + \frac{Q^{4}}{90} + \frac{3}{4}\sigma_{t}^{4}(2\pi A)^{4}\left(f_{a}^{4} + f_{b}^{4}\right)}{\left(\sqrt{2}\pi A\right)^{4}\left(f_{b}^{2} - f_{a}^{2}\right)^{2}M} + \frac{2\left(\sigma_{n}^{2} + \frac{Q^{2}}{12}\right)\sigma_{t}^{2}(2\pi A)^{2}\left(f_{a}^{2} + f_{b}^{2}\right)}{\left(\sqrt{2}\pi A\right)^{4}\left(f_{b}^{2} - f_{a}^{2}\right)^{2}M}.$$
 (51)

Finally, we are going to compute the variance of $\hat{\sigma}_t$ from the variance of $\hat{\sigma}_t^2$ using [15, pp. 113]

$$\sigma_y^2 \simeq \left|g'(\mu_x)\right|^2 \sigma_x^2, \qquad y = g(x). \tag{52}$$

Note that μ_x and σ_x are the mean and variance of x, respectively. In our case, $y = \sqrt{x}$; thus

$$g'(\mu_x) = \frac{1}{2\sqrt{\mu_x}}.$$
(53)

Combining (51) with (52) and (53), where $x = \hat{\sigma}_t^2$ and $y = \hat{\sigma}_t$, and using (32) lead to

$$\operatorname{VAR}\{\widehat{\sigma}_{t}\} \simeq \frac{\sigma_{n}^{4} + \frac{Q^{4}}{90} + \frac{3}{16}\sigma_{t}^{4}(2\pi A)^{4}\left(f_{a}^{4} + f_{b}^{4}\right)}{\left(\sqrt{2}\pi A\right)^{4}\left(f_{b}^{2} - f_{a}^{2}\right)^{2}M\sigma_{t}^{2}} + \frac{\frac{1}{2}\left(\sigma_{n}^{2} + \frac{Q^{2}}{12}\right)\sigma_{t}^{2}(2\pi A)^{2}\left(f_{a}^{2} + f_{b}^{2}\right)}{\left(\sqrt{2}\pi A\right)^{4}\left(f_{b}^{2} - f_{a}^{2}\right)^{2}M\sigma_{t}^{2}} \quad (54)$$



Fig. 3. Representation of the standard deviation of the jitter estimation as a function of the actual standard deviation of the jitter. The solid line represents the values given by (55), and the vertical bars represent the result of Monte Carlo simulations of the jitter test method.

which can be written as

$$\begin{aligned} \operatorname{VAR}\{\widehat{\sigma}_{t}\} &\simeq \frac{1}{M(\sqrt{2}\pi A)^{4} \left(f_{b}^{2} - f_{a}^{2}\right)^{2}} \frac{\sigma_{n}^{4} + \frac{Q^{2}}{90}}{\sigma_{t}^{2}} \\ &+ \frac{12}{16M} \frac{\left(f_{a}^{4} + f_{b}^{4}\right)}{\left(f_{b}^{2} - f_{a}^{2}\right)^{2}} \sigma_{t}^{2} \\ &+ \frac{1}{2M\pi^{2}A^{2}} \frac{\left(f_{a}^{2} + f_{b}^{2}\right)}{\left(f_{b}^{2} - f_{a}^{2}\right)^{2}} \left(\sigma_{n}^{2} + \frac{Q^{2}}{12}\right). \end{aligned} \tag{55}$$

Equation (54) allows the computation of the variance of the estimated jitter standard deviation obtained with the proposed estimator (29). In the approximation $f_b \gg f_a$, the proposed estimator is equal to the IEEE Standard 1057 estimator (8), and its variance is, from (54), given by

$$\operatorname{VAR}\{\widehat{\sigma}_{t}\}_{f_{b} \gg f_{a}} \frac{\sigma_{n}^{4} + \frac{Q^{4}}{90}}{M(\sqrt{2}\pi A f_{b})^{4}} \frac{1}{\sigma_{t}^{2}} + \frac{1}{M(\sqrt{2}\pi A f_{b})^{2}} \left(\sigma_{n}^{2} + \frac{Q^{2}}{12}\right) + \frac{3}{4M}\sigma_{t}^{2}.$$
 (56)

Fig. 3 shows the result of the Monte Carlo simulations for the determination of the estimator standard deviation. Again, 1000 repetitions were made. The vertical bars represent the confidence intervals for a 99.9% confidence level [15, p. 253]. It clearly supports the claim that the standard deviation of the jitter estimation obtained with (29) can be computed using (55).

The conditions of the test are the same as those used for the validation of the estimator expected value (see Table I).

For very low values of the jitter standard deviation, the approximations made using (52) cease to be valid, because the jitter is not enough to always make the mean square error measured at high frequency mse_b higher than the its value when measured at low frequency, i.e., mse_a . We can empirically consider a threshold on the value of jitter for this situation: the minimum of the estimator standard deviation. This value can be

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Fig. 4. Test bench. The personal computer controls all the instruments through General Purpose Interface Bus or Universal Serial Bus interfaces. The combination of the sinusoidal stimulus signal and the normally distributed noise is carried out inside the data-acquisition board through the use of one of its differential inputs.

obtained by calculating the derivative of (55) with respect to σ_t and equating it to 0. The result obtained is

$$\sigma_{t\min} = \sqrt[4]{\frac{\frac{Q^4}{90} + \sigma_n^4}{3\pi^4 A^4 \left(f_b^4 - f_a^4\right)}}.$$
(57)

If we encounter an application where mse_b is smaller that mse_a , we should increase the frequency difference $f_b^4 - f_a^4$, which will increase the value of mse_b in relation to mse_a . This corresponds to pushing the minimum of the estimator standard deviation (see Fig. 3) to the left [decrease in the value given by (57)].

VI. EXPERIMENTAL VALIDATION

To validate the results obtained so far, i.e., that estimator (29) is unbiased and that the standard uncertainty of that estimator can be computed using (54), we measured jitter in an actual ADC using the method under study.

Since we are interested in studying the statistical properties of the jitter estimator, we need a setup where we can control the amount of jitter present. ADC jitter is a deterministic or random delay between the ideal sampling instants and the actual sampling instant. (Here, we are just considering normally distributed random jitter.) To be able to carry on our study, we have to be able to accurately control the jitter that is present. It is not practical to have different ADCs with different values of jitter to test. We can, however, mimic the effect of jitter in the ADC on jitter in the transition time of the clock signal, which controls the sampling. The two jitters are equivalent, and in fact, when we are measuring the ADC jitter with the test recommended in the IEEE standard, we are actually measuring both jitters (as well as the stimulus signal phase noise). Here, we are going to inject the desired amount of jitter in our test setup by controlling the phase noise of the clock signal produced by a Tektronix arbitrary function generator. The clock signal used was a square-wave phase modulated by Gaussian noise generated by an Agilent function generator. In this way, we can inject different amounts of jitter by controlling the power of the Gaussian noise produced.

The ADC under test is that embedded in a National Instruments data acquisition board (model PCI-6023). This board was plugged into a Peripheral Component Interconnect slot of a personal computer, which is used to program the data acquisition board, store the acquired samples, control the four instruments used, and obtain the jitter estimate using (29). The test setup is shown in Fig. 4. The data acquisition board used was chosen, because it has an external input that can be used to connect a clock signal for the timing of the analog-to-digital conversions. Note that this is required, because we want to inject jitter in our test setup for the purpose of studying the measurement method. Measurement of jitter is not necessary. This is the reason this method, although not the best in separating the different sources of jitter present in a test setup, is appropriate when measuring jitter on waveform recorders and oscilloscopes that generally do not have the capability of using an external clock.

A very low distortion sine wave generator from Stanford Research Systems was used to generate the signal used to stimulate the ADC. We also added a given amount of additive noise to the ADC input to mimic the presence of additive noise in the ADC. Using another Agilent function generator, we added Gaussian noise to the sine wave by making use of the differential input of the data acquisition board.

We implemented the IEEE jitter test in National Instruments LabView. The application developed completely automates the

TABLE II MANUFACTURER SPECIFICATIONS FOR THE INSTRUMENTS AND ADC USED

Specification	Value
Stimulus Signal Generator (Stanford Research DS 360)	
Sine Wave Amplitude Accuracy	1%
Sine Wave Offset Accuracy	1% + 25 mV
Sine Wave Frequency Accuracy	25 ppm+4 mHz
Total Harmonic Distortion	-93 dB
Clock Generator (Tektronix AFG 3022)	
Frequency Accuracy	1 ppm
Jitter	500 ps
External Phase Modulation Bandwidth	25 kHz
Degrees of Phase Modulation per control voltage unit	Not specified
Additive/Phase Noise Generator (Agilent AG 33220A)	
Noise Bandwidth	≈ 9 MHz
Amplitude Accuracy	1 % / 1 mVpp
Analog/Digital Converter (National Instruments 6023)	
Number of Bits	12
Number of Most Significant Bits Used	8
Integral Non Linearity (INL)	1.5 LSB
Differential Non Linearity (DNL)	1 LSB
Gain Error	0.02 %
Offset Error	0.5 mV
Bandwidth (small signal)	500 kHz
Additive Noise	0.1 LSB
Jitter	Not specified

test study: from test parameter calculation to instrument control, data gathering and processing, graphical representation of results, and Monte Carlo analysis.

Before carrying out the tests, there were two constants that had to be determined: 1) the ratio between the generated additive noise and the voltage noise present in the ADC K_v and 2) the ratio between the additive noise generated and the amount of jitter present in the clock signal K_t .

In the first case, we adjusted the function generator to a given value of noise root-mean-square (RMS) voltage and measured the standard deviation of noise in the ADC using the method described in IEEE Standard 1057 for the estimation of random noise using sine fitting. Five measurements were made, and linear regression was used to arrive at $K_v = 0.2344$ (correlation of 0.9994). This test was carried out with the internal clock of the data acquisition board set with a frequency of 100 kHz and a sine wave stimulus signal frequency of 25 kHz. The test was repeated for lower values of sine wave frequency, but the value obtained for K_v was the same, showing that the combined values of jitter present in the ADC and phase noise of the internal clock were negligible. The obtained value is close to the expected value that can be computed if we take into account the specification for the bandwidth of the National Instruments PCI-6023 data-acquisition board, which is 500 kHz, and the Agilent AG33220A function generator specification for the bandwidth of noise, which is 9 MHz. These and other relevant specifications are presented in Table II. By taking the square root of the ratio between these two numbers, we get

which is very close to the measured value of 0.2344. The difference can be explained by measuring uncertainty and considering that the noise bandwidth of 9 MHz specified by Agilent for the AG3322A is just an approximated value and that the 500-kHz bandwidth for the data acquisition board is the small signal bandwidth and not the equivalent noise bandwidth.

To compute the second constant K_t , we produced different values of additive noise RMS voltage (five points from 100 mV to 1 V), with the generator connected to the external modulation input of the clock generator, and used a digital phosphor oscilloscope from Tektronix to measure the amount of jitter present in the clock (30 000 transition measurements carried out for each of the five points). The linear regression gave a value of $K_t = 266.82$ ns/V (with a correlation of 1.0000). We cannot determine an expected value for this constant from the instrument specifications, because the modulation constant (in degrees per volt) of the Tektronix is not supplied in the specification sheets.

We also used this oscilloscope to draw the histogram of the measured values and, by visual inspection, concluded that it had a good Gaussian distribution. We tried to use the Tektronix AFG3022 arbitrary function generator, instead of the Agilent AG33220A, to produce the additive noise, but it showed a poor statistical distribution of the noise voltages.

The statistical properties of the estimator were measured by repeating the jitter measurement under the same conditions a given number of times R and by computing the average and standard deviations of the different values obtained. The results were compared with the theoretical results given by (32) and (55). We carried out this analysis under several different conditions by varying the following parameters:

- 1) stimulus signal frequency;
- 2) stimulus signal amplitude;
- 3) sampling frequency;
- 4) quantization step;
- 5) additive noise standard deviation;
- 6) jitter standard deviation;
- 7) number of samples acquired.

In all cases where the jitter standard deviation was not too small, the experimental results were in agreement with the theoretical results within the confidence intervals obtained for a confidence level of 99.9%. Here, we present the results for one set of those conditions that we judge were the most illustrative and representative of the actual conditions. The values used can be found in Table III.

The results obtained for the error of the estimation (the difference between the expected and actual values) are shown in Fig. 5 for a range of injected jitters from 0 to 300 ns (the values of the jitter standard deviation). We can see that all the confidence intervals (vertical bars), with the exception of the first interval (for the absence of jitter), are around 0 (the theoretical value), which shows that the estimator is unbiased under those conditions.

The confidence interval of the first point is not around 0, because estimator (29) is biased when the amount of jitter is small. This is because, in some cases, the value of mse_a will be higher than that of mse_b . In those cases, we cannot take the

TABLE III Experimental Setup Settings

Setting	Value
Sine Wave Amplitude (A)	4 V
Sine Wave Offset (C)	0
Low Sine Wave Frequency (f_a)	2.478 kHz
High Sine Wave Frequency (f _b)	24.9878 kHz
ADC Full Scale (FS)	5 V
ADC Quantization Step (Q)	39.0625 mV
Number of Acquired Samples (M)	8192
Sampling Frequency (fs)	100 kHz
Injected Additive Noise (σ_n)	20 mV
Injected Clock Phase Noise (σ_t)	0 to 1.72 º
Number of Repetitions (R)	200
Confidence Level (v)	99.9 %



Fig. 5. Representation of the experimental results of the difference between the expected value of the jitter estimation and the injected jitter, as a function of the actual standard deviation of the jitter. The solid line represents the theoretical values, which are 0 in this case (unbiased estimator). The vertical bars represent the confidence intervals of the result of the Monte Carlo simulations of the jitter test method (99.9% confidence level).



Fig. 6. Representation of the experimental results of the standard deviation of the jitter estimation as a function of the actual standard deviation of the jitter. The solid line represents the values given by (55). The vertical bars represent the confidence intervals of the result of the Monte Carlo simulations of the jitter test method (99.9% confidence level).

square root. The same happens in Fig. 6, where the standard deviation of the estimated jitter is shown. We can see that the confidence intervals are all around the theoretical value given by (55) and are depicted as a solid line.

These results validate the assumptions made under the conditions that were used in this paper (see Table III).

VII. MINIMUM NUMBER OF SAMPLES REQUIRED

One of the important considerations when performing the jitter test is to know how many samples should be acquired. There is a compromise to be made between the test time and the estimation uncertainty. The higher the number of samples acquired M, the lower the test result uncertainty, as shown in (54), but the longer the time it will take for the test to complete, which, in the case of production line testing of ADCs, is critical.

Using the statistics calculated for the value of the jitter estimation, we can determine a confidence interval inside which the true value of the measured jitter standard deviation is with a certain confidence level [17], i.e.,

$$\widehat{\sigma}_t - K_{\nu} \sqrt{\operatorname{VAR}\{\widehat{\sigma}_t\}} \le \sigma_t \le \widehat{\sigma}_t + K_{\nu} \sqrt{\operatorname{VAR}\{\widehat{\sigma}_t\}}$$
(59)

where K_{ν} is the coverage factor corresponding to a certain confidence level ν and that depends on the statistical distribution of the estimator.

For the case in study, the square of the estimator $\hat{\sigma}_t^2$ has a distribution that tends to normal as the number of samples tends to infinite. This is demonstrated by the Central Limit Theorem [15] applied to variables mse_a and mse_a , which are the summation of a large number of random variables [see (6) and (7)].

Estimator (29) will thus have a statistical distribution that is the distribution of a variable that is the square root of a randomly distributed variable. Its probability density function (pdf) can be obtained by [15, pp. 96]

$$f_{\widehat{\sigma}_t}(y) = 2y f_{\widehat{\sigma}_t^2}(y^2) U(y) \tag{60}$$

where U(y) is 1 for positive y and 0 otherwise, and

$$f_{\widehat{\sigma}_t^2}(y) = \frac{1}{\sqrt{2\pi}\sigma_{\widehat{\sigma}_t^2}} e^{-\frac{\left(y-\mu_{\widehat{\sigma}_t^2}\right)^2}{2\sigma_t^2}}$$
(61)

is the Gaussian probability function. The pdf of the jitter estimator is thus

$$f_{\widehat{\sigma}_t}(y) = \frac{2yU(y)}{\sqrt{2\pi}\sigma_{\widehat{\sigma}_t^2}} e^{-\frac{\left(y^2 - \mu_{\widehat{\sigma}_t^2}\right)^2}{2\sigma_t^2}}.$$
 (62)

To compute the coverage factor, we need cumulative distribution function (cdf) F(x), which, by definition, is

$$F(x) = \int_{-\infty}^{x} f(y) dy.$$
 (63)

The cdf of the jitter estimator is thus

$$F_{\widehat{\sigma}_t}(x) = \int_0^x \frac{2y}{\sqrt{2\pi}\sigma_{\widehat{\sigma}_t^2}} e^{-\frac{\left(y^2 - \mu_{\widehat{\sigma}_t^2}\right)}{2\sigma_t^2}} dy.$$
(64)

Given a desired confidence level, we use (64) to find the coverage factor and, hence, the confidence interval. To simplify the calculation of this interval, we can use the fact that $F_{\hat{\sigma}_t}(x)$ is approximately equal to the cdf of a normal distribution with



Fig. 7. Representation of the difference between the cdfs of a normally distributed random variable and the cdf of the square root of a normally distributed variable. In this example, the normal distributed variable has a mean of 1 and a standard deviation of 0.2.

mean $\sqrt{\mu_{\sigma_t^2}}$ and standard deviation $\sigma_{\widehat{\sigma}_t^2}/(2\sqrt{\mu_{\widehat{\sigma}_t^2}})$. In Fig. 7, we can see the difference between the two given by

$$\operatorname{cdf}_{\operatorname{diff}}(x) = \int_{0}^{x} \frac{2y}{\sqrt{2\pi}\sigma_{\widehat{\sigma}_{t}^{2}}} e^{-\frac{\left(y^{2}-\mu_{\widehat{\sigma}_{t}^{2}}\right)^{2}}{2\sigma_{t}^{2}}} dy$$
$$-\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\frac{\sigma_{t}^{2}}{\sigma_{t}^{2}}} e^{-\frac{\left(y-\sqrt{\mu_{\widehat{\sigma}_{t}^{2}}}\right)^{2}}{\sigma_{t}^{2}}} dy. \quad (65)$$

In this example and in general, as long as the mean minus three times the standard deviation is not close to 0, the difference is small, which makes the use of normal-distribution percentiles that are adequate in determining the coverage factor for this estimator. We have, for instance, $K_{\nu} = 2.58$ for a 99% confidence level.

In Fig. 8, we show the cdf of the estimated jitter computed from experimental values. The test conditions were the same as those used in Section VI, and the injected value of jitter was 200 ns. A good agreement with the cdf of a Gaussian distribution with the same mean (199.868 ns) and standard deviation (2.236 ns) is observed.

If we wish to have a given desired confidence interval with half-length B_t

$$\widehat{\sigma}_t - B_t \le \sigma_t \le \widehat{\sigma}_t + B_t \tag{66}$$

we need to have

$$K_{\nu} \cdot \sqrt{\mathrm{VAR}\{\widehat{\sigma}_t\}} \le B_t. \tag{67}$$



Fig. 8. Representation of the cdf of the jitter estimator obtained with the experimental data. The theoretical cdf of a normal random variable with the same mean and variance is shown for comparison.

Introducing (54) into (67), we can derive an expression for the minimum number of samples required to achieve a certain bound on the estimation uncertainty, i.e., (68), shown at the bottom of the page.

This expression is useful for choosing the optimal number of samples to acquire for the application at hand. The use of a value that is too high will entail a longer test duration, whereas the use of a value that is too low will lead to greater uncertainty than desired.

VIII. CONCLUSION

We have analyzed one of the tests recommended by IEEE in [11] and [12] for the estimation of the jitter of waveform digitizers and ADCs. We have concluded that the estimator suggested is biased and inconsistent if the frequencies used in the test do not satisfy $f_b \gg f_a$ [see (25)]. We have proposed a new estimator that is asymptotically unbiased whatever the value of the frequencies used, i.e., (29). We have derived an expression for determining the uncertainty of the jitter estimates made with the referred method in the presence of additive noise, i.e., (54). Finally, we have presented an expression [i.e., (68)] that is useful to optimize the test by allowing the tester to know the minimum number of samples required to achieve a desired confidence interval on the estimates.

Several simplifying assumptions were made here, which require further work in the future, i.e., the study of what happens when the following three scenarios hold:

- 1) Samples are acquired during a noninteger number of periods of the stimulus signal.
- 2) The amount of jitter is high when compared with the sampling period.

$$M \ge \frac{K_{\nu}}{B_t^2} \frac{\left[\sigma_n^4 + \frac{Q^4}{90} + \frac{3}{16}\sigma_t^4 (2\pi A)^4 \left(f_a^4 + f_b^4\right) + \frac{1}{2} \left(\sigma_n^2 + \frac{Q^2}{12}\right)\sigma_t^2 (2\pi A)^2 \left(f_a^2 + f_b^2\right)\right]}{\left(\sqrt{2}\pi A\right)^4 \left(f_b^2 - f_a^2\right)^2 \sigma_t^2} \tag{68}$$

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3) Quantization cannot be treated as an error term independent of the stimulus signal.

As stated in the beginning, this work is just the first step in understanding the uncertainty of the jitter measurement method 4.9.2.2 of IEEE Standard 1057 (method 12.2.2 in the 2007 version) [11], [12]. Further research can be carried out on different uncertainty sources, which are given as follows:

- 1) harmonic distortion;
- 2) sine-fitting parameter uncertainty;
- 3) stimulus signal and sampling clock frequency error.

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